

# Probing superconducting phase fluctuations from the current noise spectrum of pseudogaped metal-superconductor tunnel junctions

Xi Dai, Tao Xiang<sup>†\*</sup>, Tai-Kai Ng and Zhao-bin Su<sup>†</sup>

*Physics Department, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*

<sup>†</sup>*Institute of Theoretical Physics, Chinese Academy of Sciences, P.O. Box 2735, Beijing 100080, the People's Republic of China*

<sup>\*</sup>*Isaac Newton Institute for Mathematical Sciences, the University of Cambridge, 20 Clarkson Road, Cambridge CB3 0HE, U.K.*

(March 27, 2000)

We study the current noise spectra of a tunnel junction of a metal with strong pairing phase fluctuation and a superconductor. It is shown that there is a characteristic peak in the noise spectrum at the intrinsic Josephson frequency  $\omega_J = 2eV$  when  $\omega_J$  is smaller than the pairing gap but larger than the pairing scattering rate. In the presence of an AC voltage, the tunnelling current noise shows a series of characteristic peaks with increasing DC voltage. Experimental observation of these peaks will give direct evidence of the pair fluctuation in the normal state of high- $T_c$  superconductors and from the half width of the peaks the pair decay rate can be estimated.

Direct measurements of the pair susceptibility in the normal state of a superconductor are important for understanding the pseudogap phenomenon in high- $T_c$  superconductors. For conventional superconductors, the pairing fluctuation exists only in a very narrow temperature range above  $T_c$ . However, in the underdoped high- $T_c$  materials, it is believed that the pair fluctuation is important in a substantial temperature range in the normal state [1]. This fluctuation is also important in superconductor thin films, where long ranged phase coherence is strongly suppressed [2]. The pseudogap effect was often interpreted as a precursor to the high- $T_c$  superconductivity. It was observed in many different experiments [3–9]. However, what was really found in experiments is the reduction of the low-lying density of states associated with the opening of an energy gap, which may or may not be a pairing gap, and a direct observation of the pair phase fluctuation is still absent.

The pair susceptibility in the normal state of an underdoped cuprate can be measured from the linear response of the system to an external pair field, like the measurement of the magnetic susceptibility. Such an external pair field can be introduced, for example, by coupling this underdoped cuprate to an optimally doped cuprate with a higher superconducting transition temperature via a tunnelling junction. When the temperature lies between the superconducting transition temperatures of these two materials, the tunnelling process will undoubtedly be strongly affected by the pair fluctuation in the normal side [10].

The Josephson effect occurs when both sides of the junction are in the superconducting state. A DC voltage across the junction leads to an oscillating Josephson current at frequency  $\omega_J = 2eV$ , where  $V$  is the applied voltage. If one side of the junction is not in the superconducting phase, the Josephson current is zero on average. However, the Josephson effect does not disappear completely. It manifests in the current noise spectrum and, as

will be shown later, the current noise is greatly enhanced at the Josephson frequency  $\omega_J$  if the pairing fluctuation is strong.

In this paper, we present a theoretical analysis of the tunnelling current noise spectrum of the pseudogaped metal-superconductor junction as discussed above. We assume that the pairing on both sides of the junction has  $d_{x^2-y^2}$ -wave symmetry. The extension to other pairing symmetry cases is straightforward. The main results we obtained are: (1) With an applied DC voltage, there is a characteristic Josephson peak in the noise spectrum at the intrinsic Josephson frequency  $\omega_J$ , although the coherent Josephson current is zero. (2) When an AC voltage  $V_s \cos(\omega_s t)$  is applied in addition to the DC voltage, there are a series of characteristic peaks in the zero frequency noise centred at  $\omega_J = n\omega_s$  ( $n$  an integer) with increasing  $\omega_J$ . (3) the half width of the peak is proportional to the pair decay rate  $\Gamma$  of the system, but the peak intensity drops very quickly with increasing  $\Gamma$ .

The Hamiltonian of the junction we study is defined by  $H = H_S + H_N + H_T$  with

$$H_S = \sum_{p\sigma} \varepsilon_p d_{p\sigma}^+ d_{p\sigma} + \sum_p (\Delta \varphi_p d_{p\uparrow} d_{-p\downarrow} + h.c.), \quad (1)$$

$$H_N = \sum_{k\sigma} \varepsilon_k c_{k\sigma}^+ c_{k\sigma} + \sum_{kq} [\Psi(q, t) c_{-k+q/2, \uparrow} c_{k+q/2, \downarrow} + h.c.] \\ + \sum_q \chi_{\text{pair}}^{-1}(q, t, t') \Psi^*(q, t) \Psi(-q, t'), \quad (2)$$

$$H_T = \sum_{kp\sigma} V_{kp} e^{i \int_0^t eV_s \cos(\omega_s t') dt' + i\omega_J t} c_{k\sigma}^+ d_{p\sigma} + h.c., \quad (3)$$

where  $\varphi_p = \cos p_x - \cos p_y$  and  $\Psi(x, t) = \Delta_n \exp[i\phi(x, t)]$  is the pairing field of the normal lead.  $H_S$  is the Hamiltonian for a  $d$ -wave superconductor.  $H_N$  is the effective Hamiltonian of a metal with pairing fluctuations [10,12] and  $\chi_{\text{pair}}$  is the pair propagator (or pair susceptibility) in the normal state. In  $H_T$ ,  $V_{kp}$  is the tunnelling matrix

and can be expanded with the crystal harmonics [10,14]. To the second order approximation, it is given by

$$|V_{pk}|^2 = V_0^2 + V_1^2 \varphi_k \varphi_p. \quad (4)$$

The constant term in  $|V_{pk}|^2$  has contribution to the single particle tunnelling current, but no contribution to the pair tunnelling current because of the  $d$ -wave pairing symmetry. The integration resulted from the applied AC voltage on the exponent in  $H_T$  is difficult to handle analytically. In practical calculations, it is often expanded as a sum of a serial of oscillating terms with the Bessel function:

$$e^{i \int_0^t eV_s \cos(\omega_s t') dt'} = \sum_n J_n \left( \frac{eV_s}{\omega_s} \right) e^{in\omega_s t},$$

where  $J_n(x)$  is the  $n$ 'th order Bessel function [13].

Let us first consider phenomenologically the effect of pairing phase fluctuation on the current noise. We assume that the phase fluctuation follows a diffusion process and the spatial correlation length is very short. In this case, the correlation function of the phase field is given by

$$\langle e^{i\phi(x,t)} e^{-i\phi(x',0)} \rangle \sim e^{-\Gamma|t|/2} \delta(x-x'), \quad (5)$$

where  $\Gamma$  is the pair fluctuation rate. From this equation, it can be shown that

$$\chi_{\text{pair}}(q, \omega) = 1/(i\omega + \Gamma/2). \quad (6)$$

The current noise spectrum measures the current fluctuation in the junction. It is defined by the symmetrized current-current function:

$$S(\omega) \equiv \int dt e^{i\omega t} \langle \{I(t) - \langle I(t) \rangle, I(0) - \langle I(0) \rangle\} \rangle, \quad (7)$$

where  $I(t)$  is the current across the junction and  $\{A, B\} \equiv AB + BA$ . In the case only a DC voltage is applied, the above tunnelling Hamiltonian can be treated semi-classically and the current-current correlation function contributed from the pair tunnelling is simply given by the thermal average of the phase fields:

$$\begin{aligned} \langle I(t)I(0) \rangle &\sim \int dx dx' \langle \sin[\phi(x,t) + \omega_J t] \sin \phi(x',0) \rangle \\ &\sim \cos(\omega_J t) e^{-\Gamma|t|/2} \end{aligned} \quad (8)$$

Substituting this equation into (7), we then obtain the noise spectrum contributed from the pair tunnelling

$$S_{\text{pair}}(\omega) \sim \frac{\Gamma}{(\omega - \omega_J)^2 + \Gamma^2/4} + \frac{\Gamma}{(\omega + \omega_J)^2 + \Gamma^2/4}. \quad (9)$$

This result indicates that the Josephson current cross the junction is coherent if the tunnelling time is much shorter than the fluctuation time  $\tau \sim \Gamma^{-1}$  of virtual cooper pairs.

In the limit  $\Gamma \rightarrow 0$ ,  $S(\omega)$  becomes two delta-functions centred at  $\pm\omega_J$ . For finite  $\Gamma$ , these two delta-functions are broadened and the half width of the broadened peak is proportional to  $\Gamma$ .

Eq. (9) is similar as the result obtained by Martin and Balatsky very recently [11]. However, as both sides of the junction in the experimental setup they suggested are in the normal state, the total pair decay rate is larger than the case discussed here. Since the intensity of the peak compared with the average noise drops sharply with increasing  $\Gamma$  (Figure 2), it may be difficult to observe the characteristic Josephson fluctuation in finite frequency conductance measurement suggested in their paper. While in our paper, to determine the pair decay rate only the zero frequency noise is required which is easier to measure.

In a s-wave pairing system, since no quasiparticle excitations exist within the energy gap, the single particle tunnelling can be ignored when the noise frequency is lower than the gap. For the d-wave pairing case, the contribution of the single particle tunnelling cannot be ignored because of the existence of gap nodes. To find out the contribution from both the quasiparticle and pair tunnelling to the noise, we perform a microscopic calculation the current-current correlation function using the closed time Green's function technique [15,16]. The tunnelling Hamiltonian  $H_T$  is treated as a perturbation. Since the normal Josephson effect is absent in the junction, the second order contribution of  $H_T$  contains only the quasiparticle tunnelling terms. However, the noise caused by the quasiparticle tunnelling varies very slowly with frequency. When the noise frequency is smaller than  $\Delta$ , it adds only a featureless background to the noise spectra.

The most important contribution of  $H_T$  to the noise is from the fourth order perturbation, in particular from the virtual pair tunnelling process. When the pairing fluctuation in the normal state is strong, a Cooper pair can tunnel from the superconductor to the normal lead and contributes a significant term to the noise at low frequencies. Figure 1 shows the Feymann diagram for this pair tunnelling term in the current-current correlation function. The solid line is the Green's function of electrons. The dashed line is the propagator of the pair field in the normal state.

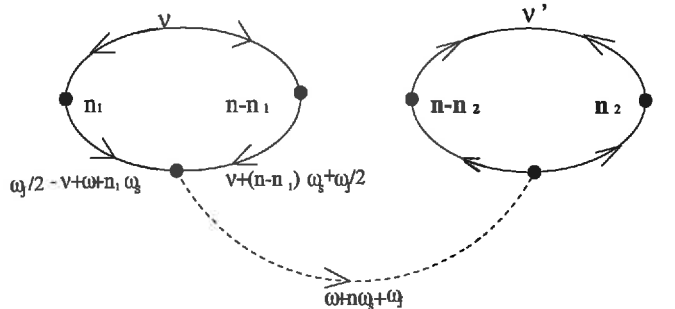


FIG. 1. The Feymann diagram for the pair tunnelling term, where the solid line represents the electronic Green's function and the dashed line represents the pairing fluctuation.

To the leading order approximation in both the quasiparticle and pair tunnelling channels, we find that the

$$N_q(\omega) = V_0^2 \sum_n J_n^2(x) \rho_s \int d\nu \rho_d(\nu) [n_f(\nu) - n_f(\nu + \omega'_n)] \coth \frac{\beta \omega'_n}{2}, \quad (11)$$

$$N_p(\omega) = V_1^4 C \sum_{n, n_1, n_2} \frac{J_{n_1}(x) J_{n-n_1}(x) J_{n_2}(x) J_{n-n_2}(x)}{\omega_n'^2 + \Gamma^2/4} \coth \frac{\beta \omega_n'}{2} \sum_q \{ \omega_n' [\phi_2^2(\omega_n', q) - \phi_1^2(\omega_n', q)] + \Gamma \phi_2(\omega_n') \phi_1(\omega_n', q) \}. \quad (12)$$

In the above equations,  $x = eV_s/\omega_s$ ,  $\omega'_n = \omega + \omega_J + n\omega_s$ ,  $\omega_n'' = \omega + eV + n\omega_s$ ,  $C = -\sum_p \Delta \varphi_p^2 / (\varepsilon_p^2 + \Delta^2 \varphi_p^2)$ ,  $\rho_s$  is the density of states of the superconductor in the normal state, and  $\rho_n(\nu)$  is the density of states of the normal lead. In Eq. (12),

$$\phi_1(\nu, q) = \pi \sum_k \varphi_k^2 (1 - n_k - n_{-k+q}) \delta(\nu - \varepsilon_k - \varepsilon_{-k+q}),$$

$$\phi_2(\nu, q) = -\sum_k \varphi_k^2 \frac{1 - n_k - n_{-k+q}}{\nu - \varepsilon_k - \varepsilon_{-k+q}},$$

and  $n_k = 1/(1 + e^{\beta \varepsilon_k})$  is the Fermi-Dirac function.

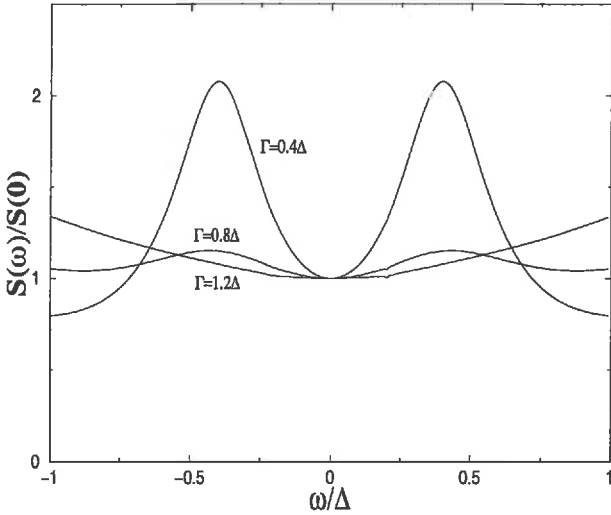


FIG. 2. The normalised current noise spectrum as a function of the noise frequency for three different  $\Gamma$ .  $\omega_J = 0.4\Delta$ .

Figure 2 shows  $S(\omega)$  normalised by  $S(0)$  as a function of  $\omega$  for the case  $\omega_J = 0.4\Delta$  and  $\omega_s = 0$ . The param-

noise spectra is given by

$$S(\omega) \approx N_q(\omega) + N_p(\omega) + N_q(-\omega) + N_p(-\omega), \quad (10)$$

where  $N_q$  and  $N_p$  are respectively the contributions from the quasiparticle and pair tunnelling:

eters used are  $\Delta_n = \Delta$ ,  $T = 0.2\Delta$ ,  $V_0 = 2V_1 = 2\Delta$ , and  $\rho_s = 1/10\Delta$ . The band width of the normal lead is assumed to be  $10\Delta$ . From the figure, it is clear that if the fluctuation rate  $\Gamma$  is sufficiently small, the noise peak centred at  $\omega = \pm\omega_J$  is very sharp. The half width of the peak is determined by  $\Gamma$ . When  $\omega_J \gg \Gamma$ , the thermally excited pair tunnelling current will oscillate a few times before it loses the phase coherence. But with increasing  $\Gamma$ , the noise peak at  $\omega_J$  becomes broadened and vanishes eventually.

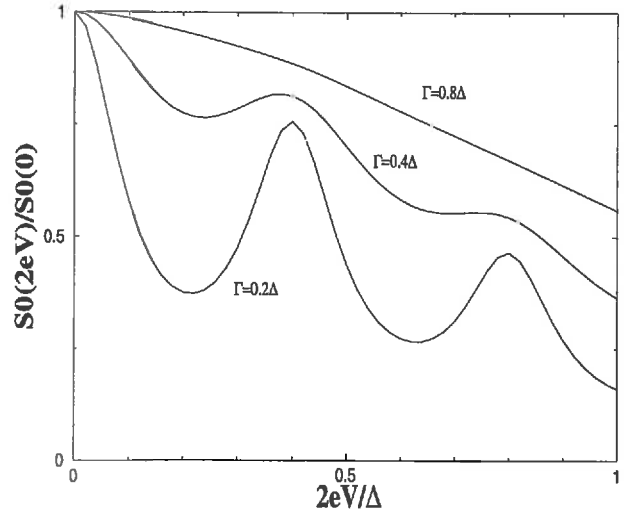


FIG. 3. The normalised current noise spectrum at zero noise frequency as a function of the intrinsic Josephson frequency  $\omega_J = 2eV$  for three different  $\Gamma$ .  $\omega_s = 0.4\Delta$ .

In the presence of both DC and AC voltages, the pair tunnelling is enhanced at the frequency  $\omega = \omega_J + n\omega_s$  and the noise spectrum shows a series of local maxima at these

frequencies. If the applied microwave frequency and the Josephson frequency satisfy the condition  $\omega_J + n\omega_s = 0$ , a local maximum will appear at the zero frequency. Figure 3 shows the zero frequency normalised current noise spectrum as a function of  $\omega_J$  for the case  $\omega_s = 0.4\Delta$  (the other parameters are the same as in Figure 2). There are a series of peaks for  $\Gamma = 0.2\Delta$ . When  $\Gamma$  is increased, these peaks become broader and weaker.

The above results indicate that the pair fluctuation in the normal lead has strong effects on the noise spectrum of the tunnelling current. The broadened peaks at the characteristic frequencies in the noise spectrum shown in Figures 2 and 3 are unique to the normal-superconductor junction with strong pair fluctuations. Experimentally, it is not easy to measure the finite frequency noise, but when the AC voltage is applied we only need to measure the zero frequency noise which is quite easy to do. Thus experimental measurements of the current noise allow us to judge how strong the pairing fluctuation is in the pseudogap phase. This is apparently important for a direct test of the superconducting precursor scenario.

In summary, we have shown that the noise spectrum of the tunnelling current between an underdoped cuprate in the normal state and an optimally doped cuprate in the superconducting state is a useful probe of the pairing fluctuation in the pseudogap phase. We have calculated the noise spectrum using the closed path Green's functions and found that the current noise is strongly enhanced at a series of characteristic frequencies determined by the Josephson pair tunnelling processes. Notice that the system is nonequilibrium when the DC voltage is applied, it is proper to use the closed time green's function technique. Experimental observations of the noise peaks at these characteristic frequencies will help us to establish the correct picture of the pair fluctuation in the pseudogap phase of high- $T_c$  cuprates.

TX acknowledges the financial support from the National Natural Science Foundation of China and from the special funds for Major State Basic Research Projects of

China.

- 
- [1] V. Emery and S. Kivelson, *Nature* **374**, 434 (1995).
  - [2] D. B. Haviland, Y. Liu and A. M. Goldman, *Phys. Rev. Lett.* **62**, 2180 (1989); A. M. Goldman and Y. Liu, *Physica D* **83**, 163 (1995).
  - [3] M. Takigawa, A. P. Reyes, P. C. Hammel, J. D. Thompson, R. H. Heffner, Z. Fisk, and K. C. Ott, *Phys. Rev. B* **43**, 247 (1991).
  - [4] D. S. Marshall, D. S. Dessau, A. G. Loeser, C-H. Park, A. Y. Matsuura, J. N. Eckstein, I. Bozovic, P. Fournier, A. Kapitulnik, W. E. Spicer, and Z.-X. Shen, *Phys. Rev. Lett.* **76**, 4841 (1996).
  - [5] A. G. Loeser, Z. X. Shen, D. S. Dessau, D. S. Marshall, C. H. Park, P. Fournier, A. Kapitulnik, *Science* **273**, 325 (1996).
  - [6] H. Ding, T. Yokoya, J. C. Campuzano, T. Takahashi, M. Randeria, M. R. Norman, T. Mochiku, K. Kadowaki and J. Giapintzakis, *Nature* **382**, 51 (1996).
  - [7] J. W. Loram, K. A. Mirza, J. R. Cooper, and W. Y. Liang, *Phys. Rev. Lett.* **71**, 1740 (1993).
  - [8] D. N. Basov, R. Liang, B. Dabrowski, D. A. Bonn, W. N. Hardy, and T. Timusk, *Phys. Rev. Lett.* **77** 4090 (1996).
  - [9] Ch. Renner, B. Revaz, J. Y. Genond, K. Kadowak and Fischer, *Phys. Rev. Lett.* **80**, 149 (1998).
  - [10] B. Janko, I. Kosztin, K. Levin, M. R. Norman and D. J. Scalapino, *Phys. Rev. Lett.* **82** 4304 (1999).
  - [11] Ivar Martin and Alexander Balatsky, cond-mat/0002043.
  - [12] B. Janko, J. Maly and K. Levin, *Phys. Rev.* **B56**, R11407 (1997); J. Maly, B. Janko and K. Levin, *Phys. Rev.* **B59**, 1354, (1999).
  - [13] G. D. Mahan, "Many-Particle Physics", Chapter 9, Plenum Press, New York, (1990).
  - [14] P. J. Hirschfeld, S. M. Quinlan and D. J. Scalapino, *Phys. Rev.* **B55**, 12742 (1997).
  - [15] L. V. Keldysh, *JETP* **20**, 1018 (1965).
  - [16] K. C. Chou, Z. B. Su, B. L. Hao, and L. Yu, *Phys. Rep.* **118**, 1 (1985).

## Recent Newton Institute Preprints

- NI00001-SMM **KZ Markov**  
*Justification of an effective field method in elasto-statics of heterogeneous solids*
- NI00002-SCE **YY Lobanov and VD Rushai**  
*Studying the evolution of open quantum systems via conditional Wiener integrals*
- NI00003-SCE **J-G Wang and G-S Tian**  
*Spin and charged gaps in strongly correlated electron systems with negative or positive couplings*
- NI00004-SCE **FV Kusmartsev**  
*Conducting electron strings in oxides*
- NI00005-ERN **SG Dani**  
*On ergodic  $Z^d$  actions on Lie groups by automorphisms*
- NI00006-SMM **V Nesi and G Alessandrini**  
*Univalence of  $\sigma$ -harmonic mappings and applications*
- NI00007-SCE **X Dai, T Xiang, T-K Ng et al**  
*Probing superconducting phase fluctuations from the current noise spectrum of pseudogaped metal-superconductor tunnel junctions*
- NI00008-ERN **B Hasselblatt**  
*Hyperbolic dynamical systems*
- NI00009-SCE **J Lou, S Quin, T-K Ng et al**  
*Topological effects at short antiferromagnetic Heisenberg chains*
- NI00010-SCE **V Zlatić and J Freericks**  
*Theory of valence transitions in Ytterbium-based compounds*
- NI00011-ERN **A Iozzi and D Witte**  
*Cartan-decomposition subgroups of  $SU(2,n)$*
- NI00012-ERN **D Witte and L Lifschitz**  
*On automorphisms of arithmetic subgroups of unipotent groups in positive characteristic*
- NI00013-ERN **D Witte**  
*Homogeneous Lorentz manifold with simple isometry group*
- NI00014-SGT **R Uribe-Vargos**  
*Global theorems on vertices and flattenings of closed curves*
- NI00015-SGT **EA Bartolo, P Cassou-Nogués, I Luengo et al**  
*Monodromy conjecture for some surface singularities*
- NI00016-SGT **IG Scherbak**  
*Boundary singularities and non-crystallographic Coxeter groups*
- NI00017-SGT **K Houston**  
*On the classification and topology of complex map-germs of corank one and  $A_e$ -codimension one*
- NI00018-SGT **PJ Topalov and VS Matveev**  
*Geodesic equivalence via integrability*
- NI00019-GTF **S Friedlander**  
*On vortex tube stretching and instabilities in an inviscid fluid*
- NI00020-SGT **VD Sedykh**  
*Some invariants of admissible homotopies of space curves*
- NI00021-SGT **IA Bogaevsky**  
*Singularities of linear waves in plane and space*

