# Proposal for a loophole-free Bell test using homodyne detection

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# GOAL: Finding a loophole-free test of Bell inequalities using homodyning

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| Bell test = evidence of the incompatibility between quantum mechanics and "local realism" |

1. Locality loophole : the experimental data admit a local realistic description if communication between the parties is possible

$\Rightarrow$ Alice's and Bob's detection events must be spacelike separated $\Rightarrow$ use photons

*e.g. G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett.* **81**, *5039 (1998).*

2. Detection loophole : the experimental data can be explained by local realistic theories wherein the detectors only click with probability $\eta$

$\Rightarrow$ the detector efficiency $\eta$ must be high

*e.g. M.A. Rowe, D. Kielpinski, V. Meyer, C.A. Sackett, W.M. Itano, C. Monroe, and D.J. Wineland, Nature* **409**, *791 (2001).*

EPR light beams and highly efficient homodyne detection may circumvent both loopholes, but ...

## The original EPR state cannot exhibit nonlocal effects with homodyning

$$|\text{EPR}\rangle = \int_{-\infty}^{\infty} e^{i(x_a - x_b)p}\, dp \;\sim\; \left\{ \begin{array}{l} \delta(x_a - x_b) \\ \delta(p_a + p_b) \end{array} \right.$$

Regularized as a 2-mode squeezed vacuum state

$$|\text{EPR}\rangle \simeq \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n \, |n, n\rangle \quad \text{with } \lambda = \tanh(r)$$

generated with optical parametric amplifier (OPA)

Wigner function $W(r) = \frac{\pi^{-2}}{\sqrt{\det \gamma}} \exp\left[-r^T \gamma^{-1} r\right] > 0$

with $r = (x_a, p_a, x_b, p_b)^T$ and $\gamma_{ij} = \langle r_i r_j + r_j r_i \rangle$

$W$ provides an explicit local hidden-variable model for homodyne measurements $[x_a(\theta), x_b(\phi)]$

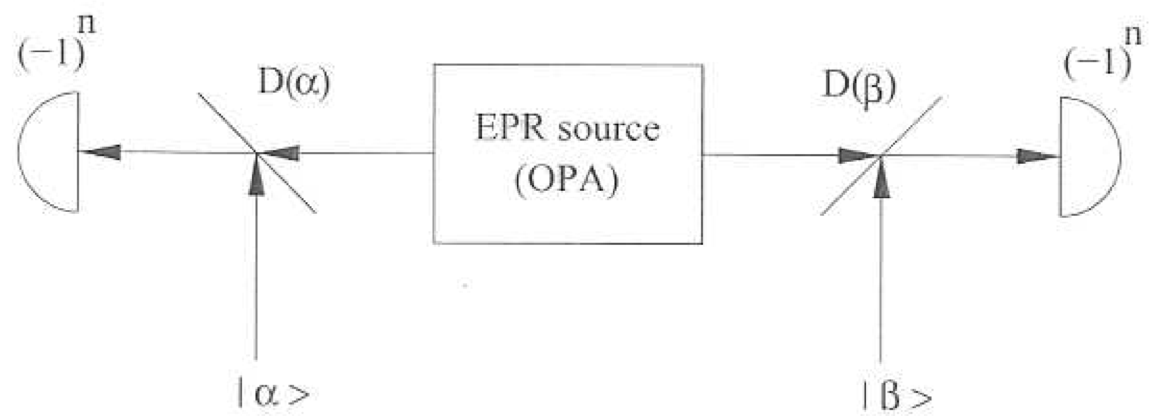
$\Rightarrow$ Explore $\left\{ \begin{array}{l} \text{(1) non-gaussian measurements} \\ \text{(2) non-gaussian states} \end{array} \right.$

# (1) Non-gaussian measurements

Use EPR state but measurement of the photon number parity $(-1)^{\hat{n}_a+\hat{n}_b}$

*K. Banaszek and K. Wódkiewicz, PRA 58, 4345, 1998*

Use $W(x,p) = \mathrm{tr}(\rho \hat{W})$ with $\hat{W} \propto \hat{D}(x,p)(-1)^{\hat{n}}\hat{D}^\dagger(x,p)$




$$E(\alpha,\beta) = \langle\, \underbrace{\hat{D}(\alpha)(-1)^{\hat{n}_a}\hat{D}^\dagger(\alpha)}_{Alice's\ parity} \otimes \underbrace{\hat{D}(\beta)(-1)^{\hat{n}_b}\hat{D}^\dagger(\beta)}_{Bob's\ parity} \,\rangle$$

$S = E(0,0) + E(J,0) + E(0,J) - E(J,J)$
violates CHSH inequality $|S| \leq 2$ by $\sim 10$ %
although $W > 0$

Detection loophole...
Experimental feasibility...

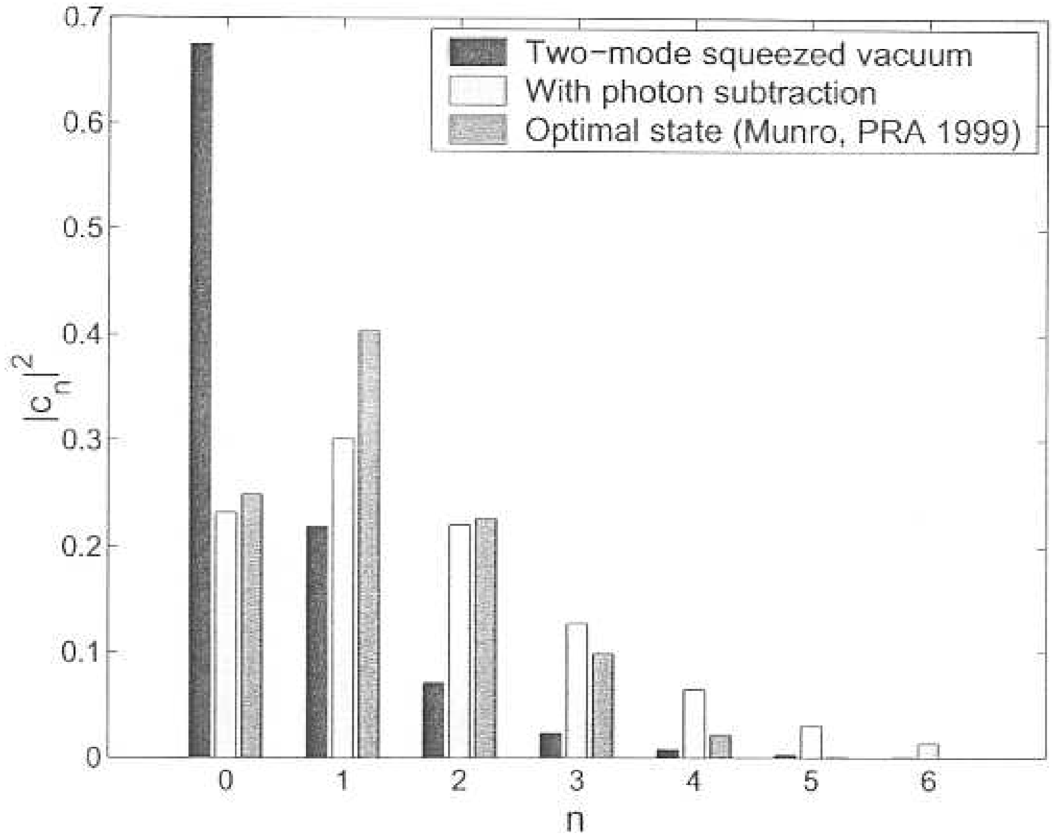
## (2) Non-gaussian states

Use more "exotic" states that have $W \ngeq 0$ (necessary but not sufficient condition)

*e.g. W.J. Munro, PRA 59, 4197, 1999*

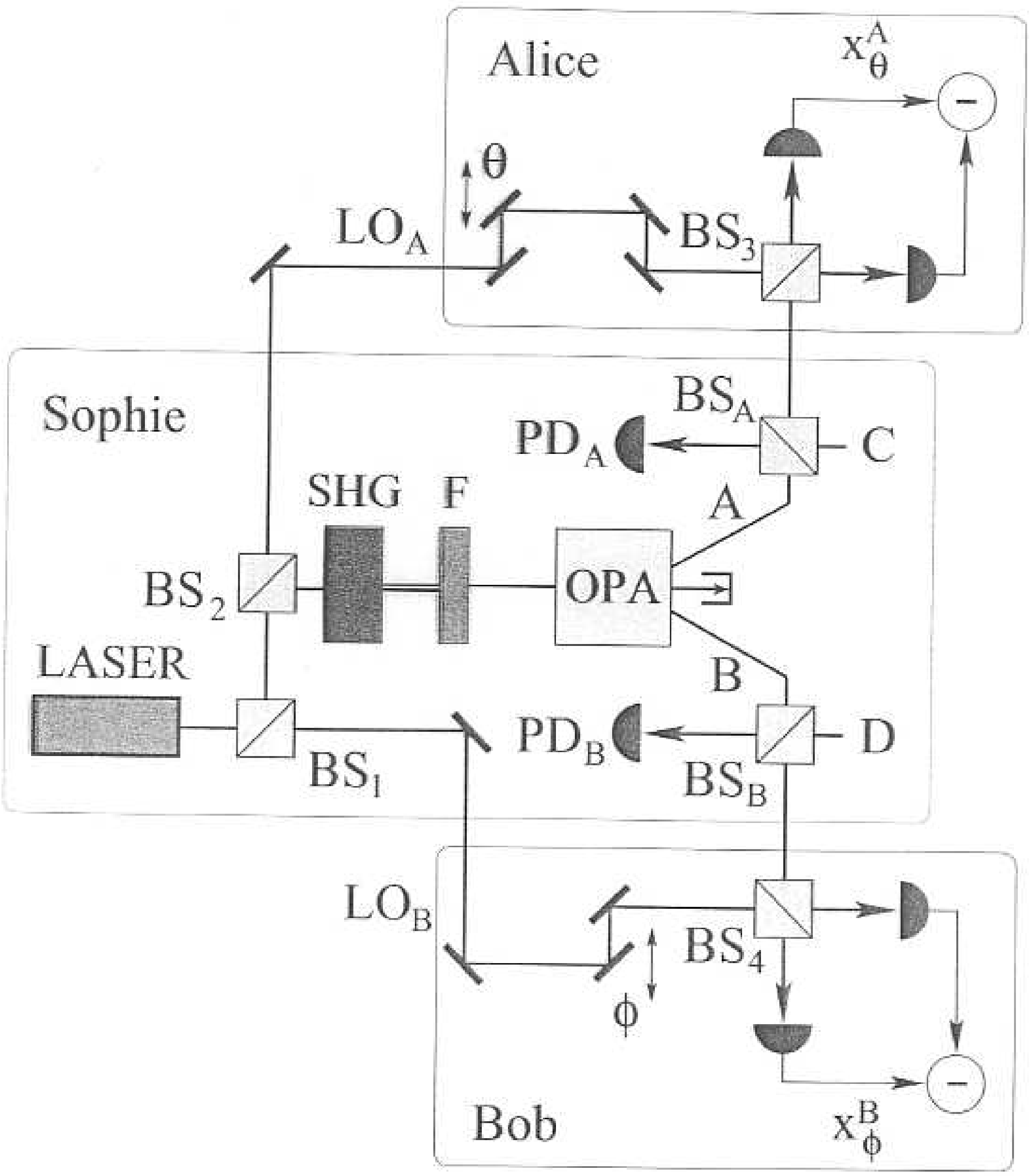
$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n \, |n,n\rangle$$ (correlated photon-number states)

Binarization of measured quadrature: sign$(x)$
Optimal source violates CHSH by $\sim 3.8$ %




Experimental feasibility...

Find a reasonable compromise between experimental feasibility and stringent requirements of loophole-free Bell test

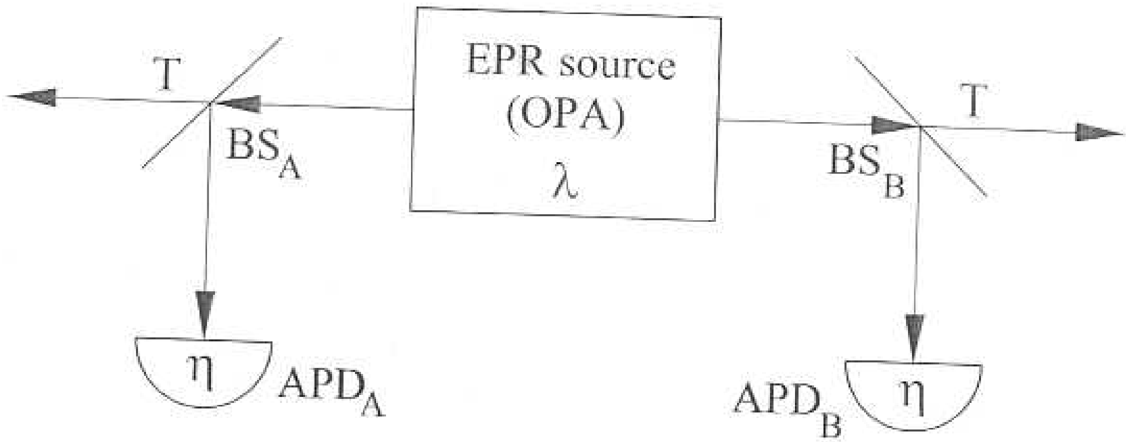



Sophie prepares a non-gaussian state by subtracting a photon on each mode, conditionally on a double-click

Alice measures $x_a(\theta_1)$ or $x_a(\theta_2)$ $\rightarrow$ binarizes it   sign ($x_a$)

Bob measures $x_b(\phi_1)$ or $x_b(\phi_2)$ $\rightarrow$ binarizes it   sign ($x_b$)

Simplified model (ideal photodetector $\eta = 1$)




$$|\Psi_{\text{in}}(\lambda)\rangle = \sqrt{1-\lambda^2} \sum_{n=0}^{\infty} \lambda^n \, |n,n\rangle$$

$$\begin{aligned} |\Psi_{\text{out}}\rangle &\propto \hat{a}_A \, \hat{a}_B \, |\Psi_{\text{in}}(T\lambda)\rangle \\ &\propto \sum_{n=0}^{\infty} (n+1)(T\lambda)^n \, |n,n\rangle \end{aligned}$$

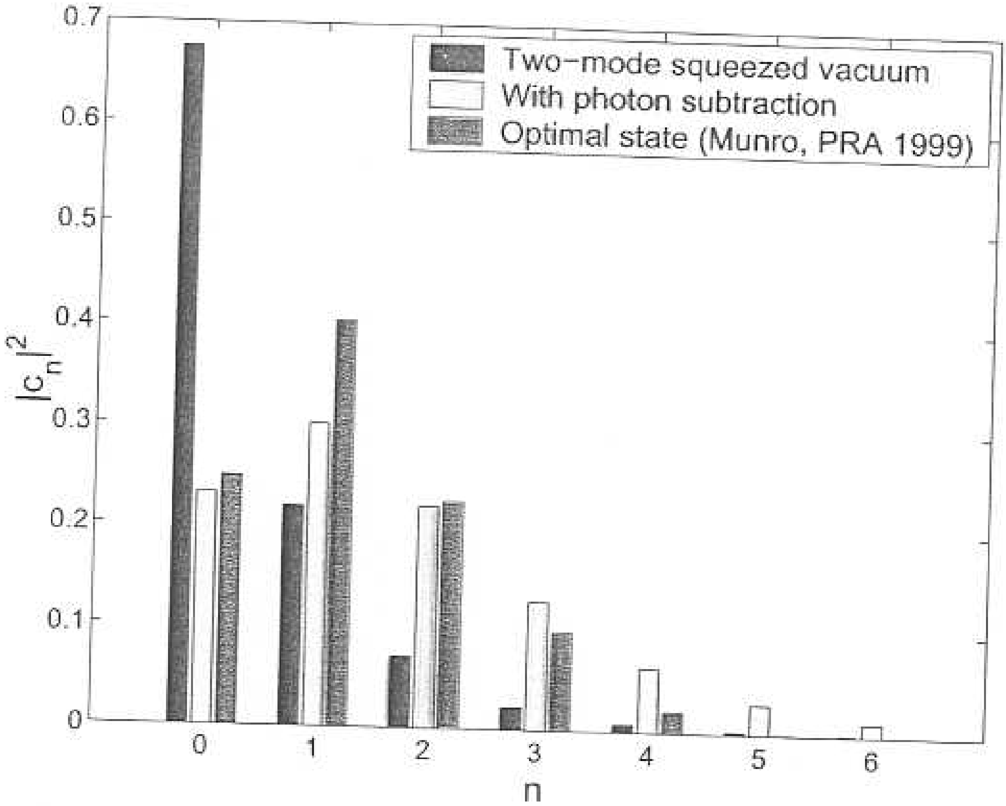
$T\lambda =$ effective squeezing

Reduce squeezing $\lambda$ by taking $T \to 1$
(then, imperfect APD $\simeq$ ideal APD)

Works well with imperfect APDs ($\eta \simeq 10$ %)
with no single-photon resolution

$P_{\text{APD}} \propto (1-T)^2$ so there is a compromise with the need to accumulate enough statistics

$$|\Psi_{\text{out}}\rangle \propto \sum_{n=0}^{\infty} \underbrace{(n+1)(T\lambda)^n}_{c_n} |n,n\rangle \quad \text{with } T\lambda = 0.57$$




The photon subtraction reduces the relative contribution of the vacuum, making $|\Psi_{\text{out}}\rangle$ more "non-gaussian", and thus a good candidate for a Bell test

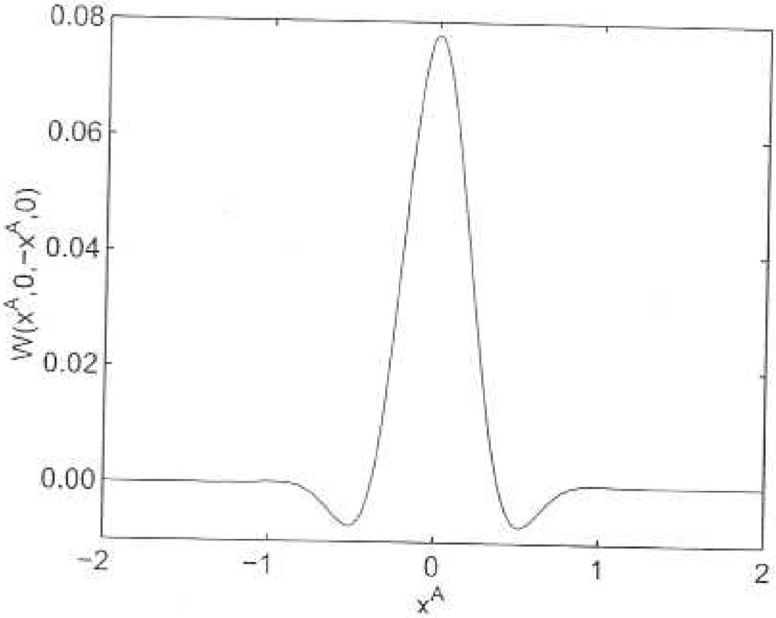
$$E(\theta,\phi) = \int_{-\infty}^{\infty} \text{sign}(x_\theta^A x_\phi^B)\; P(x_\theta^A, x_\phi^B)\; dx_\theta^A\, dx_\phi^B$$

with $P(x_\theta^A, x_\phi^B) \equiv |\langle x_\theta^A, x_\phi^B | \Psi_{\text{out}}\rangle|^2$

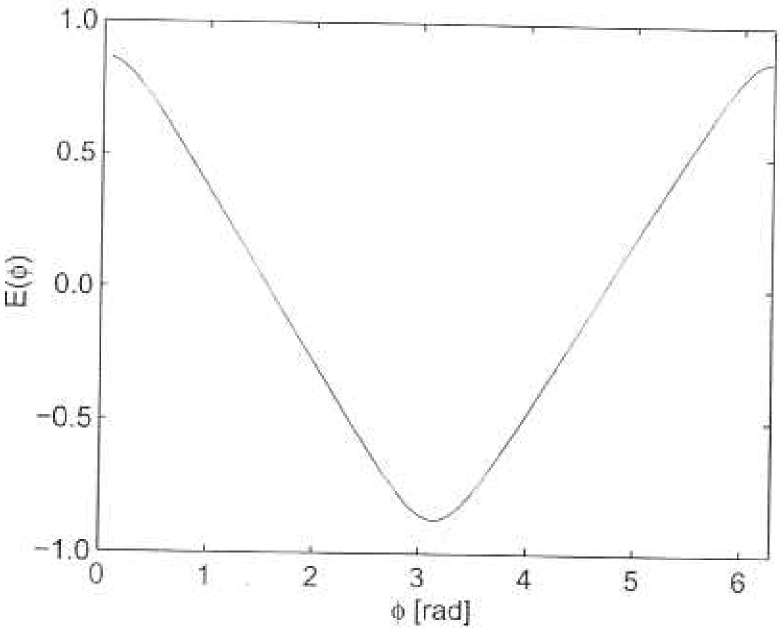
$$S = E(\theta_1,\phi_1) + E(\theta_1,\phi_2) + E(\theta_2,\phi_1) - E(\theta_2,\phi_2)$$

## Case $T\lambda = 0.5$

One-dim cut of $W(x_a, p_a, x_b, p_b)$ along the line $x_a = -x_b$, $p_a = p_b = 0$ for the non-gaussian state $|\Psi_{\text{out}}\rangle$

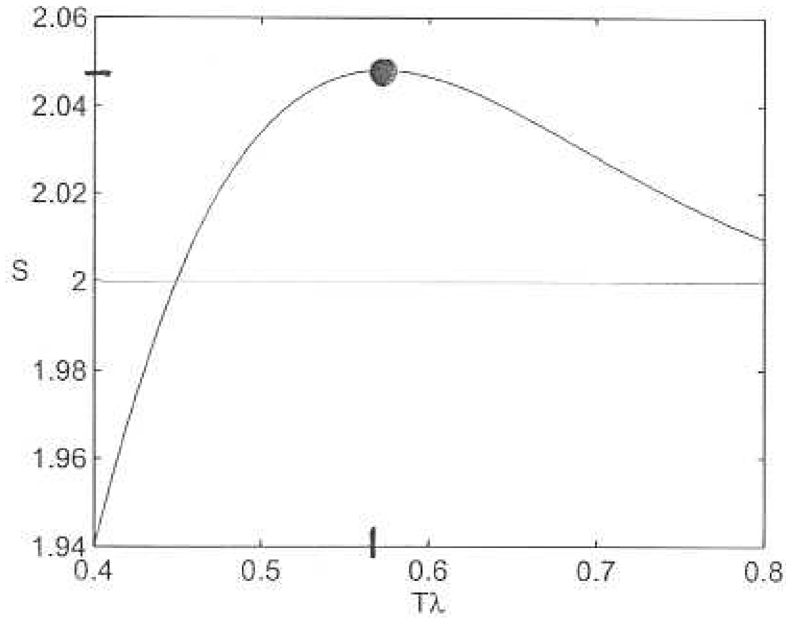



Dependence of the correlation function $E(\theta, \phi)$ on the phase-sum $\varphi = \theta + \phi$

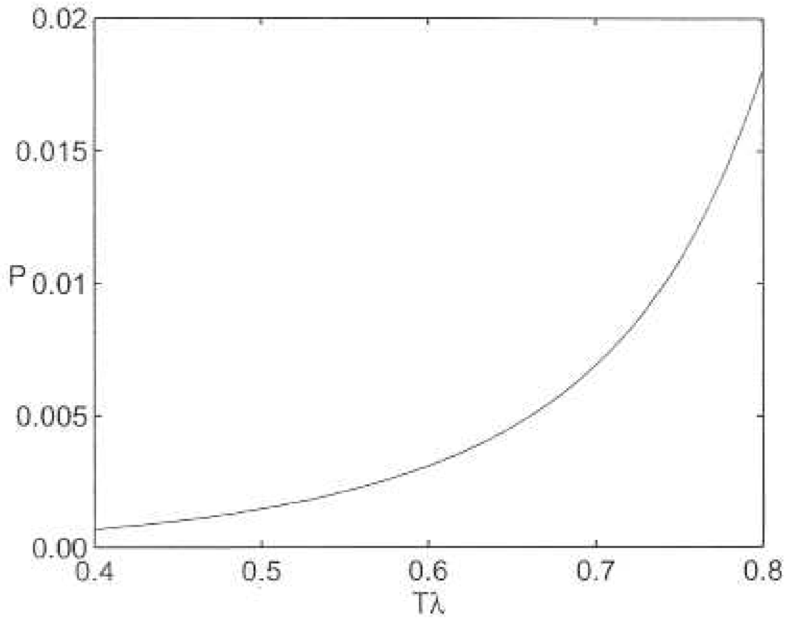


Maximum violation of $\sim 2.3$ % for $T\lambda \simeq 0.57$
e.g. $T \simeq 0.95$ and $\lambda \simeq 0.60$ (6 dB)

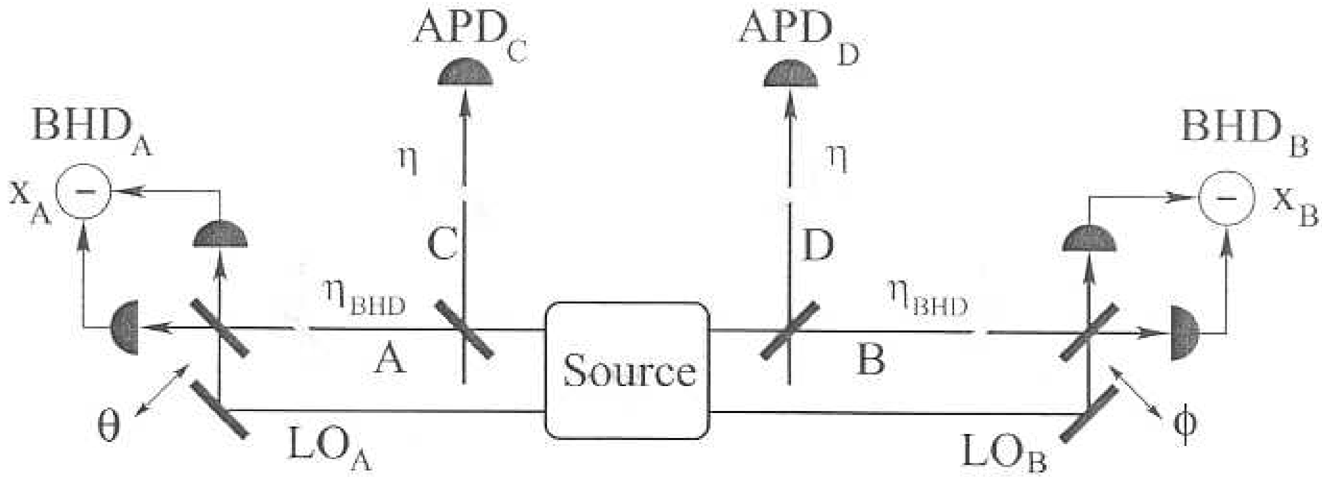
$\star$ optimal angles $\theta = 0$, $\pi/2$ and $\phi = -\pi/4$, $\pi/4$




Probability of success



Exact calculation (imperfect APD & BHD)




Gaussian completely positive map:

$$\gamma_{\text{in}} = \gamma_{AB}^{\text{EPR}} \oplus I_{CD}$$

$$\gamma_{\text{in}} \longmapsto \gamma_{\text{out}} = S_2\, S_1\, \gamma_{\text{in}}\, S_1{}^T\, S_2{}^T \,+\, G_2$$

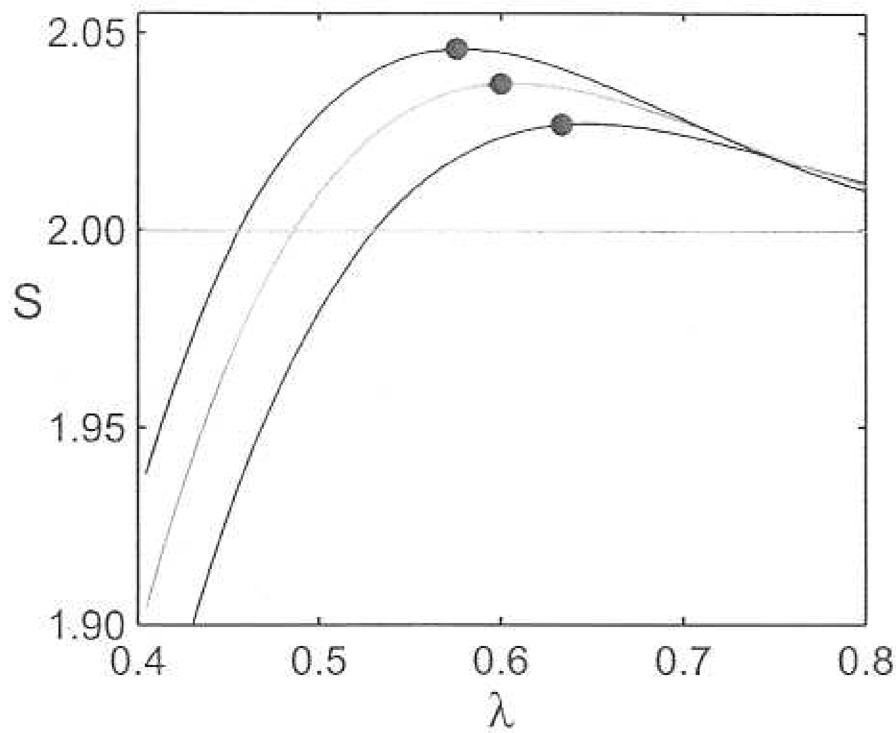
with $S_1 = \text{BS}_{AC} \oplus \text{BS}_{BD}$
$S_2 = \sqrt{\eta_{\text{BHD}}}\, I_{AB} \oplus \sqrt{\eta}\, I_{CD}$
$G_2 = (1-\eta_{\text{BHD}})\, I_{AB} \oplus (1-\eta)\, I_{CD}$

$$\text{Tr}_{CD}\Big\{\rho_{\text{out}}\, \Big[I_A \otimes I_B \otimes (I_C - |0\rangle\langle 0|) \otimes (I_D - |0\rangle\langle 0|)\Big]\Big\}$$

$$\Rightarrow (2\pi)^2 \int \underbrace{W_{\text{out}}(\cdots)}_{\gamma_{\text{out}}}\, O(\cdots)\, dx_C\, dp_C\, dx_D\, dp_D$$

$\Rightarrow$ linear combination of 4 gaussian Wigner f.
$\Rightarrow$ analytical calculation of $E(\theta,\phi)$, hence of $S$

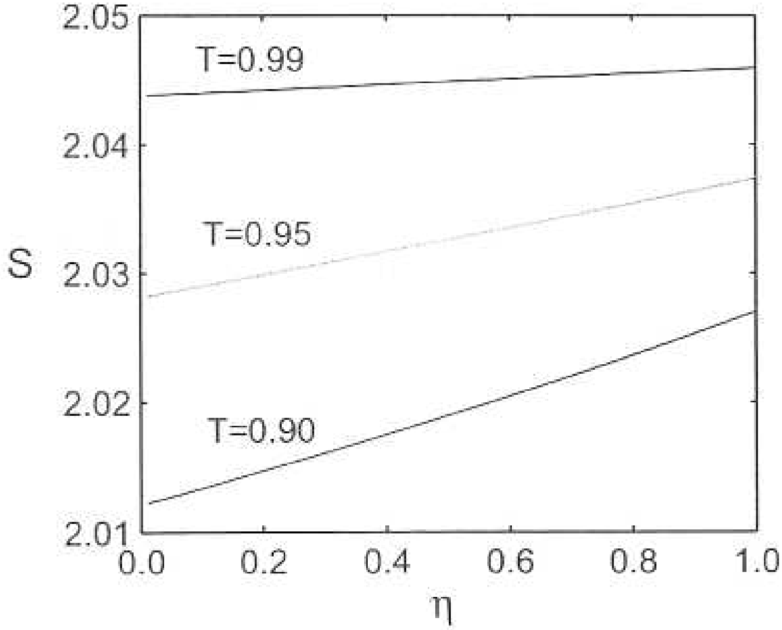
Violation of CHSH inequality $|S| \leq 2$
(perfect detectors $\eta = \eta_{\text{BHD}} = 1$)



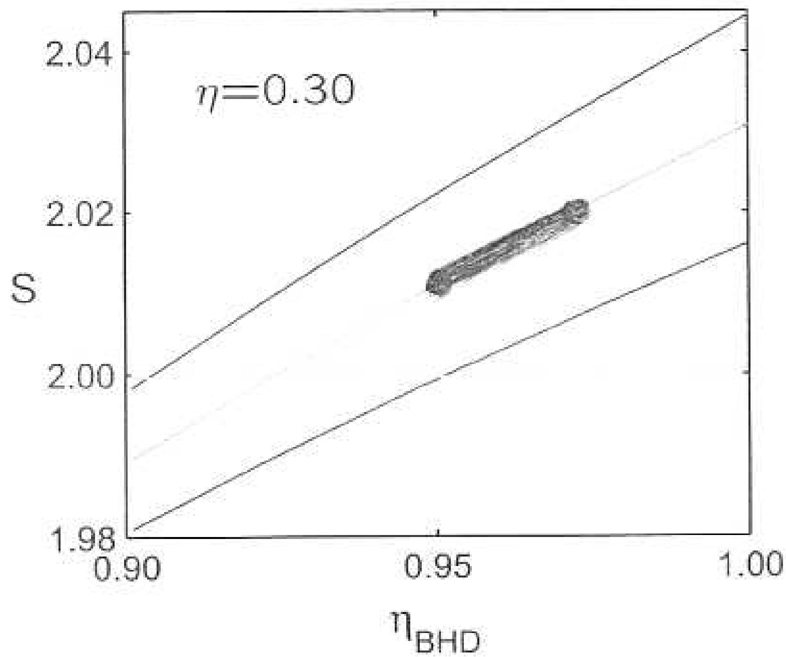

$T = 0.99 \rightarrow$ 5.6 dB (2.3 % violation)
$T = 0.95 \rightarrow$ 6.0 dB (1.9 % violation)
$T = 0.90 \rightarrow$ 6.4 dB (1.4 % violation)

- simplified model ($T\lambda \simeq 0.57$)

Imperfect photodetectors ($\eta < 1$)




Imperfect homodyne detectors ($\eta_{\mathrm{BHD}} < 1$)



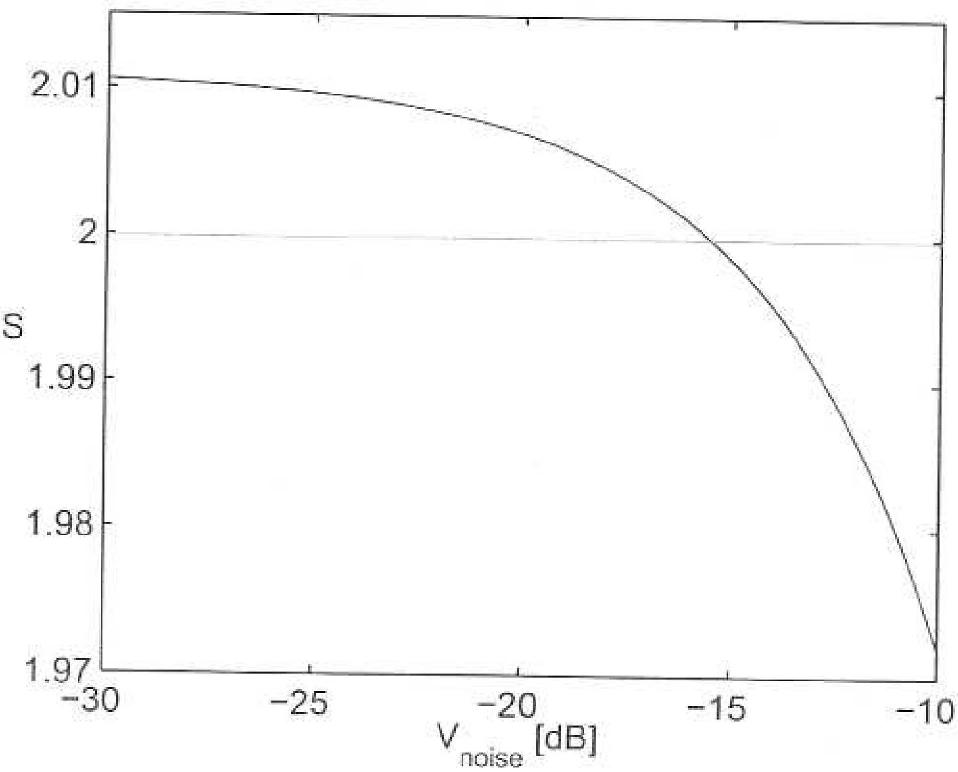

Realistic parameters:

$T = 0.95$, $\lambda = 0.6$ (6 dB), $\eta = 0.30$

$\eta_{\mathrm{BHD}} = 0.95 - 0.97 \Rightarrow$ violation of $0.5 - 1$ %

Electronic noise of the homodyne detectors

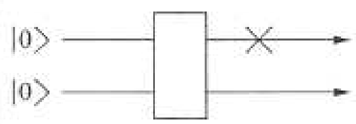
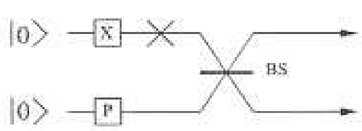
$T = 0.95$, $\lambda = 0.6$ (6 dB), $\eta = 0.30$, $\eta_{\text{BHD}} = 0.95$

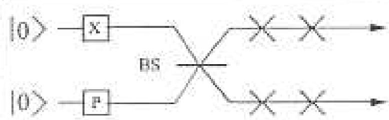
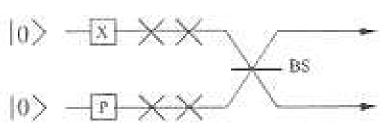
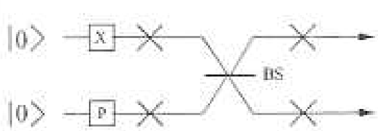


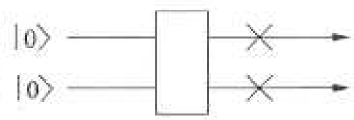
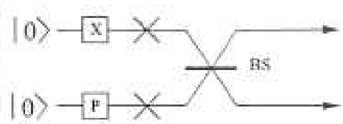
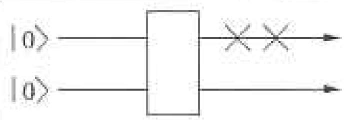
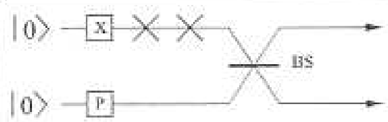
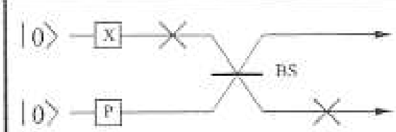

with sign binarization, the effect of added noise is equivalent to a decreased efficiency $\eta_{\text{BHD}}$

$\Rightarrow$ should be 15-20 dB below shot noise

# Some alternative schemes

| | Schemes: one subtraction | S |
|---|---|---|
| a) |  | 2 |
| b) |  | 2 |

| | Schemes: four subtractions | S |
|---|---|---|
| a) |  | 2.06 |
| b) |  | 2.05 |
| c) |  | 2 |

| | Schemes: two subtractions | S |
|---|---|---|
| a) |  | 2.046 |
| b) |  | 2 |
| c) |  | 2 |
| d) |  | 2.02 |
| e) |  | 2.01 |

# Conclusions

Existence of an experimental window for a loophole-free Bell test seems very plausible !

$\star$ $T$=0.95, $\lambda$=0.6 (6 dB)

$\star$ $\eta$ = 30 % or lower
but should filter out the other modes impinging on APDs!

$\star$ $R \sim 1$ MHz (1 $\mu$s is enough for pulse analysis and random bit generation)

$\Rightarrow$ $P \simeq 2 \times 10^{-4}$ ($\sim$ 200 counts per sec)

$\star$ $\eta_{\mathrm{BHD}} \gtrsim 95$ %

$\star$ $N_{\mathrm{el}} \lesssim 15 - 20$ below shot noise

*J. Wenger, R. Tualle-Brouri, Ph. Grangier, PRL 92, 153601 (2004): experimental generation of single-mode non-gaussian states by photon subtraction*

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| Local realism may be definitively disproved by observing Bell violation (in percent range) after about 1 hour statistics |