Predictability error growth of turbulent flows

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Abstract

Recently, alternative viewpoints were suggested that is in contrast to the conventional picture of predictability error growth in the spectral domain. We survey key historical and current literatures and suggest that the traditional perspective has not been invalidated.

Key words: Turbulent Flows, Predictability

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1. Introduction

Instability and unpredictability are fundamental to turbulent flows. The atmospheric turbulence is an example of a very high Reynolds number flow and as such, one is confronted with a multiple scale problem with a wide range of interacting scales over several decades. Early work on the problem of atmospheric predictability has been carried out by Thompson (1957). The major advancement was achieved by Lorenz (1969) that deduced quantitatively the predictability of atmospheric turbulence. With only low-order representations of atmospheric flow, Lorenz was able to demonstrate limited predictability could be achieved.

To develop a methodology to treat a wide interacting scales of atmospheric flows, Leith and Kraichnan (1972) and Leith (1971) turned to the techniques of statistical turbulence theory. They employed a particular version of the closure theories, the test-field model of turbulence, to evaluate the growth of prediction error for inertial range turbulence for both two- and three- dimensional turbulence. As in other publications, the tool of analysis of predictability is the growth in the difference between the Eulerian velocity fields of pairs of flows chosen from statistical identical ensembles. Using improved closure model for the evolution of the energy in wave-number space, Leith and Kraichnan verified Lorenz's results, which has lead to the picture of the evolution of errors in the wave-number spectral domain.

The statistical closure approaches are appropriate for dealing with the atmospheric turbulence since by construction they are designed for describing fully developed turbulence at high Reynolds number. At the time of its introduction, the closure method was the only one that could address the multiple scale problem associated with the high Reynolds number flows. Yet, the methodology has intrinsic difficulties in taking into account many features that need to be incorporated into a realistic atmospheric predictive model.

2. Alternative perspective

Recently, alternative viewpoints were suggested that is in contrast to the conventional picture of predictability error growth in the spectral domain. As an illustrative example, Tribbia and Baumhefner (2004, TB hereafter) revisited the predictive problem, prompted by renewed interest in observation strategies and the consideration that some two decades have passed. In this important and influential paper, the authors used the NCAR Community Climate Model Version 3 (CCM3, see Kiehl *et al*., 1998), which has well-documented capabilities as an atmospheric climate model and a medium-range forecast tool. TB run the model at a range of horizontal resolutions, from T42 to T170. The initial data for the atmospheric model are taken from the National Center for Environmental Prediction (NCEP) operational analysis, with experiment dates taken from the boreal winter of 1995/96. To provide insight on the predictive issue within the CCM3, TB carried out both “the identical-model twin” and “the imperfect model twin experiments”.

In the identical-model twin framework, TB compared and contrasted the error growth experiments using initial errors confined to long and short scales. In these numerical integrations, a fixed resolution model (T63) was first initialized with the observed analyses for a single day and integrated forward in time ("the control run"). In their Fig. 2, the kinetic energy spectrum for 500 mb is plotted for 2D wavenumber *n* indicated on abscissa. The amplitude of each wavenumber is on the ordinate. The solid line is the spectrum of full field, and dotted (day 0), dashed-dotted (day 1), and dashed lines (day 7) are the spectrum of the difference fields from ensemble members (a) full perturbation in all scales, (b) perturbations only in wavenumber ≤ 30, and (c) perturbations only in wave-number > 30. These experiments offered an estimate of the rate of error growth due to imperfections in the current analysis system.

In the imperfect model twin experiments, TB takes the control integration from a T170 integration. They inspected the error growth in the T42, T63, T106, and T170 versions of CCM3 in cases in which the initial condition error is quite small. The dominant source of error is the difference in resolution between the high-resolution control and the coarser-resolution forecast models. All three truncations exhibit a similar spectral shape and growth pattern. TB found that the errors rapidly disperse from scales technically beyond model resolution to a small amplitude, spectrally uniform distribution of errors in resolved scales. The errors in resolved scales further amplified in a quasi-exponentially growth of baroclinically active scales.

Based on these results, TB argued that a different picture emerges if errors in the largest scales are nearly eliminated. In this case there is a lag in time of the growth of errors in the large scale reflective of the necessity of injecting errors through a cascade process into the baroclinically active scales. As opposed to the traditional paradigm of predictability error growth, once in the synoptic scales, the errors organize within the synoptic structures and amplify exponentially, extracting energy from the large-scale background flow, not from the small scales. In this alternative view,

the inverse cascade becomes of lesser important than in the traditional one. The role of the inverse cascade reduces to that of seeding disturbances in the baroclinically active region of the spectrum.

3. Historical and recent work

We suggest that the outcomes from the application climate models could be quite distinctive from those from an idealized situation where the turbulent flow was assumed as homogeneous. While the lesson from the TB study is that one must be carefully in extrapolating the theoretical model to a practical application, we stress that the original predictive theory remains correct. We will inspect some of historical and current work below to illustrate this point.

Theoretical studies of two dimensional turbulence showed that small scales were distributed according to $E(k) \sim k^{-3}$ (Batchelor, 1969; Kraichnan, 1967; Leith, 1968). Then the similarity between quasi-geostrophic turbulence (QGT; see for example, Charney, 1971; Herring, 1988) and 2-d turbulence suggested a similar spectrum for the small scales (smaller than the baroclinic instability) of QGT. In a recent paper, Boffetta and Musacchio (2001) carried out a direct numerical simulation of stationary 2-d turbulence with resolution $N=1024$. These authors found that their numerical simulation leads to the result that is in “remarkable” agreement with the Leith-Kraichnan (1971) prediction obtained from the test-field-model.

It is also instructive to look into the predictive problem for 2-d decaying turbulence, where the role of coherent vortices is essential for a correct interpretation of the results (Basdevant et al., 1981; McWilliams, 1984). Phenomenological treatment (Batchelor, 1969) [the Test Field Model] predicts a t^{-2} decay of enstrophy for unforced 2-d turbulence, whereas it decays much slower at $t^{-0.8}$, t^{-1}, $t^{-1,1}$, according to numerical simulations (Chasnov, 1997; Herring et al., 1999; Clercx and Nielsen, 2000; Ossia and Lesieur, 2001). Boffetta *et al.* (1997) paid special attention to make a qualitative and quantitative comparison with previous results obtained from the closure theories. Within the limits of Eulerian based definitions for the error, e.g., average energy or enstrophy of the error field, these authors found that the predictability time estimates look quite similar and in qualitative agreement with closure approximations (Boffetta *et al.* (1997).

Charney (1971) presented a model for the minus three spectra of horizontal velocity and temperature at high wavenumbers in three-dimensional quasi-geostrophic flow. He pointed out, especially in the range $k=7\text{-}20$, that this 2-d picture of motions is not realistic in the atmosphere. Herring (1980) provided further investigation of quasi-geostrophic turbulence within the context of statistical homogeneity. He found for decaying turbulence that the high wavenumbers tend toward three-dimensional isotropy, as predicted by Charney. However, for wavenumbers smaller than the energy peak, the flow tend toward an approximate two-dimensional state, with the crossover wavenumber near the peak energy wavenumber. The high wavenumber energy spectrum is found to be log modified minus 3 scaling, where *k* is the *three dimensional* wavenumber. Restricted to rather short integration time and used the homogeneous version of system, Herring (1984) made a comparison between 2-d error growth to an equivalent problem for the quasi-geostrophic system. He found that the rate of error growth for the quasi-geostrophic system is smaller than that for 2-d turbulence if the rate of overtaking of the peak energy wave number by the error cross over wave number is used as a measure. Otherwise the error profiles of the two systems were closely similar. Herring offered a tentative suggestion for the reason for the smaller quasi-geostrophic error growth. This system has an additional degree of freedom which is excited by the equipartioning property. While this degree of freedom does not contribute to strain and is inefficient in effecting energy (and enstrophy) transfer, Herring reasoned that in the same argument, error growth is not enhanced. The predictability behavior of a quasi-geostrophic model (Morss *et al.,* 2009) also showed expected results from the traditional view.

In 3-d turbulence, Herring *et al* (1973) compared the numerical simulations with the results of the direct interaction approximation. Recently, Aurell *et al*. (1996) used a shell model of turbulence to study the predictability. The results suggest the picture of the evolution of errors in the wave-number spectral domain.

4. Conclusion

In conclusion, we have argued that the historical and current literatures suggest that the traditional perspective has not been invalidated. The alternative viewpoints obtained are likely a consequence of other physics in the model, which are not included in the study of predictive problem of “pure” 2-d. and quasi-3d (also suggestive in a model 3-d) turbulent flows.

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