Isaac Newton Institute for Mathematical Sciences

Moduli Spaces

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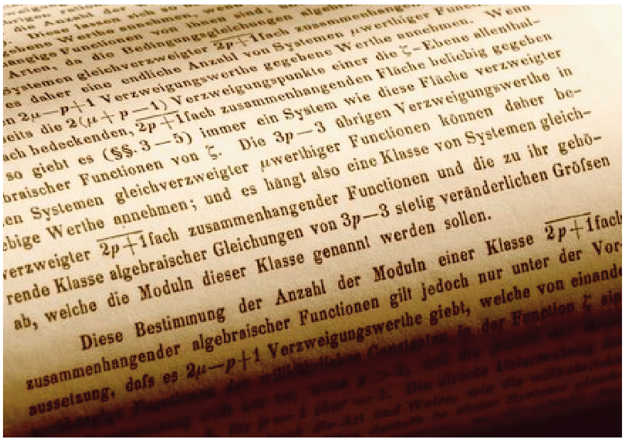
Moduli spaces are geometric versions of parameter spaces. That is, they are geometric spaces which parametrise somthing – each point represents one of the objects being parametrised, such as the solution of a particular equation, or a geometric structure on some other object. In the language of physics, a moduli space is a model of the degrees of freedom of some classical system. The term "moduli" is derived from a paper of Bernhard Riemann published in 1857.



From Crelle's Journal 54 (1857) p.134
(www.degruyter.de)

In geometry one usually wants to define invariants of the spaces, or geometric objects, under consideration. An example might be "the number of loops in the space". In general we want to associate numbers, or something more complicated, to each such space, and those numbers should be invariant however we look at the space: if we are given two spaces that happen to be the same (though this may be unknown to us, or very hard to see) then their invariants will be the same.

"Classical" invariants, which usually means those defined by linear mathematics, have been studied exhaustively and have led to great progress in the classification of various geometric structures. For instance such invariants completely classify compact manifolds (spaces which are smooth, and not infinitely big, in some sense) in all dimensions except 3 and 4: if two such manifolds have the same set of classical invariants then they are in fact the same manifold.

This approach is not sufficient to handle spaces with additional structure, such as algebraic varieties or analytic spaces, and in these cases one must study more complicated invariants, for instance defined by nonlinear equations on the spaces concerned. The idea is to construct a moduli space parametrising solutions of these equations. One can consider this moduli space itself to be the invariant, though it is rather unwieldy. Usually one takes simple classical (or linear) invariants of the moduli space, and these turn out to be extremely deep and powerful invariants of the original space.

This procedure of studying the invariants or geometry, not of the original space, but of an auxiliary moduli space constructed from it, was later interpreted as a type of quantum field theory, leading to an incredibly fruitful interaction between geometry and theoretical physics over the past 25 years. Physics suggested many of the "right" equations to study, and made predictions about the resulting invariants often coming from various "dualities" in their theories. In turn mathematics and geometry provided a fertile testing ground for their theories and conjectural dualities.

The interaction with "string theory" in physics has been especially productive for algebraic geometry. String theory requires various complicated high dimensional geometries that arise in algebraic geometry, and in fact precisely those spaces ("Calabi–Yau manifolds") which have resisted attack by classical invariants in mathematics (in particular pluricanonical linear systems).

The geometry involved is very complicated, and the definition and computation of the relevant invariants is a hot topic today. This programme will bring together the leading researchers in precisely these areas, to make progress in calculating these invariants and to use them to shed more light on differential geometry, topology and physics as well as on algebraic geometry.