Isaac Newton Institute for Mathematical Sciences

New Contexts for Stable Homotopy Theory

2 September – 20 December 2002

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Algebraic topology started in the late 19th century with the work of Henri Poincaré. In the beginning its objective was to analyse geometric mathematics, such as the global properties of smooth manifolds which arise in connection with the differential equations of physics, by converting the slippery high-dimensional geometry into more amenable algebra. In particular, Einstein's theory of relativity interpreted physics in terms of higher-dimensional topological manifolds and spurred on the development of algebraic topology. For example, this conversion of geometry into algebra via homology or cohomology theory has often been used to prove the non-existence of specific types of symmetries in the original geometry. Another example is the fixed point theorem of Brouwer, which states that every continuous map of the closed unit disc to itself must have a fixed point. Later more powerful fixed point theorems of Lefschetz and others gave methods for counting the number of fixed points of a continuous map from a manifold to itself by homological techniques.

Stable homotopy theory is the ultimate context in which to perform the type of conversion from geometrical to algebraic data which Poincaré began.

Algebraic topology was applied with astounding success by Lefschetz, Hodge and others to tackle complex algebraic geometry, which is the study of the geometry of spaces of zeros of polynomial functions in many variables. Not all algebraic geometry is concerned with complex numbers – in algebraic geometry "in characteristic p" the zeros of the polynomials are purely algebraic in context and do not appear to be amenable to the homological techniques at all. Nonetheless, one of the most celebrated mathematical developments of the 20th century was the discovery and development by Grothendieck and his school of cohomology theories which behaved in characteristic p in a similar manner to the classical cohomology used by Lefschetz. In particular, new types of Lefschetz fixed point theorems were discovered which enable one to count the number of solutions of a polynomial equation in characteristic p. The Grothendieck school flourished in the 1960's and 1970's and left a legacy of powerful new techniques, famous successes such as Deligne's proof of the Weil Conjectures and a series of mysterious cohomological problems concerning Grothendieck's nebulous notion of a "motive" (or motif).

In the 1990's Voevodsky revitalised work on these questions by discovering new cohomology theories – in particular the much sought after "motivic cohomology" – by deep and ingenious constructions which take place in the algebraic geometry version of the world of stable homotopy theory. This won him the Fields Medal in 2002. These innovations led to the solution of a number of longstanding problems, such as the Milnor Conjecture, in algebraic geometry. In addition these developments have spurred on new discoveries in number theory and algebraic topology.

The motivic world is only one of several new manifestations of stable homotopy theory, each of which have had recent notable successes.

This programme at the Newton Institute is intended to further this synergistic development by bringing together the practitioners of stable homotopy in all its current diverse disguises – from arithmetic to physics to topological modular forms – to explore new applications of their current techniques.

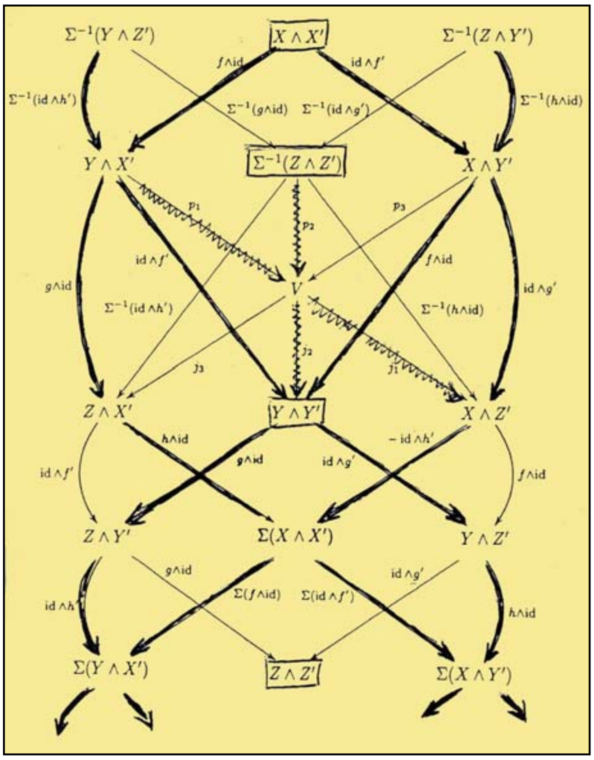

Diagram relating the smash product and triangulation in a stable homotopy category