Phase plots visualize a complex function f by depicting its color-coded argument (phase) as an image on the domain. Though meromorphic functions are uniquely determined by their phase plots up to a positive constant, more sophisticated color schemes help to make the reconstruction easier.

The phase plot of the *Euler Gamma function* (top left) is enhanced by equidistant contour lines of $\log\Gamma$ and $\arg\Gamma$. Most "tiles" generated by the shading have an almost square shape, which reflects the conformality of the mapping $z \mapsto \Gamma(z)$. The points where all colors meet are poles of Γ.

The image top right depicts a special solution to the 2D *Ginzburg-Landau equation*

$$\frac{\partial u}{\partial t} = (1 + \mathrm{i}\alpha)\,\Delta u + \lambda\, u - (1 + \mathrm{i}\beta)\,|u|^2\, u$$

Introduced to model the phenomenon of superconductivity, equations of this type also describe oscillating chemical reactions, Bose-Einstein condensates, liquid crystals, pattern formation, and self-organizing systems.

In the lower left corner we see an *approximation of the inverse tangent* function proposed by Isaac Newton. Starting from the Maclaurin series of $\operatorname{atan} t$ for $-1 < t \leq 1$, Newton derived the representation

$$\operatorname{atan} z = \frac{z}{1+z^2} \sum_{k=0}^{\infty} \frac{(2k)!!}{(2k+1)!!} \left(\frac{z^2}{1+z^2} \right)^k$$

valid in the domain $(\operatorname{Im} z)^2 - (\operatorname{Re} z)^2 < 1/2$. This is a pioneering example of analytic continuation. Depicted is the 20th partial sum of the series.

The fourth image (lower right) is an enhanced phase of a *Cauchy integral* with constant density along the black spiral S with endpoints a and b. The function is a well-defined continuous branch of

$$f(z) = c\,\log \frac{z-a}{z-b}$$

in the complement of S. It has an analytic extension to a Riemann surface with logarithmic branch points at a and b.

For more information on phase plots, see *Visual Complex Functions*, Springer, 2012 by Elias Wegert.