A Hamiltonian Cycle in the Square of a 2-connected Graph in Linear Time

Stephen Alstrup[1], Agelos Georgakopoulos[*2], Eva Rotenberg[1] and Carsten Thomassen[†3]

[1]Department of Computer Science, University of Copenhagen, Denmark. s.alstrup@di.ku.dk, roden@di.ku.dk
[2]Mathematics Institute, University of Warwick, UK. A.Georgakopoulos@warwick.ac.uk
[3]Department of Applied Mathematics and Computer Science, Technical University of Denmark. ctho@dtu.dk

Abstract

Fleischner's theorem says that the square of every 2-connected graph contains a Hamiltonian cycle. We present a proof resulting in an $O(|E|)$ algorithm for producing a Hamiltonian cycle in the square G^2 of a 2-connected graph $G = (V, E)$. More generally, we get an $O(|E|)$ algorithm for producing a Hamiltonian path between any two prescribed vertices, and we get an $O(|V|^2)$ algorithm for producing cycles $C_3, C_4, \dots, C_{|V|}$ in G^2 of lengths $3, 4, \dots, |V|$, respectively.

1 Introduction

Fleischner [5] proved in 1974 that the square of every 2-connected graph is Hamiltonian, solving a conjecture from 1966 by Nash-Williams. This remarkable result has stimulated much work on paths and cycles in the square of a finite graph, e.g [2], [6], [11] and [17]. Fleischner's theorem has also been extended to infinite locally finite graphs with at most two ends by Thomassen [18], and to (compactifications of) all locally finite graphs by Georgakopoulos [7]. The theorem is also of potential interest in theoretical computer science. Indeed, Hamiltonian cycles in powers of graphs have been used to make efficient labelling schemes for distance, see [1].

A short proof of Fleischner's theorem was obtained by Říha (1991) [19]. The technique in that proof has some resemblance with the technique in [18]. More recently, a simpler proof was presented by Georgakopoulos (2009) [8], see also [3].

*Supported by EPSRC grant EP/L002787/1 and ERC Starting grant RGGC (grant agreement No 639046). The second author would like to thank the Isaac Newton Institute for Mathematical Sciences, Cambridge, for support and hospitality during the programme 'Random Geometry' where work on this paper was undertaken.

†Partially supported by ERC Advanced grant GRACOL

Lau (1980) [13] was the first to give an efficient constructive algorithm, more precisely an $O(|V|^2)$ algoritm, for finding a Hamiltonian cycle in the square of a 2-connected graph.

In this paper, we present a simple proof of Fleischner's Theorem based on the ideas of [8], which results in a linear time algorithm for finding a Hamiltonian cycle in the square G^2 of a 2-connected graph G. The algorithm is then used to give a new proof of the result in [2] that G^2 is *Hamiltonian connected*, that is, it has a Hamiltonian path between any two prescribed vertices, and we get an $O(|E|)$ algorithm for producing such a path. The algorithm is also used to give a new proof of the result in [11, 17] that G^2 is *pancyclic*, that is it has a collection of cycles $C_3, C_4, \dots, C_{|V|}$ of lengths $3, 4, \dots, |V|$, respectively. The algorithm results in an $O(|V|^2)$ algorithm for producing such a collection of cycles. In fact, we may in $O(|V|^2)$ time produce such cycles C_i with nested vertex sets, that is $V[C_i] \subset V[C_{i+1}]$, such that $x \in V[C_3]$ for any prescribed vertex x of a graph whose block-cutvertex tree is a path.

These results follow from our main theorem:

Theorem 1.1. *There is a linear time algorithm finding a Hamiltonian cycle in the square of any 2-connected graph.*

Proof. The algorithm comprises the following steps:

1. Find a minimally 2-connected spanning subgraph G of the 2-connected graph under consideration.
2. Find a proper ear decomposition of G.
3. Pick a vertex of degree 2 in each ear, which exists by Lemma 3.2.
4. 'Implement' the proof of [8].

A linear time algorithm for step 1 above was found by Han et. al. [9, Theorem 12]. A linear time algorithm for step 2 was found by Schmidt [15]. Given an ear in our decomposition, one can check the degrees of all vertices and choose one of degree 2 for each ear, ensuring linear running time of step 3 above. Finally, we will show in Section 4 why step 4 runs in linear time.

□

2 Preliminaries

A *proper ear decomposition* of an undirected graph G is a partition of its set of edges into a sequence $C^0, \dots, C^k$ where C^0 is a cycle and C^i is a path for every $i \geq 1$, such that for every $i \geq 1$, $C^i \cap \bigcup_{j<i} C^j$ consists of two end vertices of C^i.

A graph is *k-connected* if no deletion of $k-1$ vertices disconnects the graph. An edge e of a graph G is *k-essential*, if $G - e$ is not k-connected. A minimally k-connected graph is one where every edge is k-essential.

An *Euler tour* (respectively *Euler walk*) of a multigraph is a walk which uses every edge exactly once, and has a last vertex which is the same (respectively not the same) as the first vertex. A multigraph has an Euler tour if and only if every vertex has even degree, and an Euler tour may be found in linear time using Hierholzer's algorithm [10]. A multigraph is *Eulererian* if and only if it has an Euler tour.

The square of a graph $G = (V, E)$ is the graph $G^2 = (V, E')$ where $(u, v) \in E'$ if and only if u and v are connected by a path of length at most 2 in G.

A *Hamiltonian cycle* is a cycle that contains all vertices of the graph. Extending hamiltonicity, a graph G is *pancyclic* if it contains a cycle of length n for every $n \in [3, 4, \ldots, |V(G)|]$. A graph is *vertex pancyclic* if, for every vertex x, these cycles can be chosen to pass through x.

3 Every ear has a degree 2 vertex

Dirac [4] gave a detailed investigation of minimally 2-connected graphs. His work inspired deep results on minimally k-connected graphs, see e.g. [12, 14]. We use the definition introduced by Dirac [4, Definition 6]: Given two vertices of a minimally 2-connected graph, they are *compatible* if no path between them has a chord. We also use Dirac's observation that every 2-connected subgraph of a minimally 2-connected graph is minimally 2-connected.

Lemma 3.1. *Let G be a minimally 2-connected graph, let u and v be vertices of G and let P_1, P_2, P_3 be three internally disjoint paths between u and v in G. Then each of the three paths will contain at least one vertex of degree 2.*

Proof. We observe that u and v must be compatible: Let P_4 be any path between u and v, and consider the union $G' = P_1 \cup P_2 \cup P_3 \cup P_4$ which must be a 2-connected subgraph of G, and hence minimally 2-connected by the above observation. Assume for contradiction e is a chord of P_4, then e lies on at most one of P_1, P_2, P_3. But then, $G' - e$ is still 2-connected, in contradiction with minimality. Now [4, Corollary 2 to Theorem 6] directly states that every path from u to v has a vertex of degree 2 in G. □

Lemma 3.2. *Let $C^0, \ldots, C^k$ be a proper ear decomposition of a minimally 2-connected graph G. Then every C^i contains a vertex of degree 2, and C^0 contains at least two vertices of degree 2.*

Proof. Consider a proper ear decomposition of a minimally 2-connected graph as stated. Then the union of the first i ears $C^0 \cup \ldots \cup C^{i-1}$ always forms a 2-connected graph. If u, v are the end vertices of C^i, there already exist two internally disjoint paths between them. Together with C^i they form three internally disjoint paths, each of which must contain a vertex of degree 2 by Lemma 3.1. Thus, C^i contains a vertex of degree 2.

For C^0 we have a stronger statement. If $k = 0$, then all vertices of $C^0 = G$ are of degree 2. Otherwise, let u, v be the end vertices of C^1. Then there are two internally disjoint paths in C^0 between u, v. Together with C^1 we have three internally disjoint paths, as before, and each must contain a vertex of degree 2. But then, since two of the paths lie entirely on C^0, we must have at least 2 vertices of degree 2 on C^0. □

4 A Hamiltonian cycle in linear time

Let G be a minimally 2-connected graph. In this section, we use the ear decomposition found above in order to construct a Hamiltonian cycle in the square of G. This part of our algorithm draws heavily from the proof of [8].

Let $C^0, C^1, \ldots, C^k$ be a proper ear decomposition of our minimally 2-connected subgraph G, where C^0 is a cycle and each other C^i has both its endvertices in ears with smaller indices. By Lemma 3.2, every C^i contains an interior vertex y^i of degree 2, and it is easy to pick such a vertex for each i in linear time. Furthermore, by Lemma 3.2, C^0 contains two vertices of degree 2, say x and y^0.

We enumerate the vertices of each C^i as $x_0^i, x_1^i, \ldots, x_{\ell_i}^i$ in the order they appear on C^i, starting with the end-vertex lying in the ear with the smallest index. For C^0 we just start the enumeration at $x_0^0 = x = x_{\ell_0}^0$.

We start our proceedure by turning G into an Eulerian multigraph $G_{\between}$ by adding parallel edges to some existing edges of G and deleting some edges of G as follows.

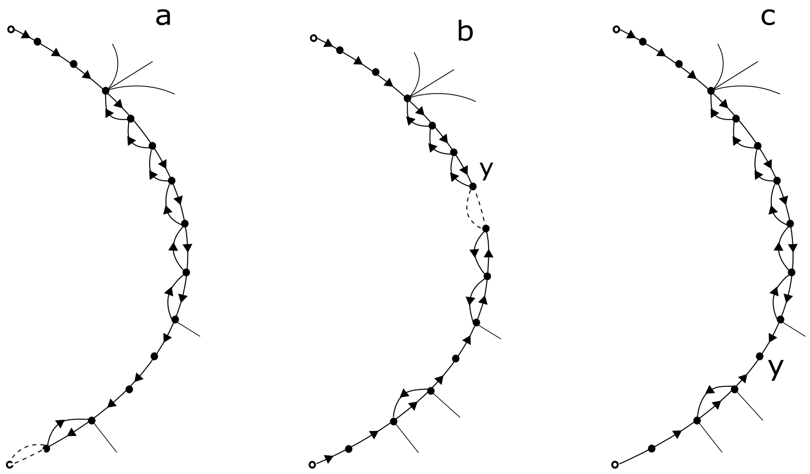

Figure 1: We go through the ears in decreasing order, double some edges and delete at most one (dotted line), thus, turning the graph into an Eulerian graph.

For $i = k, k-1, \ldots, 0$, we go through the interior vertices $x_1^i, x_2^i, \ldots$ of C^i in the given order, and if the degree $d(x_j^i)$ is odd in the current graph, we introduce a new edge parallel to the $x_j^i x_{j+1}^i$ edge. If the last edge of C^i is doubled, we delete both its copies (see Figure 1 case a). If the last edge is not doubled, but one (and hence both) of the edges incident with y^i is doubled, then we delete the pair of parallel edges incident with y^i which comes second as we move along C^i from x_0^i (see Figure 1 case b). Clearly, this proceedure has a linear running time.

Note that every vertex v has even degree after we are finished: its incident edges were last affected at the unique step i where v is an interior vertex of C^i, and we made the degree $d(v)$ even in that step. The only vertex we never considered is x, which now has even degree by the handshaking lemma. Moreover, $G_{\between}$ is connected because every vertex of C^i is still connected to $C^0, C^1, \ldots, C^{i-1}$ after any edge deletions. Thus $G_{\between}$ is Eulerian.

Next, we orient the edges of $G_{\between}$ as follows. We go through the ears C^i of G again (in fact, this step of our algorithm can be combined with the previous step). If the last edge of C^i has been deleted, we orient all edges of $C^i \cap G_{\between}$ from x_0^i towards $x_{\ell_i - 1}^i$ (see Figure 1 case a). We orient any parallel copies of those edges in $G_{\between}$ in the oposite direction. Otherwise, we orient all edges of $C^i \cap G_{\between}$

towards y^i, and any parallel copies of those edges in the oposite direction (see Figure 1 case b and case c).

Note that after we are done, though vertices may have arbitrarily high out-degree, every vertex v has at most 2 incoming edges, since only edges of the ear C^i containing v as an interior vertex can be directed towards v by construction; here we used the fact that the first edge of each C^i is never doubled, and if the last one is doubled it is immediately deleted. Moreover, if $v \neq x$, then v has at least 1 incoming edge.

We now describe an Euler tour J of the underlying undirected graph $G_{\between}$ such that for every vertex v having two incoming edges vw, vz, these edges are consequtive in J. This can easily be achieved by first replacing these two edges vw, vz by a single wz edge for every vertex v having two incoming edges, then finding an Euler tour in the resulting auxiliary graph $G^{\lhd}$, and finally replacing the wz edge back into the pair wv, vz for every v as above. The fact that $G^{\lhd}$ is Eulerian too is easy to check given the construction of $G_{\between}$: only connectedness needs to be proven. When the ear falls into case c (see Figure 1), its special vertex y^i might be disconnected entirely in $G_{\between}$, all other vertices are still connected. Namely, for each vertex $v \in C^i$ $(v \neq y^i)$, if it has indegree 2, it has at least one outneighbour on C^i, and thus, the substitution does not disconnect it from C^i. Furthermore, the only vertices on C^i that can be endvertices of later ears, $C^{j>i}$, are those of degree > 2, and thus, y^i is not an endvertex of another ear.

Finally, we transform the Euler tour J into a Hamiltonian cycle by *lifting* some pairs of edges of $G_{\between}$ into edges of G^2. More precisely, we traverse the Euler tour J, replacing every 2-edge subwalk $u \leftarrow w \rightarrow v$ in J by the edge (u, v). We make a single exception for the unique subwalk having x as its middle vertex.

Note that this operation is unambiguous, as whenever we have the subwalk $u \leftarrow w \rightarrow v$, the edges in question are incomming to both u and v.

We claim that H is a Hamiltonian cycle of G^2. Indeed, note that for every vertex $w \neq x$, the number of times that H visits w equals the number of subwalks uwv in J containing an incomming edge of w, and there is exactly one such subwalk by the construction of $G_{\between}$ and J. Namely, each vertex $w \neq x$ had indegree either 1 or 2 in $G_{\between}$. If its indegree was 2, then those two edges will form a subwalk of J and all other edge pairs will be substituted. If its indegree was 1, then its only in-edge will appear somewhere in the Euler tour, and will not be substituted (but followed by an out-edge), while all other edge pairs will be substituted by edges in the square of G. The vertex x has degree 2 in G and in $G_{\between}$, and thus appeared in one unique place in J which was not lifted. Note that both edges of H incident with x are edges of G.

5 A Hamiltonian path in linear time

Given vertices u, v of a 2-connected graph G, it was shown by Chartrand, Hobbs, Jung, Kapoor, and Nash-Williams [2] that G^2 contains a Hamiltonian path from u to v. We shall now describe an efficient algorithm to find it.

Theorem 5.1. *There exists a linear time algorithm for finding a Hamiltonian path between any two prescribed vertices u, v in the square of a 2-connected graph G.*

Proof. We use the following trick from [2]. Take the union of five disjoint copies of G. Add two new vertices x, y. Let x be joined to the five copies of u, and let y be joined to the five copies of v. The resulting graph H is 2-connected, and therefore our algorithm can produce, in linear time, a Hamiltonian cycle C in H^2. One of the five copies of G does not contain a neighbor of x, y in C. The intersection of that copy with C is a Hamiltonian path from u to v in G^2. □

6 Cycles of all lengths in quadratic time

Hobbs [11] proved that the square of a 2-connected graph is pancyclic. Thomassen [17] proved the same under the weaker assumption that the block-cutvertex tree is a path. If G is a graph, then its *block-cutvertex tree* is the tree whose vertices are the blocks and cutvertices of G. There is an edge between a block and a cutvertex if and only if the cutvertex is contained in the block. The block-cutvertex tree can be found in linear time [16].

The first part of the following result was first proven in [17].

Lemma 6.1. *If G is a graph whose block-cutvertex tree is a path, then G^2 has a Hamiltonian cycle. Moreover, there exists a linear time algorithm for finding a Hamiltonian cycle in G^2.*

Proof. If G is 2-connected we use Theorem 1.1. So assume that G is not 2-connected. We let u, v be two non-cutvertices in distinct endblocks of G. We now use the following trick from [17]. Take the union of four disjoint copies G_1, G_2, G_3, G_4 of G. Add two new vertices x, y. Let x be joined to the two copies of u in G_1, G_2, and let y be joined to the two copies of v in G_3, G_4. Let the copy of v in G_1 (respectively G_2) be joined to the copy of u in G_3 (respectively G_4). The resulting graph H is 2-connected, and therefore our algorithm can produce, in linear time, a Hamiltonian path P between x, y in H^2. As proved in [14], the intersection of P with one of the four copies of G gives rise to a Hamiltonian cycle in G^2. □

Theorem 6.2. *There exists a $O(n^2)$ algorithm for producing cycles $C_3, C_4, \ldots, C_n$ of lengths $3, 4, \ldots n$, respectively in the square of a graph G on n vertices whose block-cutvertex tree is a path. Moreover, if x_0 is any vertex in G, then the cycles can be chosen such that $x_0 \in V(C_3) \subset V(C_4) \subset \ldots \subset V(C_n)$.*

Proof. Again, we use the idea in [17]. First, we may find the block-cutvertex graph in linear time using [16], and use the linear time algorithm of [9] on each block to obtain a spanning subgraph such that every block is minimally 2-connected. Dirac [4] proved that such a graph has at most $2n - 4$ edges. Then, it follows from Lemma 6.1 that we may find a Hamiltonian cycle C_n in linear time. If G has only one block, we delete any edge of G and use the linear algorithm in [16] to find the block-cutvertex tree which is a path. As pointed out by Dirac [3], each block is minimally 2-connected. If G is not 2-connected, we let x be a cutvertex contained in an endblock B of G. If x has degree at least 2 in B we delete any edge in B incident with x, and use the linear algorithm in [16] to find the block-cutvertex tree which is a path. Finally, if B has only two vertices x, y, then we delete y. (If $y = x_0$, we consider the other endblock of the current graph instead of B.) Then we use the algorithm in Lemma 6.1 to find a Hamiltonian cycle C_{n-1}. We repeat the argument.

We first spend $O(m)$, $m = |E|$, time obtaining a sparse graph, and then spend one iteration per edge in the sparse graph, where each iteration requires $O(n)$ time. Thus, the total time consumption is $O(m + n^2) = O(n^2)$. □

7 A modification of the algorithm

In the two previous theorems we used our initial algorithm on some modified graphs using the tricks in [2, 17]. For the sake of completeness we remark that we can instead modify the algorithm so that we can work on the graph without modifying it.

Both algorithms rely on the following lemma about the Eulerification of the graph G with ear-decomposition $C^0, \ldots, C^k$ in Section 4.

Lemma 7.1. *Given an edge e of C^0, we may choose freely whether we want to double e by introducing a parallel edge, or not, while still maintaining the property that all vertices but one have a given degree parity, all vertices different from the starting vertex have indegree 1 or 2, and the starting vertex x_0^0 has indegree 0 or 1.*

Proof. When we choose not to double the first edge of C^0, we partition all edges of C^0 in two sets; those which have been doubled, and those which have not. By interchanging these sets and doubling exactly those edges that were not doubled before, the parity of the degree of vertices is unchanged. The edge e is doubled in exactly one of them. After choosing an appropriate doubling scheme, proceed as before, either deleting both copies of the last edge $(x_{l_0-1}^0, x_0^0)$, or both copies near y^0, or none, and orient as before. □

Consider first the case where we wish to find a Hamiltonian path between two prescribed vertices u, v in the square of a 2-connected graph G. It was shown in [2] that G^2 contains a Hamiltonian path from u to v.

Second proof of Theorem 5.1. As in the proof in Section 4, we wish to find an Euler walk J between u and v. As before, we shall choose J such that any vertex of indegree 2 has both incoming edges consecutive in J. Finally, when we traverse J, lifting the subwalk $z \leftarrow w \rightarrow z'$ to the edge $(z, z') \in G^2$, we aim to obtain a Hamiltonian path P between u and v in G^2. To avoid having u, v occur twice in P, we shall ensure that each of u, v has at most one incoming edge, and any such incoming edge to u or v shall be an end of J. All other occurrences of u, v in J will be lifted.

To ensure that u and v are the only odd-degree vertices of the graph, and have indegree at most 1, we do the following. We can ensure that u, v both lie on the cycle C^0 of the proper ear decomposition by starting the depth first search of [15] with a cycle containing u and v. Then, we may choose u or v as x_0^0. We may assume without loss of generality that $x_0^0 = u$ and either $y^0 = v$, or y^0 is later than v along C^0. If $y^0 = v$, by Lemma 7.1, one may choose not to double the last edge on C^0, and then delete both copies of the edge near $y^0 = v$ that has been doubled (one of them must be doubled). If $y^0 \neq v$, by Lemma 7.1, we may choose not to double the edge after v, thus ensuring that v has indegree 1 while still maintaining that u has indegree ≤ 1.

To make sure that any incoming edges to u, v are at the ends of J, we shall first delete any incoming edge (u', u) or (v', v) from the graph without disconnecting the graph. Then we find an Euler walk J' from u' to v', and finally, extend J' with the two in-edges: $J = (u, u')J'(v', v)$. We shall ensure that deleting any in-edge near u or v will not disconnect the graph, except that one of u, v may become an isolated vertex. For, if u has an in-edge, it is part of a double edge. Deleting an edge which is part of a double edge does not affect connectedness. If $v \neq y^0$ has even degree just before we double edges of C^0 (we call that graph G_1), then its in-edge (after we have doubled edges in C^0) will be part of a double edge. Again, the deletion of such an edge does not affect connectedness. If $v = y^0$, then v has only one incident edge after doubling edges on C^0, so deleting that edge will preserve connectedness except that v becomes an isolated vertex. So, there only remains the case that v has odd degree in G_1. As we can interchange between u and v we may assume that also u has odd degree in G_1. As u has odd degree in G_1, it has also odd degree in $G_1 - E(C^0)$. Therefore u is connected by a path P in $G_1 - E(C^0)$ to another vertex x of odd degree in G_1. That vertex x must be in C^0, as all vertices in G_1 but not on C^0 have even degree in G_1. Now we apply Lemma 3.1 to the three paths in $C^0 \cup P$ between x and u. Hence we may choose y^0 and an ordering of C^0 such that y^0 is later than x which is again later than (or equal to) v. We have earlier seen that deleting the directed edge (u', u) (if it exists) does not affect connectedness. It remain to prove that deleting the directed edge (v', v) (if it exists) also does not affect connectedness. So assume that (v', v) exists. Then v' is the predecessor of v on C^0. Note that if we delete a double edge on C^0, then that edge is on the segment from x to u by the choice of y_0. This shows that, after deleting (v', v), we still have a path from v to v' using P and two appropriate paths on C^0. □

Finally we point out how to prove Lemma 6.1 directly.

Lemma 7.2. *Let G be a 2-connected graph, and let e be an edge of G. We may in linear time find a Hamiltonian cycle in $(G - e)^2$.*

Proof. First, find a minimally 2-connected spanning subgraph, G'. If $e \notin G'$, we are done. Otherwise, we may in linear time find a proper ear decomposition $C^0, \ldots, C^k$ with $e \in C^0$, and an enumeration $x_0^0, \ldots x_{l_0-1}^0$ of the vertices of C^0, such that $e = (x_{l_0-1}^0, x_{l_0}^0)$ is the last edge before x_0^0. Again, this follows from [15] by taking e as the first edge of the depth first search. Then, use Lemma 7.1 to ensure e is doubled, and thus, since it was the last edge before x, both copies are deleted. The Hamiltonian cycle found by the algorithm will now avoid the edge e. □

To prove Lemma 6.1 directly, assume G is a graph whose block-cutvertex tree is a path, and assume G is not 2-connected. Let x, y be non-cutvertices belonging to distinct endblocks. Then, $G' = G \cup (x, y)$ is 2-connected, and so by Lemma 7.2 we may find a Hamiltonian cycle in $(G' - (x, y))^2 = G^2$ in linear time.

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