Application of discrete Ricci flow to medical imaging

Case study: Registration of prone and supine surfaces in CT colonography

Colorectal cancer, a leading cause of cancer-related death in Western countries [1], is largely preventable if precursor adenomatous polyps are detected and excised before becoming malignant. A colonoscopy detects them by inserting a flexible video camera in the colon. Recently there has been increased interest in Computed Tomography Colonography (CTC). CTC is a less invasive technique which uses computed tomography imaging and advanced visualisation software to enable a radiologist to simulate conventional colonoscopy.

In CTC, the patient is scanned twice, once prone and then supine. When the patient is repositioned a polyp remains fixed to the colon surface while other material moves thereby enabling, in theory, the radiologist to differentiate between polyps and residual stool. However, because the colon itself undergoes considerable deformation at the same time, matching corresponding points on the colonic surface before and after repositioning is difficult. Now an interdisciplinary team led by David Hawkes (UCL), Steve Halligan (UCH) and Greg Slabaugh (Medicsight PLC) is developing a technique to solve this problem using, among other things, the Ricci flow from differential geometry. This work was presented at an Open for Business day at the Isaac Newton Institute during the 6-month programme *Inverse Problems* in 2011.

The mapping based on Ricci flow transforms a hard problem into one that can be solved efficiently...

Mathematical techniques

The colon is a tortuous tubular structure with a 3D centreline which deforms between prone and supine views making standard registration methods (the identification of corresponding points on the two views) ineffective. The key question is how to map colon surfaces embedded in R3 onto a cylinder (Figure 1). If this can be done, then the registration can be carried out in the cylindrical domain using a simpler transformation model. The role of the Ricci flow is to unfold the colon surface.

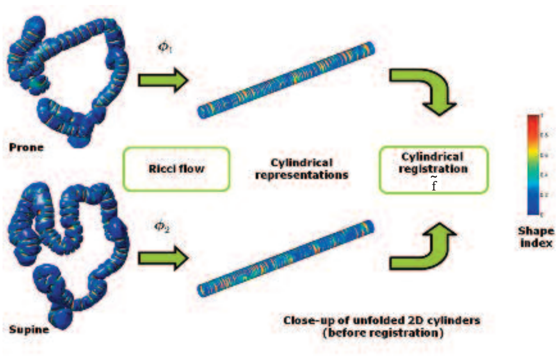

Figure 1. *Use of Ricci flow to map the 3D prone and supine surfaces to cylindrical representations, followed by a deformable registration. The colours represent the shape index, which has a higher response (visualised in yellow and red) at the ridge-like haustral folds.*

In differential geometry Ricci flow is a process that deforms and smooths out the metric of a Riemannian manifold in a way that is mathematically analogous to the diffusion of heat. Roth *et al*. use a discrete form of Ricci flow [2] to map the prone and the supine colonic mesh surfaces conformally to a cylinder. A cylindrical registration using existing rigid and deformable registration methods may then be applied to align the prone and supine cylinders. The shape index is a local descriptor that characterises the surface shape using differential geometry and the registration process seeks the transformation that minimises the sum of squared differencesd of the indices. Anatomically, the haustral folds (ridge-like circumferential structures in the colon which have a higher shape index) serve as a key feature when aligning the two cylindrical representations of the colon. With the two cylindrical representations and their registration known, the complete transformation from the prone to supine surface is obtainable (see Figure 1).

Experimental validation

Using reference images with known polyp locations in prone and supine scans, the registration's accuracy was measured quantitatively. For data of 13 patients each containing one polyp and independent of the data used to develop the registration technique, the polyp registration error was 5.7 (± 3.4) mm. A separate validation on the same data, but using 1175 expert identified haustral fold locations, had a registration error of 7.7 (± 7.4) mm.

While 3D registration error is an important metric for registration performance, it falls short of capturing the clinical utility of the registration. In the intended application, and as illustrated in Figure 2, a polyp that appears in the field of view within one image (e.g., prone) would ideally be flagged, its position determined by the registration, in the field of view in other image (e.g., supine) when displayed using volume rendered 3D 'virtual colonoscopy' software.

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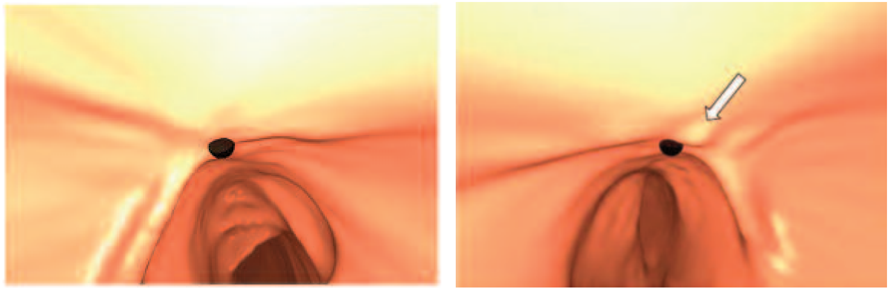

Figure 2. *Using the prone-supine registration, a polyp prompt in the prone image (left) is automatically transferred to the supine image (right). Successful registration means that the prompt intersects the actual polyp (shown with the white arrow).*

In another validation using a different dataset, for over 80% of polyps [3], automated prone-supine registration brought the observer to a location where the polyp was in immediate view when rendered in 3D.

Research highlight and impact

The work of Roth *et al.* exemplifies the application of modern mathematics to clinically important practical problems. The mapping based on Ricci flow transforms a hard problem to one that can be solved efficiently, although work still remains to improve the method's robustness and to deal with patients with varying levels of colonic distension between prone and supine views.

The technique has been published in medical imaging conference proceedings and in [4]. Subsequent work [5] improved the method. Additionally the method has been featured in the radiology press (Aunt Minnie). A presentation in 2011 at the Isaac Newton Institute for Mathematical Sciences provided an excellent opportunity for the research to be presented to an international audience of mathematicians and scientists.

References

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