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The Mirror Transform of Type I Vacua in Six Dimensions

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We study certain compactifications of the type I string on $K3$. The three topologically distinct choices of gauge bundle for the type I theory are shown to be equivalent to type IIB orientifolds with different choices of background anti-symmetric tensor field flux. Using a mirror transformation, we relate these models to orientifolds with fixed seven planes, and without any antisymmetric tensor field flux. This map allows us to relate these type I vacua to particular six-dimensional F theory and heterotic string compactifications.

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1. Introduction

String vacua in six dimensions with N=1 supersymmetry provide an interesting arena for studying non-perturbative string dynamics. Many of the difficulties that arise in four-dimensional compactifications, such as space-time superpotentials, are absent. Among the simplest ways of constructing six dimensional models are compactifications of the heterotic string, or type I string on $K3$. Some of these theories have dual realizations in terms of six-dimensional M or F theory compactifications. Orientifolds provide another interesting class of string compactifications in six dimensions.

An orientifold is a generalized orbifold where the quotient group includes a usual space-time symmetry together with world-sheet orientation reversal, and possibly some other internal symmetry transformation [1,2]. Orientifolds have proven useful for constructing models with a number of interesting features such as extra tensor multiplets, or the analogue of discrete torsion [3-7]. Type I string theory on $K3$ provides us with a special class of orientifolds in six dimensions, since these models can be regarded as type IIB on $K3$ modded out by the world-sheet orientation reversal symmetry, Ω. By the usual duality between type I and $\mathrm{Spin}(32)/Z_2$ heterotic string theory, these models are equivalent to $\mathrm{Spin}(32)/Z_2$ heterotic string theory on $K3$.

On the other hand, a type IIB orientifold can often be related to a compactification of F theory [9]. An F theory compactification is a particular background of the type IIB string described in terms of an elliptically-fibered space, which we shall take to be a Calabi-Yau manifold. The type IIB string is compactified on the base of the Calabi-Yau manifold, with the variation of the complexified string coupling specified by the variation of the complex structure modulus of the torus fiber. Alternatively, we can view this background as type IIB on a space which has positive first Chern class, but where seven-branes are used to construct a consistent vacuum. In certain cases, there are regions of the moduli space of the F theory compactification where the dilaton-axion background coincides with the background of a type IIB orientifold [10,11]. In these cases, we have two descriptions of the physics in terms of either F theory, or the orientifold. In many of these cases, we expect F theory to capture some of the non-perturbative physics of the orientifold. This is one of the reasons F theory has proven so useful. In cases where the F theory compactification has an $E_8 \times E_8$ heterotic string dual, this relation provides a means of connecting the orientifold to the $E_8 \times E_8$ heterotic string.

The orientifolds which can be related to F-theory this way all have the property that in any element of the quotient group, the world-sheet parity transformation, Ω, is always accompanied by $(-1)^{F_L}$, the internal symmetry that changes the sign of all the left Ramond states. This is due to the fact that in F-theory monodromies along closed curves on the base must be elements of the SL(2,Z) self-duality group of type IIB, and although neither Ω nor $(-1)^{F_L}$ is an element of SL(2,Z), $(-1)^{F_L} \cdot \Omega$ is an element of SL(2,Z). Thus, the type IIB orientifolds which correspond to compactification of type I string theory cannot be directly related to F-theory, since the former has Ω as one of the elements of the quotient group. However, these two classes of orientifolds can sometimes be related by using T-dualities. For example, let us consider the eight-dimensional orientifold constructed by quotienting type IIB on a two-torus by $(-1)^{F_L} \cdot \Omega$, together with an inversion of the torus. This particular orientifold can be related to type I on the two-torus by T-dualizing both circles of the torus [10]. Our discussion in this paper will focus on a generalization of this procedure to six dimensions, *i.e.* studying the relation between compactifications of the type I string on $K3$, and orientifolds of the type IIB string on $K3$ which can be related to F-theory vacua.

Understanding how to relate IIB orientifolds to type I vacua in six and lower dimensions is of interest for several reasons. Relating type I theories to the special class of orientifolds which can be related to F theory compactifications, allows us to see relations between string vacua which are otherwise hard to determine. In addition, even in six dimensions, there are regions of the moduli space of heterotic strings on $K3$ for which neither F-theory, nor the heterotic string is expected to provide a good description. For example, consider the $E_8 \times E_8$ heterotic string with $(12-n, 12+n)$ instantons embedded in the gauge group. This model has a dual description in terms of F theory compactified on a Calabi-Yau three-fold with base F_n [12]. The base of the three-fold is a P^1 fibered over a P^1, with a twist determined by the integer n. The heterotic coupling is given by the ratio of the volumes of the two P^1 factors. If we choose to keep this ratio of order one, then the perturbative heterotic string is no longer a good description of the physics. F theory provides a good description when the size of both P^1 factors is large. When both spheres have size of order unity, we expect the F theory description to receive significant corrections. However, when the F theory compactification can be related to an orientifold theory, the orientifold should continue to provide a good description of the theory in this region as long as the coupling constant in the orientifold theory remains small. If we can apply T-duality to the orientifold description to convert it into a type I compactification, we should find a description in terms of weakly coupled type I string theory for this region of the moduli space. Compactifications to four dimensions have a correspondingly richer class of degenerations, where analogous questions arise.

The cases where T-duality has been used to relate an orientifold to a type I theory have primarily been orientifolds of type IIB compactified on a torus. The aim of this paper is to relate certain compactifications of type I on $K3$ to orientifolds of type IIB on $K3$, where the $K3$ is not necessarily a toroidal orbifold. The gauge bundle of the type I theory falls into one of three topologically distinct classes, as described in [8]. In the following section, we relate these type I vacua to IIB orientifolds which may have a non-trivial $B_{\mu\nu}$ background flux, with the quotient group generated by Ω. To connect the type I vacua to F theory, we will want to get rid of the background flux, and also have the quotient group generated by $(-1)^{F_L} \cdot \Omega$, accompanied by a geometric transformation on $K3$. In section three, we describe how to use a mirror transformation to achieve both these goals. In turn, these models are in the moduli space of F theory compactifications on three-folds with base F_n, where $n = 0, 1, 4$. The case with $n = 2$ is known to be connected to the $n = 0$ model [13,12]. In this way, we will connect the type I models with these particular F theory compactifications, and their corresponding dual $E_8 \times E_8$ heterotic string compactifications. Aspinwall has recently obtained similar results using quite different techniques [14].

2. Orientifolds and Type I Vacua

To build a compactification of the Spin$(32)/Z_2$ heterotic or type I string theory on $K3$, we need to specify a Spin$(32)/Z_2$ gauge bundle. The instanton number of the bundle together with the number of five-branes filling space-time must be 24 in order to satisfy anomaly cancellation. It has been shown in [8], and subsequently in [14], that the Spin$(32)/Z_2$ gauge bundle on $K3$ can belong to one of three topological classes, characterized by an element $\widetilde{w}_2$ of $H^2(K3, Z)$, known as the "generalized second Stiefel-Whitney class." Actually, since $\widetilde{w}_2$ is defined modulo a shift by twice a lattice vector of $H^2(K3, Z)$, it is really an element of $H^2(K3, Z_2)$. We will usually view $\widetilde{w}_2$ as an integer class, which is well-defined mod 2. As in [14], we shall find it convenient to use Poincare duality to represent $\widetilde{w}_2$ as an element of $H_2(K3, Z)$. The physical significance of $\widetilde{w}_2$ can be described as follows. Let C denote a two sphere in $K3$, and C_N and C_S denote its northern and southern hemispheres. Let g_N and g_S denote the holonomies along the boundaries of C_N and C_S in the *vector representation* of $SO(32)$. Since the boundaries of C_N and C_S denote the same curves, $g_N g_S^{-1}$ must be unity in Spin$(32)/Z_2$, but not necessarily so in $SO(32)$.

$\widetilde{w}_2$ associated with the gauge bundle is defined such that in the vector representation of $SO(32)$,

$$g_N g_S^{-1} = \exp(i\pi\widetilde{w}_2 \cdot C), \tag{2.1}$$

where $\cdot$ denotes intersection number. Just as the usual second Stiefel-Whitney class describes an obstruction to a bundle admitting spinors, $\widetilde{w}_2$ describes an obstruction to a bundle admitting vectors. It is worth recalling that $H_2(K3, Z)$ is an even, self-dual lattice. Up to diffeomorphism, there are then three independent topological classes of gauge bundles, which correspond to the cases [8,14],

1. $\widetilde{w}_2 = 0$,
2. $\widetilde{w}_2 \cdot \widetilde{w}_2 = 0 \bmod 4$, and
3. $\widetilde{w}_2 \cdot \widetilde{w}_2 = 2 \bmod 4$.

The first case corresponds to a conventional $SO(32)$ bundle, while the second is pertinent to the models described in [2], when the fixed-points are blown up. We will meet all three cases in the following discussion.

Using a well known fact, we can regard the type I theory on $K3$ as type IIB theory on $K3$ modded out by the world-sheet parity transformation, Ω. In order to be able to mod out the type IIB theory by Ω, we must ensure that the original field configuration is invariant under Ω. Since the rank two anti-symmetric tensor $B_{\mu\nu}$ originating in the Neveu-Schwarz Neveu-Schwarz sector is odd under Ω, one might naively think that the components of $B_{\mu\nu}$ along $K3$ need to be set to zero in order to get an Ω invariant configuration. However, this is not strictly necessary. To see this, note that the components of B along $K3$ belong to $H^2(K3, R)$ modulo elements of $H^2(K3, Z)$ because of the periodicity of the flux of $B_{\mu\nu}$. Using Poincare duality we can take these to be elements of $H_2(K3, R)$ modulo elements of $H_2(K3, Z)$. In this notation, if B is $\Lambda/2$ for some element Λ of $H_2(K3, Z)$, then it is invariant under Ω, since $\Lambda/2$ and $-\Lambda/2$ are identified under the action of $H_2(K3, Z)$. Thus, we can mod out such a configuration of type IIB theory on $K3$ by Ω to get an unconventional type I theory on $K3$. Since in this theory, such a B-flux cannot be continuously deformed to zero, we shall refer to this as a discrete B-flux. We shall now show that the presence of such a flux actually leads us to a type I theory with $\widetilde{w}_2 = \Lambda$.

Let C be any two sphere inside $K3$, and let us consider a world-sheet instanton in type I theory, which represents an elementary string world-sheet with spherical topology wrapped once around C. In the presence of a B-flux, $\Lambda/2$, through the two-cycles of $K3$, the phase associated with such an instanton will be given by,

$$\exp(i\pi\Lambda \cdot C). \tag{2.2}$$

This instanton contribution represents an amplitude for a closed string to be produced from the vacuum, then propagate for a finite interval of time during which it wraps around C, and finally disappear into the vacuum. Now, since in type I string theory, a closed string carries the same quantum number as a pair of open strings with their ends glued to each other, we should be able to calculate this phase by considering the propagation of a pair of open strings, spanning the northern and the southern hemispheres C_N and C_S respectively. This corresponds to replacing the sphere by two disks, spanning C_N and C_S. The phases associated with these disk amplitudes are given by g_N and g_S^{-1} respectively.[1] Thus, the total phase is given by $g_N g_S^{-1}$. This, in turn, is given by (2.1). Comparing (2.1) and (2.2), we now find that

$$\widetilde{w}_2 = \Lambda, \tag{2.3}$$

up to a shift by twice a lattice vector in $H_2(K3, Z)$.

3. The Mirror Transformation

Now that we have identified a type IIB orientifold that corresponds to a type I compactification with non-trivial $\widetilde{w}_2$, we want to dualize the model to obtain an orientifold without any discrete B-flux, and with the orientifold group generated by $(-1)^{F_L} \cdot \Omega$ together with an involution on the new $K3$. For this we need to review a few facts about the conformal field theory describing propagation of type II strings on $K3$. Let us recall that $H_2(K3, Z)$ has the structure of an even self-dual lattice $\Gamma^{(3,19)}$ with signature (3,19), which can be decomposed in the following way:

$$\Gamma^{(3,19)} = 3\,\Gamma^{(1,1)} \oplus 2\,\Gamma^{(E_8)}. \tag{3.1}$$

In the conformal field theory which describes type II strings on $K3$, it is natural to introduce a lattice $\Gamma^{(4,20)}$ of signature (4,20) by adding an extra copy of $\Gamma^{(1,1)}$ to $\Gamma^{(3,19)}$:

$$\Gamma^{(4,20)} = \Gamma^{(1,1)} \oplus \Gamma^{(3,19)}, \tag{3.2}$$

[1] We get g_S^{-1} instead of g_S since the ends of the open string spanning C_S carry charge opposite to that spanning C_N.

where this extra $\Gamma^{(1,1)}$ can be associated with the 0- and the 4-cycles of $K3$ [15]. Locally, the moduli space of this conformal field theory can be identified with the Grassmannian of 4-planes of signature (4,0) in $R^{(4,20)}$, and is parametrized by an $O(4,20)$ matrix N modulo multiplication from the right by an $O(4) \times O(20)$ matrix [16]. For a given N, we obtain a 4-plane of signature (4,0) in this twenty-four dimensional space by using the projection operator:

$$\frac{1}{2}(LNN^T + I), \tag{3.3}$$

where L is the metric of signature (4,20) and I is the 24×24 identity matrix. To see that this is a projection operator, note that by definition, $N^T LN = L$. After rotating the lattice using the $O(4,20)$ matrix N^T, we can turn this projection operator into the operator $(I + L)$, which makes it clear that the operator (3.3) defines a (4,0) subspace [17]. We shall find it convenient to use this description.

Using the freedom of multiplying N by an $O(4) \times O(20)$ matrix from the right, we can choose N to be of the form:

$$N = N_B N_S N_K, \tag{3.4}$$

where N_K is an element of $O(3,19)$ that rotates the basis vectors of $\Gamma^{(3,19)}$, N_S is an element of $O(1,1)$ that rotates the basis vectors of $\Gamma^{(1,1)}$, and N_B can be represented as an upper triangular matrix[2] that mixes the basis vectors of $\Gamma^{(1,1)}$ with the basis vectors of $\Gamma^{(3,19)}$. Then the parameter labelling N_S can be identified as the size of $K3$, those labelling N_K correspond to the rest of the geometric moduli of $K3$, while the parameters appearing in N_B characterize the B-flux.

Let us now choose a $\Gamma^{(1,1)}$ component of the lattice $\Gamma^{(3,19)}$ that is orthogonal to Λ. In order not to confuse this with the part of the lattice associated with the 0- and 4- cycles, we shall denote this by $\Gamma^{(1,1)\prime}$. Orthogonality of Λ to $\Gamma^{(1,1)\prime}$ guarantees that N_B does not act on $\Gamma^{(1,1)\prime}$. Furthermore, by continuously adjusting the geometric moduli of $K3$, we can ensure that N_K does not act on $\Gamma^{(1,1)\prime}$. In that case N leaves $\Gamma^{(1,1)\prime}$ invariant. Now, let us make a mirror transformation that converts the pair of two cycles associated with $\Gamma^{(1,1)\prime}$ into the zero and four cycles of the mirror $K3$, which we shall denote by $K3'$. In this case invariance of $\Gamma^{(1,1)\prime}$ under N can be interpreted as the absence of any background B-flux

[2] In order to represent N_B as an upper triangular matrix we need to choose the two basis vectors of $\Gamma^{(1,1)}$ as the first and the last row/column, with the basis vectors of $\Gamma^{(3,19)}$ labelling the rest of the rows and columns. Later in eq. (3.8), we shall construct this matrix explicitly for a given B-flux.

in this mirror transformed model. If we had started from a generic point in the moduli space of the original $K3$ then after the mirror transformation, we would have got a $K3'$ with B-flux, but this flux will not be discrete in the sense described earlier, since it can be continuously deformed to zero, while maintaining invariance under the mirror image of Ω. Hence, we have arrived at a model without any discrete B-flux.

This mirror transformation converts the four cycle and the zero cycle of the original $K3$ into a two cycle and its dual two cycle of the mirror $K3'$. Studying the action of various discrete symmetries on the massless fields in the theory, one can easily verify that the mirror transformation conjugates Ω to

$$(-1)^{F_L} \cdot \Omega \cdot \sigma, \tag{3.5}$$

where σ is a Nikulin involution of $K3'$ that leaves this particular pair of two cycles invariant, but changes the sign of all other two cycles [18].[3] The question we shall be addressing is the following: if we start from a particular B flux background $\Lambda/2$ in the original type IIB theory, then which particular Nikulin involution do we get after the mirror transformation?

Let us recall that a Nikulin involution is characterized by three integers (r, a, δ) defined as follows. If S_+ denotes the sublattice of $H_2(K3', Z)$ that is invariant under σ, and S_+^* denotes the dual lattice of S_+, then

1. r denotes the rank of S_+,
2. a denotes that S_+^*/S_+ has the structure of $(Z/2Z)^a$ and,
3. $\delta = 0$ if x^2 is integer for all $x \in S_+^*$, otherwise $\delta = 1$.

For our purpose, it will be useful to translate this into a statement about the action of $(-1)^{F_L} \cdot \Omega \cdot \sigma$ on the lattice of allowed charges under the rank two tensor gauge fields arising in the Ramond-Ramond (RR) sector of the theory. There are twenty-four such (anti-)self-dual tensor fields coming from the RR sector, of which twenty-two come from the components of the rank four RR gauge field $D_{\mu\nu\rho\sigma}$ along the two-cycles of $K3'$. The

[3] To see this, note that in the original six dimensional theory, the only rank two anti-symmetric tensor fields invariant under Ω come from the Ramond-Ramond antisymmetric tensor field $B'_{\mu\nu}$ and its magnetic dual. Under the mirror transformation these get mapped to the components of the rank two anti-symmetric tensor field $D_{\mu\nu\rho\sigma}$ along this particular pair of two cycles in $K3'$. Since $D_{\mu\nu\rho\sigma}$ is even under $(-1)^{F_L} \cdot \Omega$, σ must leave this particular pair of two cycles invariant and change the sign of all other two cycles.

charges associated with these fields clearly belong to the lattice $H_2(K3', Z)$. The other two come from the RR two-form gauge field $B'_{\mu\nu}$ and its magnetic dual. The allowed charges associated with these fields belong to the self-dual Lorentzian lattice $\Gamma^{(1,1)'}$. Together they span a lattice $\Gamma^{(4,20)'}$, which of course is isomorphic to the the lattice of charges $\Gamma^{(4,20)}$ of the original type IIB theory on $K3$.

Now the RR four-form field is invariant under $(-1)^{F_L} \cdot \Omega$. On the other hand, the RR two-form field is odd under this transformation, since it is odd under $(-1)^{F_L}$ but invariant under Ω. Thus acting on the lattice of tensor field charges, $(-1)^{F_L} \cdot \Omega \cdot \sigma$ acts as σ on the $H_2(K3', Z)$ part of the lattice, and reverses the sign of the $\Gamma^{(1,1)'}$ part of the lattice. Thus S_+, which was earlier defined as the sublattice of $H_2(K3', Z)$ invariant under σ, can also be regarded as the sublattice of $\Gamma^{(4,20)'}$ invariant under $(-1)^{F_L} \cdot \Omega \cdot \sigma$.

So far we have not learned anything new, but now we shall use the fact that a mirror transformation acts as an automorphism of the charge lattice [15]. Thus S_+ can also be regarded as the sublattice of $\Gamma^{(4,20)}$ invariant under Ω in the original type IIB theory, since Ω, by construction, is the image of $(-1)^{F_L} \cdot \Omega \cdot \sigma$ under the mirror map. This allows us to forget all reference to the mirror $K3'$ and work with the original type IIB theory on $K3$. First let us consider the case where there is no B flux in the original theory. Since Ω changes the sign of the rank four gauge-field, but leaves the rank two RR tensor $B'_{\mu\nu}$ invariant, its action on the charge lattice will be to change the sign of the $H_2(K3, Z)$ part of the lattice, leaving the $\Gamma^{(1,1)}$ part associated with the zero and four cycles invariant. Thus we can identify S_+ with $\Gamma^{(1,1)}$. Since S_+ is an even self-dual lattice of signature (1,1), we find, from the definitions of (r, a, δ), that in this case,

$$(r, a, \delta) = (2, 0, 0). \tag{3.6}$$

Let us now consider the case where there is a background B-flux equal to $\Lambda/2$, for some element Λ of $H_2(K3, Z)$. Let e_1 and e_2 denote the basis vectors of $\Gamma^{(1,1)}$ associated with the zero and the four cycles (*i.e.* $B'_{\mu\nu}$ charges) with the inner product matrix:

$$e_1^2 = e_2^2 = 0, \quad e_1 \cdot e_2 = 1. \tag{3.7}$$

Also let Λ, together with a set of vectors f_i, be the generators of $H_2(K3, Z)$. Note that since Λ is defined modulo a shift by twice a lattice vector, we can always choose Λ to be a primitive vector of the lattice. The $O(4,20)$ matrix N_B representing the effect of this B-flux induces the following rotation of the basis vectors,

$$\begin{aligned} e_1' &= e_1, \\ e_2' &= e_2 + \frac{1}{2}\Lambda - \frac{1}{8}\Lambda^2 e_1, \\ \Lambda' &= \Lambda - \frac{1}{2}\Lambda^2 e_1, \\ f_i' &= f_i - \frac{1}{2}(f_i \cdot \Lambda) e_1. \end{aligned} \tag{3.8}$$

This rotation preserves the inner products between different basis vectors, reflecting the fact that it is an $O(4,20)$ rotation. The charge vectors lie on the new lattice generated by the primed vectors; however $B'_{\mu\nu}$, its magnetic dual, and the components of $D_{\mu\nu\rho\sigma}$ along the two cycles of $K3$ continue to couple to the components of the charge vector along the unprimed vectors e_1, e_2 and $\{\Lambda, f_i\}$ respectively. The vectors e_1' and e_2' generate a $\Gamma^{(1,1)}$ component of the new lattice. However, the action of Ω no longer leaves this $\Gamma^{(1,1)}$ invariant, since it leaves e_1 and e_2 (*not* e_1' and e_2') invariant, changing the signs of Λ and the f_i's. In other words, if we consider an element of the new $\Gamma^{(4,20)}$ lattice of the form:

$$n_1 e_1' + n_2 e_2' + n\Lambda' + m_i f_i', \tag{3.9}$$

with n_i, m_i, n integers, then invariance under Ω requires that this vector must lie in the e_1-e_2 plane. Using (3.8), this gives the following constraints:

$$m_i = 0, \qquad n + \frac{1}{2}n_2 = 0. \tag{3.10}$$

Using equations (3.8) and (3.10), we can rewrite (3.9) as,

$$(n_1 - \frac{1}{4}n\Lambda^2)e_1 - 2ne_2. \tag{3.11}$$

Therefore, S_+ is the lattice of vectors of the form given in (3.11), for integer n and n_1. Clearly this has rank two, so here $r = 2$. With the help of the inner product (3.7) and the fact that Λ^2 is even, we see that a general element of the dual lattice S_+^* is of the form:

$$\frac{1}{2}(k_1 - \frac{1}{4}k\Lambda^2)e_1 - ke_2, \tag{3.12}$$

for integer k_1 and k. Comparing (3.11) and (3.12), we see that S_+^*/S_+ is isomorphic to $(Z/2Z)^2$. This gives $a = 2$. Finally, the inner product of the element (3.12) with itself is given by

$$-k(k_1 - \frac{1}{4}k\Lambda^2). \tag{3.13}$$

Thus if $\Lambda^2 = 0 \bmod 4$, this inner product is always integer and we have $\delta = 0$. On the other hand, if $\Lambda^2 = 2 \bmod 4$, then this inner product is half integer for odd k, and we have $\delta = 1$.

Using the identification of Λ with $\widetilde{w}_2$, we now have the following identification between type I compactification on $K3$ and type IIB on $K3/(-1)^{F_L} \cdot \Omega \cdot \sigma$, with σ denoting Nikulin involution (r, a, δ):

1. $\widetilde{w}_2 = 0$: (r, a, δ)=(2,0,0).
2. $\widetilde{w}_2 \cdot \widetilde{w}_2 = 0 \bmod 4$: (r, a, δ)=(2,2,0).
3. $\widetilde{w}_2 \cdot \widetilde{w}_2 = 2 \bmod 4$: (r, a, δ)=(2,2,1).

The orientifolds appearing in the above relations are known to be dual to F-theory on elliptically fibered Calabi-Yau 3-folds on base F_4, F_0 and F_1, respectively [12,11]. Had we arrived at any other Nikulin actions and hence different orientifolds, we would certainly have been in trouble. The type I models with which we started had a single tensor multiplet, but F theory compactified on other Voisin-Borcea spaces has more (or in some cases less) than one tensor multiplet. The particular F theory models that we have found are, in turn, dual to the $E_8 \times E_8$ heterotic string compactified on $K3$, with the 24 instantons embedded in the two E_8 factors of the gauge group according to the distribution (8,16), (12,12) and (11,13), respectively [12]. Finally, using T-duality between the two heterotic string theories, these models can be identified with the Spin(32)/Z_2 heterotic string theory on $K3$, with $\widetilde{w}_2 = 0$, $\widetilde{w}_2 \cdot \widetilde{w}_2 = 0 \bmod 4$ and $\widetilde{w}_2 \cdot \widetilde{w}_2 = 2 \bmod 4$, respectively [8,14]. Therefore, we see that, using the T-duality relations we have discovered, we have travelled a full circle in duality transformations via the route type I - orientifold - F-theory - $E_8 \times E_8$ heterotic - Spin(32)/Z_2 heterotic - type I compactifications.

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