Isaac Newton Institute for Mathematical Sciences

# Discrete Analysis

## 10 January to 8 July 2011

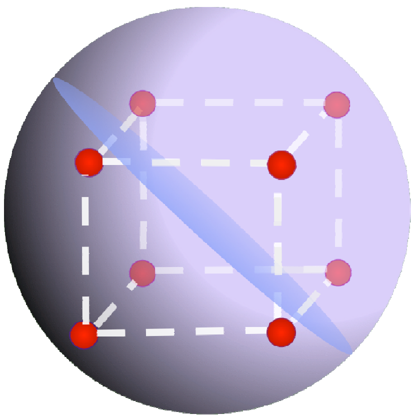
***Organisers:*** *Professor K Ball* (UCL), *Professor F Barthe* (Toulouse), *Professor B Green* (Cambridge) *and Professor A Naor* (New York)

One of the most important principles in geometry and mathematical analysis is the isoperimetric principle: among objects with a given volume, solid spheres have the least surface area. This principle was recognised in classical times and explains, amongst other things, the spherical shape of raindrops, which form the shape that minimises surface tension. Among other things, the principle certainly implies that large regions have large surface area.

It is perhaps surprising that this principle (or extensions of it) has numerous applications beyond the obvious physical ones. To see why, consider the following example. Suppose you run a random walk inside a large, possibly irregularly-shaped, box: at each time step you move a small amount in a random direction inside the box. If large regions in the box have large surface area, then you will always have a good chance of leaving a region you currently lie in. Thus, the random walk is unlikely to get stuck for long periods in one part of the box. Contrast this with a box shaped like a dumbbell, comprising two large rooms connected by a narrow corridor. If you start in one room, it will take an enormously long time for the random walk to find the corridor, negotiate it and enter the other room.

Suppose you are studying a domain of the first type. Then a random walk will spread rapidly through the domain and thus enable you to sample efficiently from different parts of the domain just by following the random walk for a few steps. For many theoretical problems in computing it is important to be able to sample effectively from a large number of objects which we can think of as points. In many cases, the objects do not form a continuous collection like the points inside a box, but a discrete collection such as the corners of the box (shown in red on the figure). In this setting we can still talk about "surface area": the surface area of a collection of corners is the number of edges of the box which cross to corners outside the collection. Another way of saying this is as follows: we cut the collection in two (for example by the plane shown in blue) and we define the surface area to be the size of the cut, or the number of "stitches" that cross it.

During the last decade or so, there has been a dramatic growth in our understanding of discrete versions of the isoperimetric principle and of related inequalities such as the logarithmic Sobolev inequality, which measures the rate at which a random walk increases the entropy of a random system. The aim of this programme is to bring together mathematicians and theoretical computer scientists who have been involved in developing and using these tools and others in the theory of algorithms and in additive number theory.



*A cut in the discrete cube*