PATTERN FORMATION IN LARGE DOMAINS

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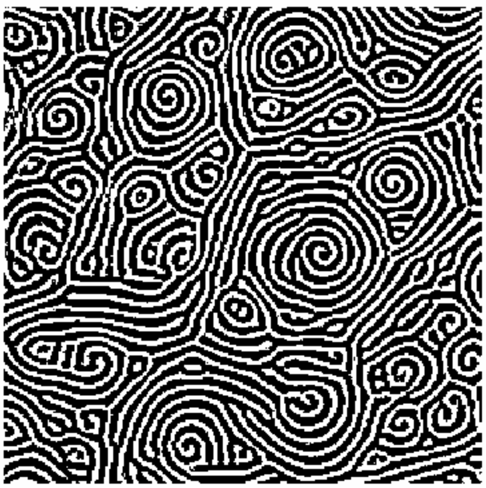
August to December 2005

Studies of order and disorder in the natural world have unearthed a remarkably diverse collection of systems that naturally organise themselves into states with degrees of symmetry, that is, patterns, with a characteristic spatial scale that is small compared to the size of the domain.

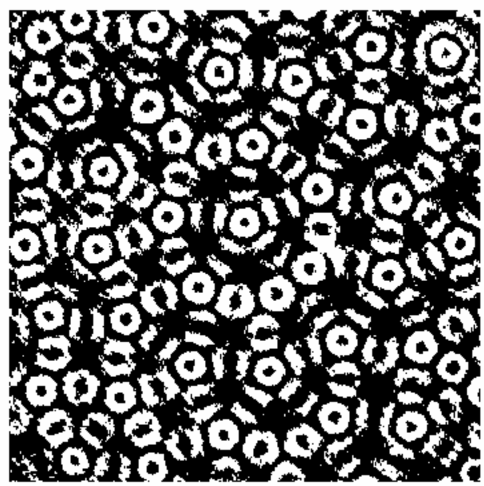


A `spiral defect chaos' state in a thermal convection experiment for parameter values where the theory predicts straight stripes (reproduced courtesy of E. Bodenschatz).

Patterns are observed in a wide variety of natural systems, including animal coat markings, cloud formations and sand dune ripples. They can also be studied in laboratory experiments, such as thermal convection in a layer of fluid heated from below, nonlinear optics, chemical reactions, surface catalysis, and the `Faraday experiment' of a vertically vibrating layer of fluid or sand. Remarkably, it turns out that these different systems generate very similar patterns, and there is an expectation that this similarity can be understood in terms of some universal mathematical model.

Current theoretical work gives central rôles to the symmetry of the system and the symmetry of the pattern, and rigorous results concerning the existence of patterns have been given in various situations. Recent advances in experimental techniques and computing power have enabled attention to be focussed on larger domains. Unexpectedly, this has led to the identification of new kinds of spatially-extended structures (for example `spiral defect chaos' and `quasipatterns', illustrated in the figures) for which theoretical understanding is very much lacking, in some cases, it seems, for deep mathematical reasons. Mathematical models have been used to describe various properties of these new patterns, though the reasons why these models seem to work are not fully understood. One theme of the programme is the development of a rigorous approach to these new kinds of patterns. Indeed, a fundamental question is, why are regular patterns seen at all in large domains?

These questions of symmetry and regularity appear in a host of related spatially-extended systems; most obviously in networks of coupled oscillators. As well as being put forward as models of a plethora of real-world phenomena, such networks illuminate some of the toughest theoretical questions.



A spatially quasiperiodic planform of surface waves (`a quasipattern') produced by the Faraday experiment (reproduced courtesy of J. Fineberg).

A short instructional course will both introduce the central topics, and enable younger researchers to understand the limitations of current approaches. A key aim of the programme is to bring together researchers from a variety of different backgrounds, including experimentalists, those involved with numerical simulations, and pure and applied mathematicians. Together, the challenge will be to develop new methods of understanding complex spatial structures.