Vacuum Entanglement

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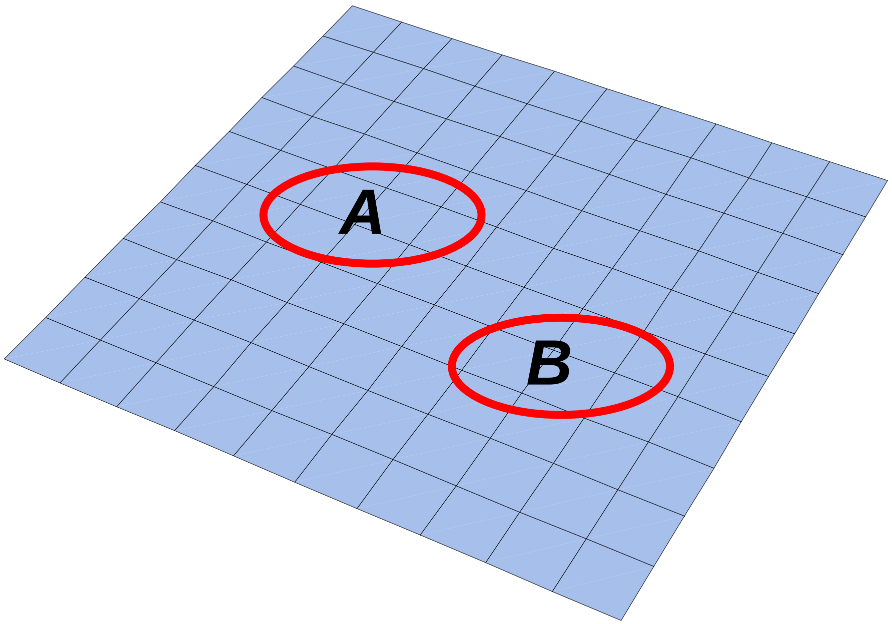
Vacuum Entanglement



Motivation:

Fundamentals: **SR** $\overset{\textbf{QI}}{\longleftrightarrow}$ **QM**

QI: natural set up to study Ent.
causal structure $\rightarrow$ LO.
$\mathcal{H}^{\infty}$, many body Ent.
.
Q. Phys.: Can Ent. shed light on "quantum effects"? (low temp. Q. coherences, Q. phase transitions, DMRG, Entropy Area law.)

See also Latorre's, Verstraete's & Plenio's talks

Background

Continuum results:

BH Entanglement entropy:

Unruh (76), Bombelli et. Al. (86), Srednicki (93), Callan & Wilczek (94) .

Albebraic Field Theory:

Summers & Werner (85), Halvarson & Clifton (00).

Entanglement probes:

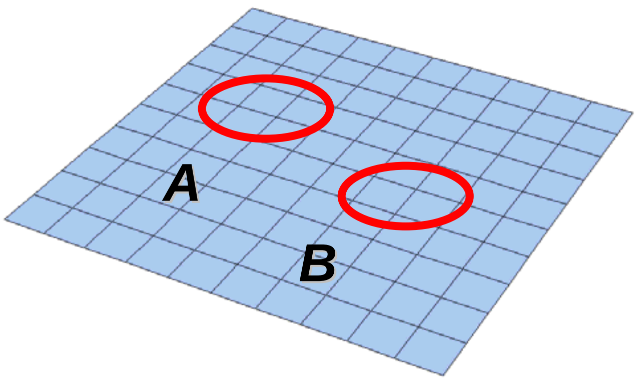
Reznik (00), Reznik, Retzker & Silman (03).

Discrete models:

Harmonic chains: *Audenaert et. al (02), Botero & Reznik (04).*

Spin chains: *Wootters (01), Nielsen (02), Latorre et. al. (03).*

Linear Ion trap: Retzker, Cirac & Reznik (04).

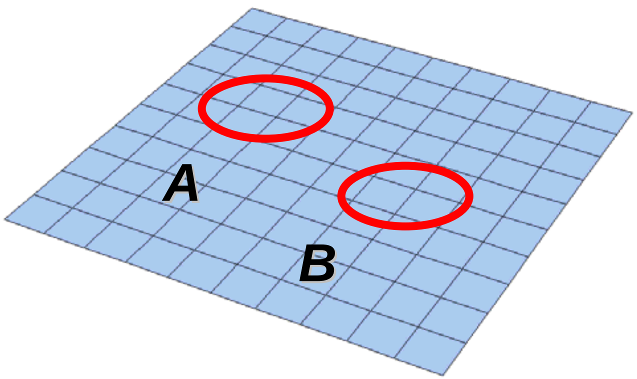


(I) Are A and B entangled?

(II) Are Bell's inequalities violated?

(III) Where does ent. "come from"?

(IV) Can we detect it?



(I) Are A and B entangled?

Yes, for arbitrary separation.

("Atom probes”).

(II) Are Bell’s inequalities violated?

Yes, for arbitrary separation.

(Filtration, “hidden” non-locality).

(III) Where does it “come from”?

Localization, shielding.

(Harmonic Chain).

(IV) Can we detect it?

Entanglement Swapping.

(Linear Ion trap).

Plan :

(1). Field entanglement: local probes.

Reznik (00), Reznik, Retzker, Silman (03).

(2). Harmonic chain: spatial structure of ent.

Botero, Reznik (04).

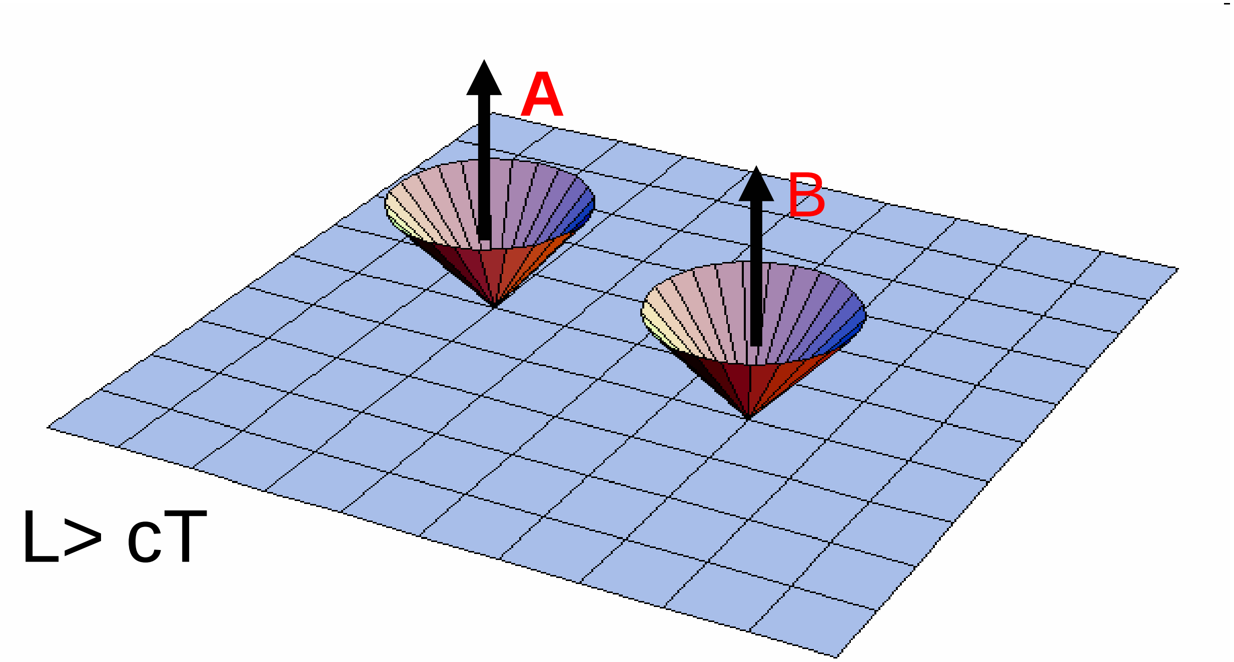
(3). Linear Ion trap: detection of ground state ent.

Retzker, Cirac, Reznik (04).

Probing Field Entanglement

RFT→ Causal structure

QI → LOCC

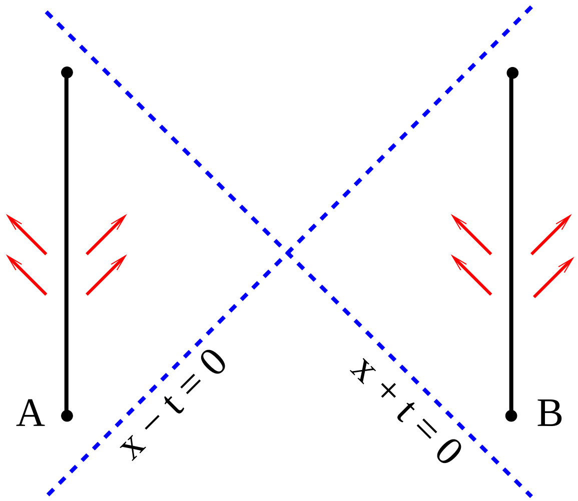


A pair of causally disconnected localized detectors

B. Reznik quant-ph/0008006,0112044

B. Reznik, A. Retzker & J. Silman quant-ph/0310058

Causal Structure + LO



For L>cT, we have $[\phi_A, \phi_B]=0$

Therefore $U_{INT}=U_A \otimes U_B$ → **LO**

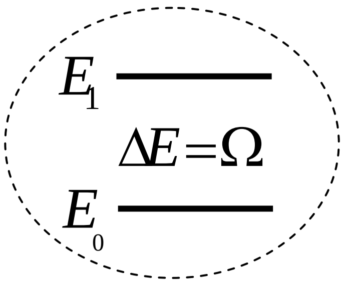
$\Delta E_{Total}=0$, but

$\Delta E_{AB}>0$. (Ent. Swapping)

Vacuum ent $\longrightarrow$ Detectors' ent.

Lower bound.

Field – Detectors Interaction



Two-level system

Interaction:

$$H_{INT}=H_A+H_B$$

$$H_A=\varepsilon_A(t)(e^{+i\Omega t}\,\sigma_A^{+} + e^{-i\Omega t}\sigma_A^{-})\;\phi(x_A,t)$$

↑ Window Function

Initial state:

$$|\Psi(0)\rangle = |\Downarrow_A\rangle\,|\Downarrow_B\rangle|VAC\rangle$$

Note: we do not use the rotating wave approximation.

Unruh (76), B. Dewitt (76), particle-detector models.

Probe Entanglement

$$\rho_{AB}^{(4\times 4)} = \mathrm{Tr}_F\, \rho^{(4\times\infty)}$$

$$\overset{?}{\neq} \sum_i p_i\, \rho_A^{(2\times 2)} \otimes \rho_B^{(2\times 2)}$$

Calculate to the second order (in ε) the final state, and evaluate the reduced density matrix.

Finally, we use Peres's (96) partial transposition criterion to check inseparability and use the Negativity as a measure.

$$U_{Interaction} = U_A \otimes U_B = (1 - i\varepsilon \int dt H_A - \frac{1}{2}\varepsilon^2 T \iint dt dt' H_A H_A)(\ldots)$$

$$\left|\Psi(T)\right\rangle = U_{Interaction} \left|\downarrow_A\right\rangle \left|\downarrow_B\right\rangle \left|0\right\rangle$$

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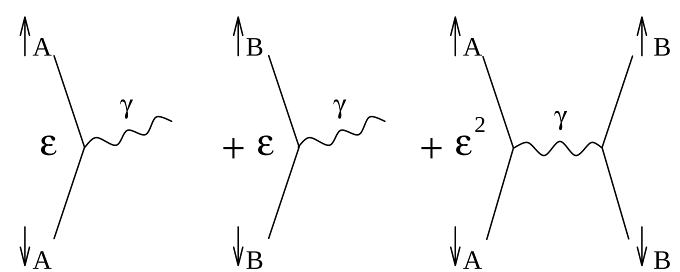
$$\left|\Psi(T)\right\rangle = U_{Interaction} \left|\downarrow_A\right\rangle \left|\downarrow_B\right\rangle \left|0\right\rangle$$

$$\begin{array}{cccc} \Uparrow\Uparrow & \Downarrow\Downarrow & \Uparrow\Downarrow & \Downarrow\Uparrow \end{array}$$

$$\rho_{AB}(T) = \begin{bmatrix} 1 & \left\langle 0 \middle| X_{AB} \right\rangle & & \\ \left\langle X_{AB} \middle| 0 \right\rangle & \left\| X_{AB} \right\|^2 & & \\ & & \left\| E_A \right\|^2 & \left\langle E_A \middle| E_B \right\rangle \\ & & \left\langle E_B \middle| E_A \right\rangle & \left\| E_B \right\|^2 \end{bmatrix} + O(\varepsilon^5)$$

$$\left| X_{AB} \right\rangle \sim \Phi_A \Phi_B \left| 0 \right\rangle \sim \left| 0 or 2, photons \right\rangle$$

$$\left| E_A \right\rangle \sim \Phi_A \left| 0 \right\rangle \sim \left| 1, photon \right\rangle$$



$$U_{Interaction} = U_A \otimes U_B = (1 - i\varepsilon \int dt H_A - \frac{1}{2}\varepsilon^2 T \iint dt dt' H_A H_A)(\ldots)$$

$$\left|\Psi(T)\right\rangle = U_{Interaction} \left|\downarrow_A\right\rangle \left|\downarrow_B\right\rangle \left|0\right\rangle$$

$$\begin{matrix} \Uparrow\Uparrow & \Downarrow\Downarrow & \Uparrow\Downarrow & \Downarrow\Uparrow \end{matrix}$$

P.T,

$$\rho_{AB}(T) = \begin{bmatrix} 1 & \left\langle 0 | X_{AB} \right\rangle & & \\ \left\langle X_{AB} | 0 \right\rangle & \left\| X_{AB} \right\|^2 & & \\ & & \left\| E_A \right\|^2 & \left\langle E_A | E_B \right\rangle \\ & & \left\langle E_B | E_A \right\rangle & \left\| E_B \right\|^2 \end{bmatrix} + O(\varepsilon^5)$$

P.T.

$$\left|X_{AB}\right\rangle \sim \Phi_A \Phi_B \left|0\right\rangle \sim \left|0 or 2, photons\right\rangle$$

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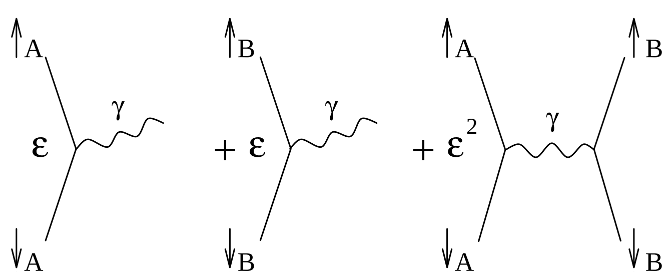
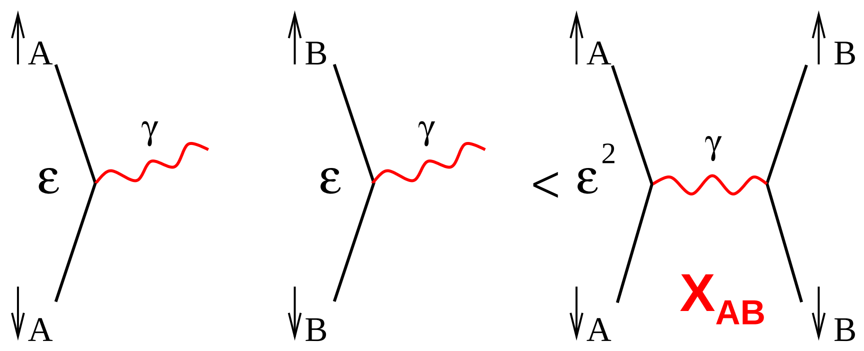


Emission < Exchange

$||E_A|| \, ||E_B|| < ||\langle 0|X_{AB}\rangle||$

$|\Downarrow\Downarrow\rangle + \langle X_{AB}|VAC\rangle \, |\Uparrow\Uparrow\rangle$ "+"...



$$\int_0^\infty \omega d\omega [\tilde{\varepsilon}(\Omega+\omega)]^2 < \int_0^\infty \frac{d\omega}{L} Sin(\omega L)\widetilde{\varepsilon_A}(\Omega+\omega)\widetilde{\varepsilon_B}(\Omega-\omega)$$

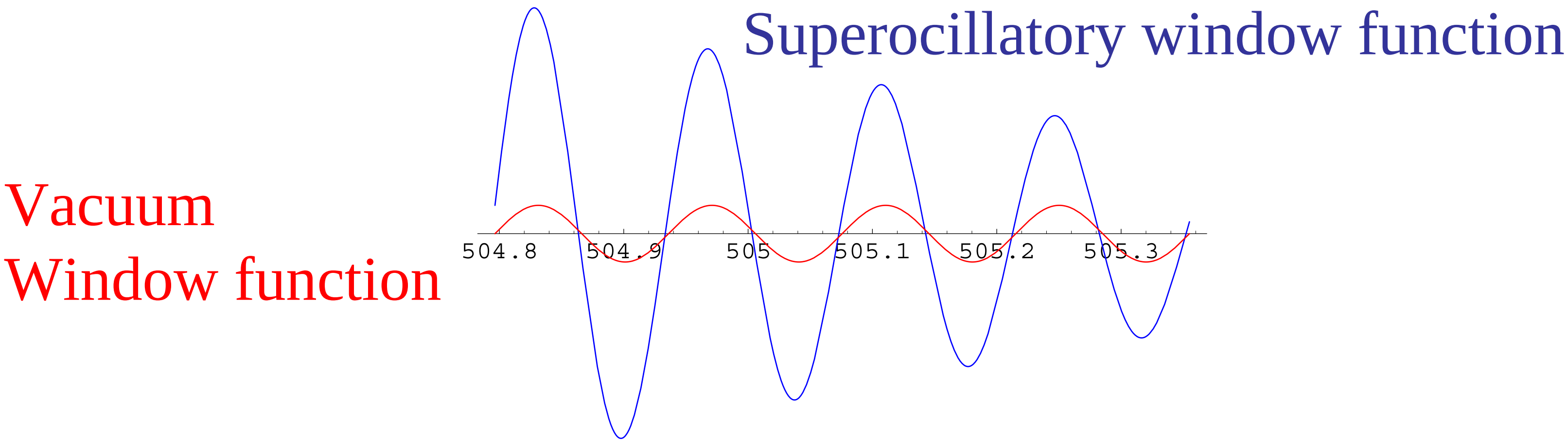
Off resonance

Vacuum "window function"

→ Superocillatory functions *(Aharonov (88), Berry(94)).*

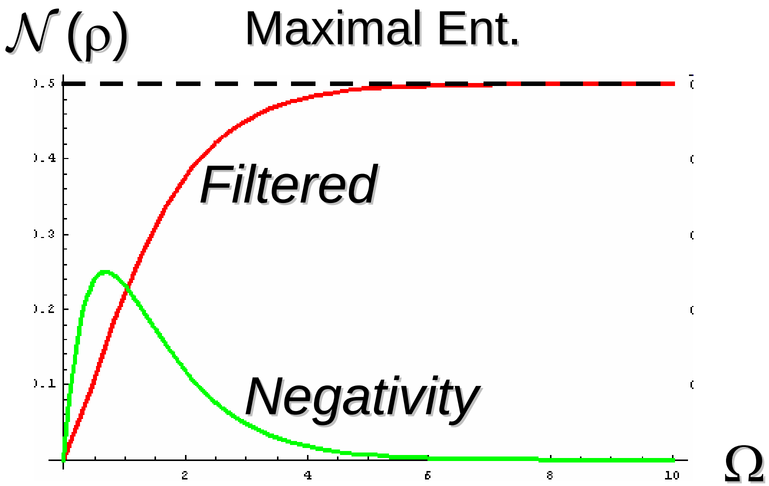
Entanglement for every separation

We can tailor a superoscillatory window function for every L to resonate with the vacuum "window function" $\sin(L\omega)$



→ Exchange term $\propto$ exp(-f(L/T))

Bell's Inequalities

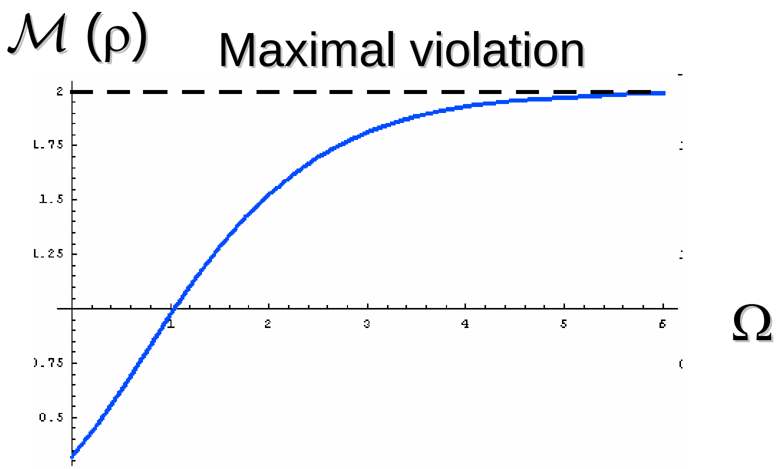


No violation of Bell's inequalities.

But, by applying local filters

$$|\Downarrow\Downarrow\rangle + \langle X_{AB}|VAC\rangle\, |\Uparrow\Uparrow\rangle \text{ “+”}\ldots$$

$$\rightarrow \eta^2\, |\Downarrow\rangle|\Downarrow\rangle + \langle X_{AB}|VAC\rangle|\Uparrow\rangle|\Uparrow\rangle \text{ “+”}\ldots$$



CHSH ineq. Violated iff
$\mathcal{M}(\rho)>1$, (Horokecki (95).)

“Hidden” non-locality.
Popescu (95). Gisin (96).

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Summary (1)

1) Vacuum entanglement can be distilled!

2) Lower bound: $E \geq e^{-(L/T)^2}$
(possibly $e^{-L/T}$)

3) High frequency (UV) effect: $\Omega \approx L^2$.

4) Bell inequalities violation for arbitrary separation
maximal "hidden" non-locality.

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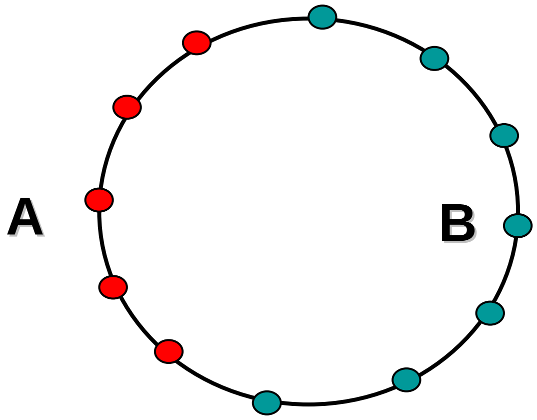
Spatial structure of entanglement in the Harmonic Chain

A. Botero & B. Reznik 0403233

The Harmonic Chain model



Circular Harmonic chain

$H_{chain} \xrightarrow{\infty} H_{scalar\ field}$

$= \int \pi(x)^2 + (\nabla\phi)^2 + m^2\phi^2(x))dx$

$\psi_{chain} \propto e^{-q_i\, Q^{-1}\, q_j/4}$

I) Gaussian state → *Exact* calculation of Ent.

II) Mode-wise decomposition → Identify *spatial* Ent. Structure

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Gaussian (pure) Entanglement

The reduced density matrix of a Gaussian state
Is a Gaussian density matrix.

Ψ_{AB} → ρ_A ↔ Covariance Matrix M ← Second moments $\langle q_i q_j \rangle$, $\langle p_i p_j \rangle$

↓

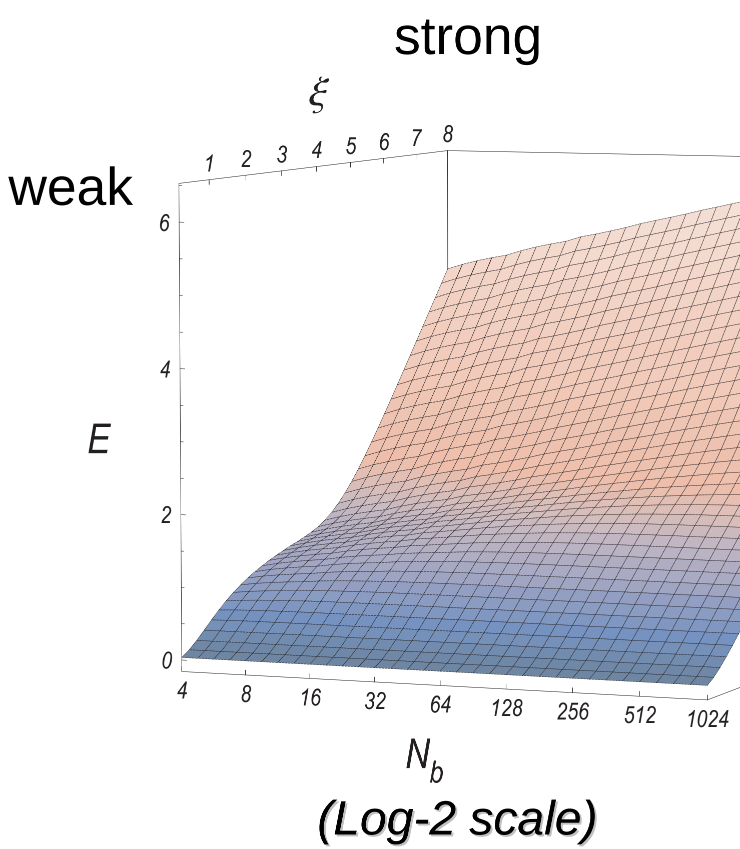
Symplectic Spectrum λ_i

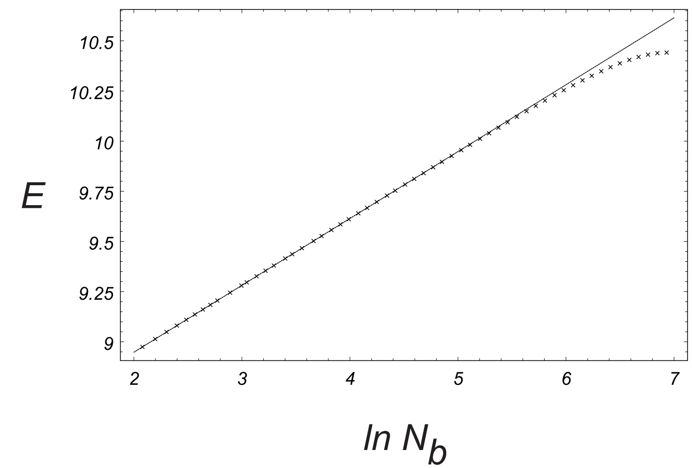
↓

Entanglement Entropy

$$E = \sum (\lambda + 1/2)\log(\lambda + 1/2) - (\lambda - 1/2)\log(\lambda - 1/2)$$

Entanglement: block vs. the rest

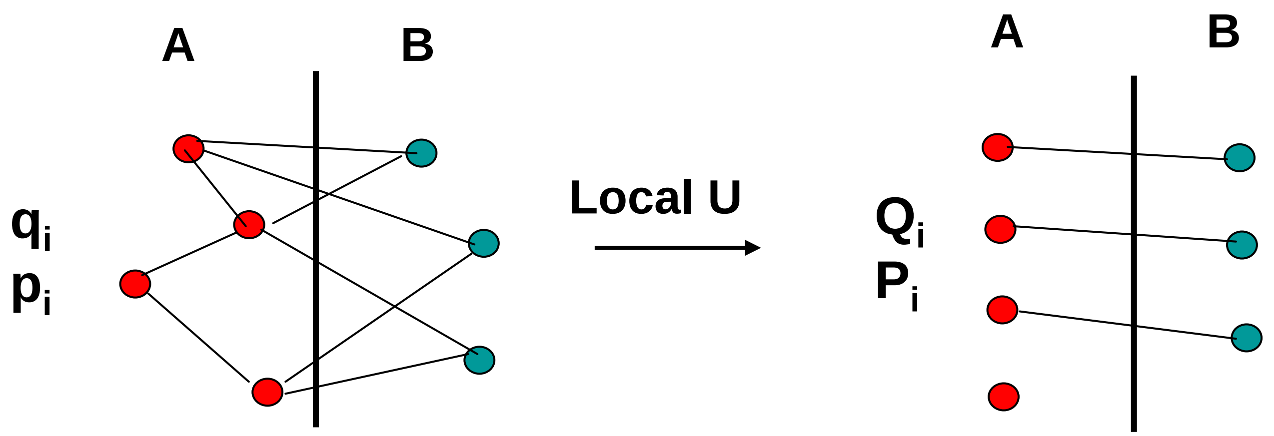




$E \simeq 1/3 \ \ln N_b + E_c(\alpha, N_b)$

→ c=1, bosonic 1-d FT

Mode-Wise decomposition theorem



Local U

$$\psi_{AB} = \sum c_i |A_i\rangle |B_i\rangle \longrightarrow \psi_{AB} = \psi_{11}\psi_{22}\cdots\psi_{kk} \quad \psi_{0,..}$$

Schmidt | **Mode-Wise decomposition**

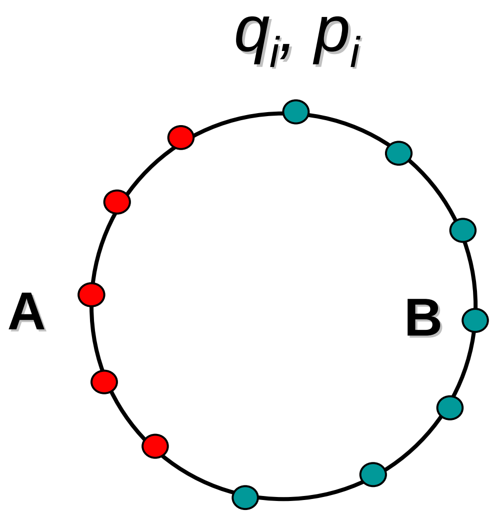
$$\psi_{kk} \propto \sum e^{-\beta k\, n} |n\rangle |n\rangle$$

Two modes squeezed state

Botero, Reznik *0209026* (bosonic modes)

Botero, Reznik *0404176* (fermionic modes)

Spatial Entanglement Structure



Circular Harmonic chain

local		**collective**
q_i	$\rightarrow$	$Q_m = \sum u_i q_i$
p_i	$\rightarrow$	$P_m = \sum v_i p_i$

Participation function:

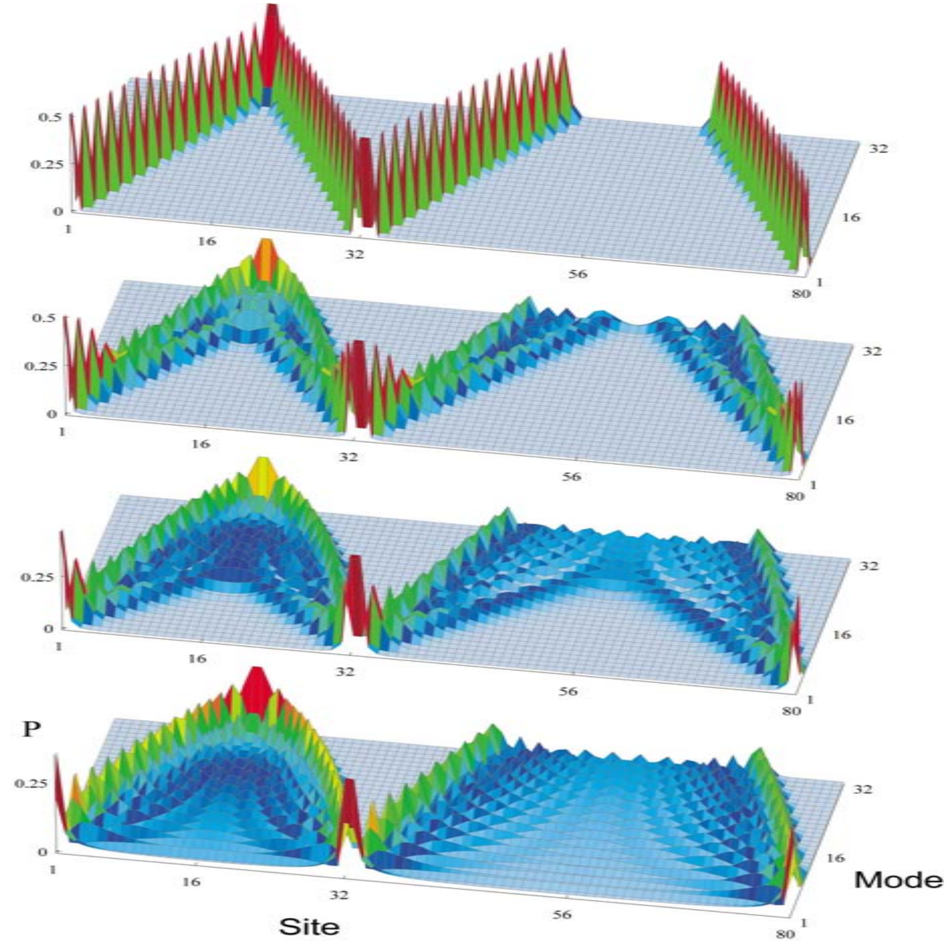
$P_i = u_i v_i, \quad \sum P_i = 1$

quantifies the contribution of local (q_i, p_i) oscillators to the collective coordinates (Q_i,P_i)

Site Participation Function

Weak coupling

Strong coupling

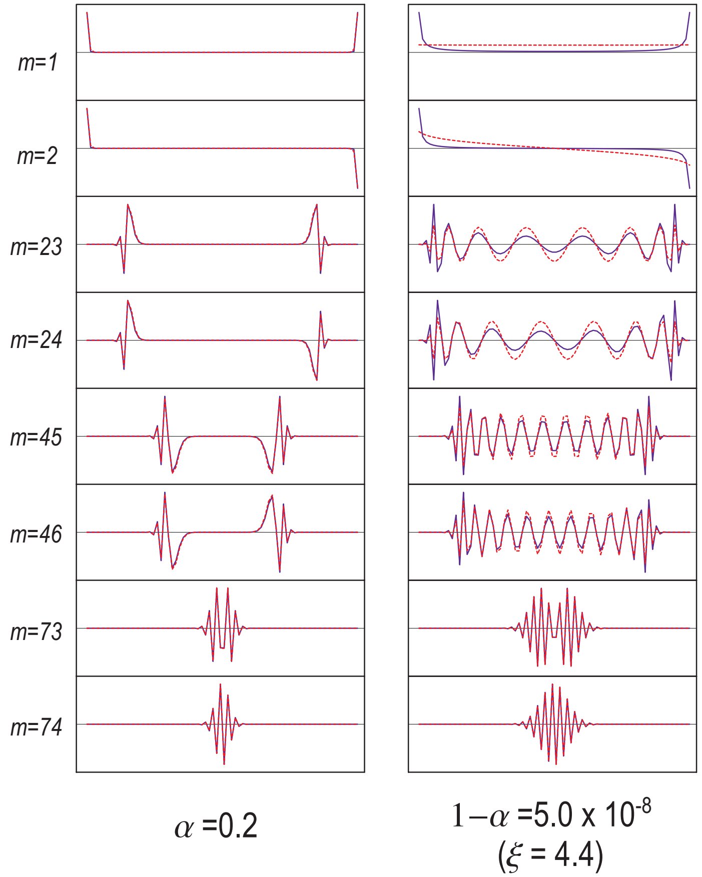


N=32+48 osc. Modes are ordered in decreasing Ent. Contribution, from front to back.

Mode Shapes

Outer modes

Inner modes

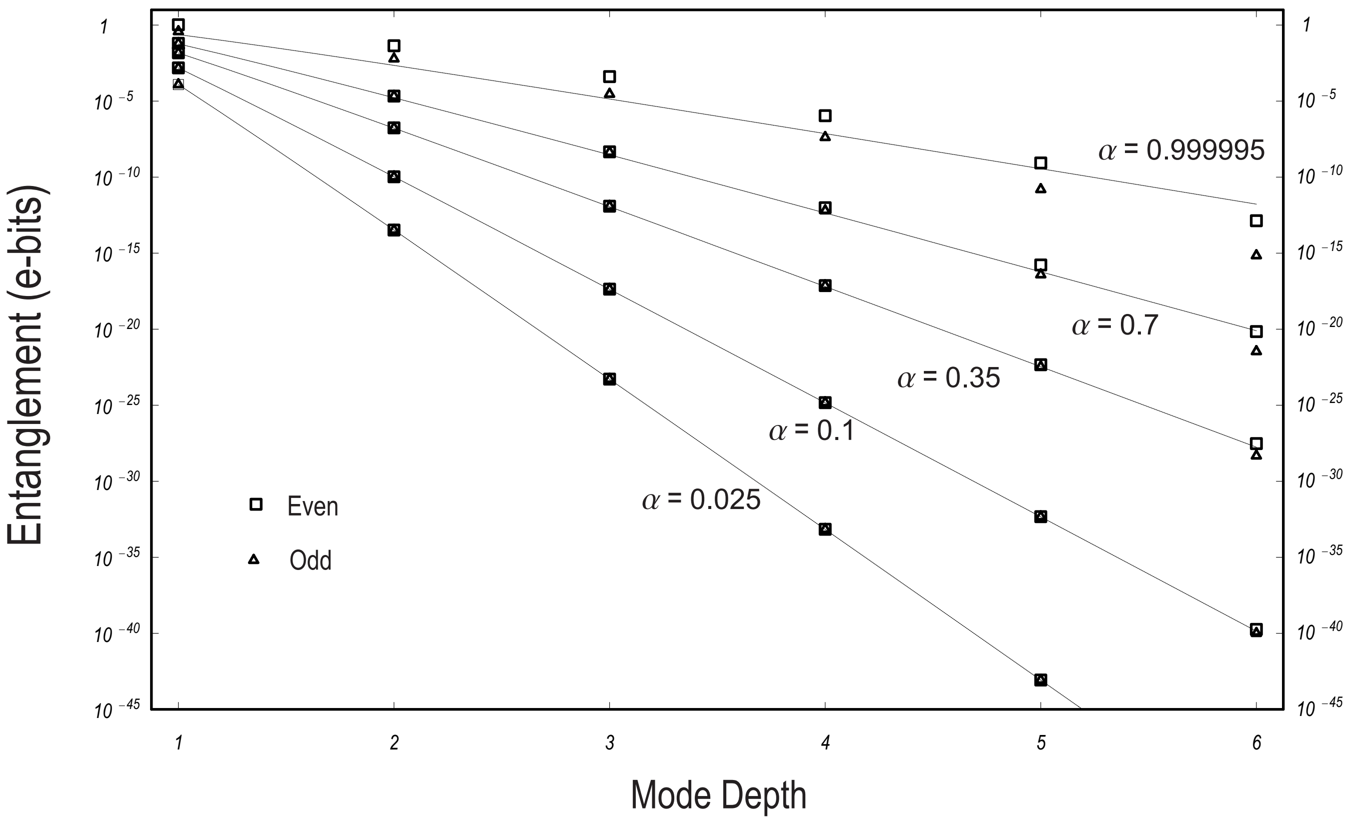


weak **strong**

→continuum

Solid – u
Dashed -v

Entanglement Contribution



Entanglement as a function of mode number decays exponentially.

Summary (2)

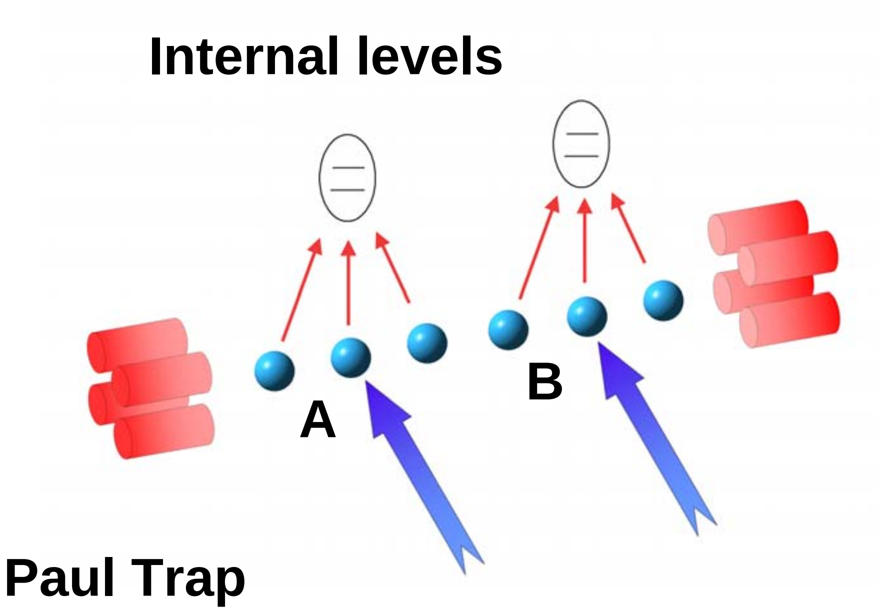
Logarithmic dependence in the continuum limit with the overall 1/3 coefficient as predicted by conformal field theory.

Inclusion of an ultraviolet cutoff leads to Localization of the highest frequency inner modes.

Mode shape hierarchy with distinctive layered structure, with exponential decreasing contribution of the innermost modes.

Can we detect Vacuum Entanglement?

Detection of Vacuum Entanglement in a Linear Ion Trap



$$H=H_0+H_{int}$$

$$H_0=\omega_z(\sigma_z^A+\sigma_z^B)+\sum \nu_n\, a_n^\dagger\, a_n$$

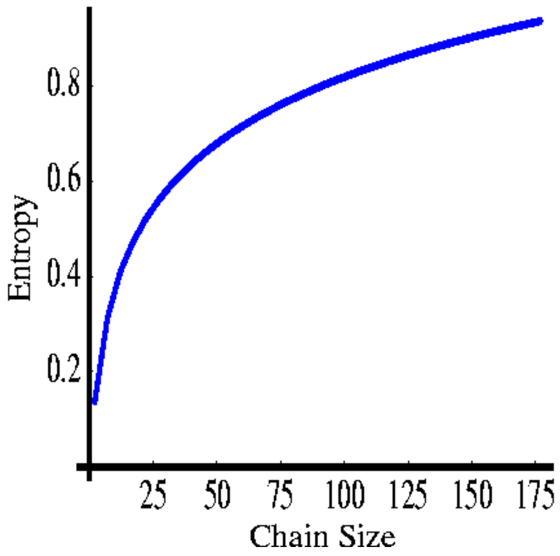
$$H_{int}=\Omega(t)(e^{-i\phi}\sigma_+^{(k)}+e^{i\phi}\sigma_-^{(k)})x_k$$

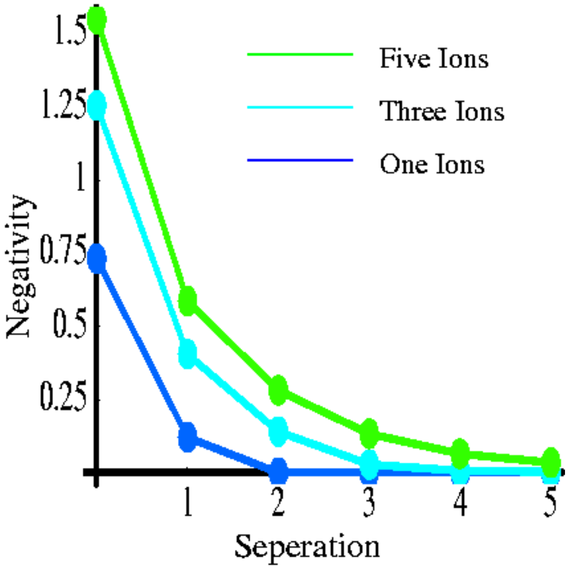
$$1/\omega_z << T << 1/\nu_0$$

A. Retzker, J. I. Cirac, B. Reznik 0408059

Entanglement in a linear trap

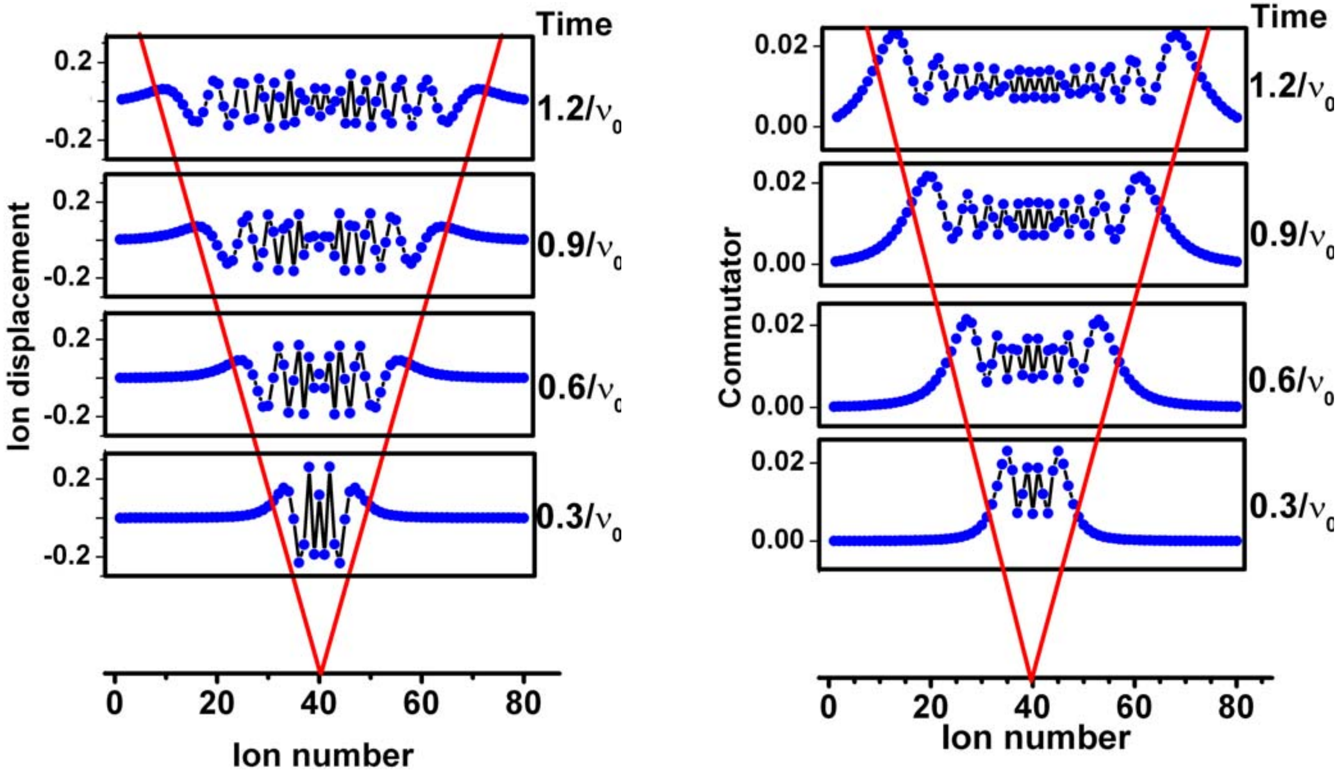
$$|vac\rangle = |0_c\rangle|0_b\rangle \propto \sum_n e^{-\beta n}|n\rangle_A|n\rangle_B$$





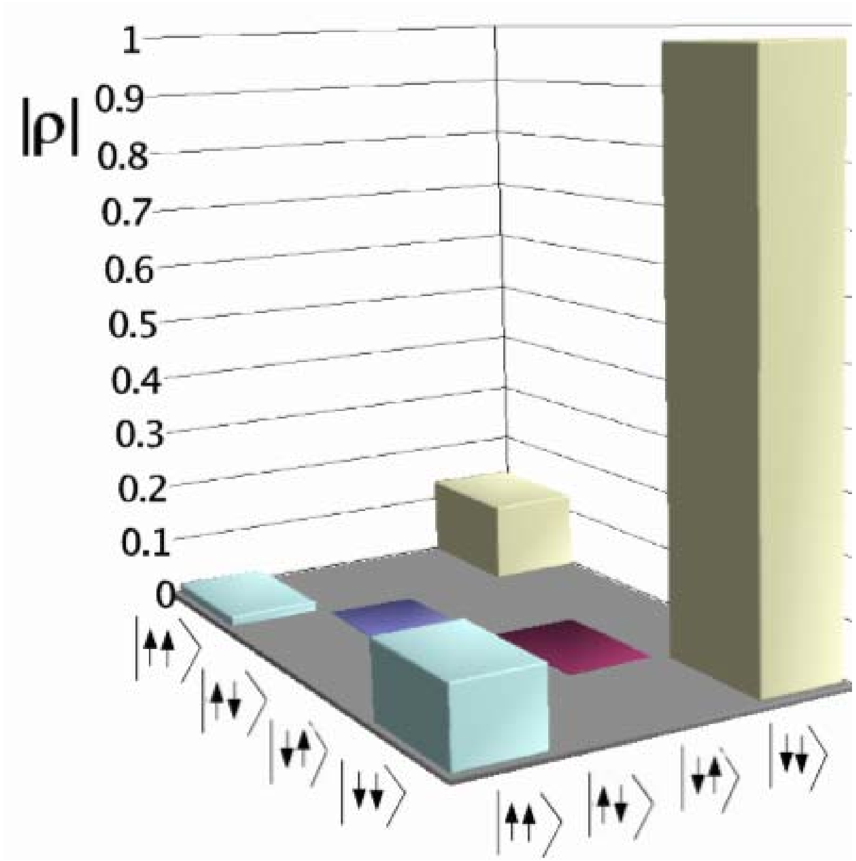
Entanglement between symmetric groups of ions as a function of the total number (left) and separation of finite groups (right).

Causal Structure



→ $U_{AB}=U_A \otimes U_B + O([x_A(0),x_B(T)])$

Two trapped ions



Final internal state

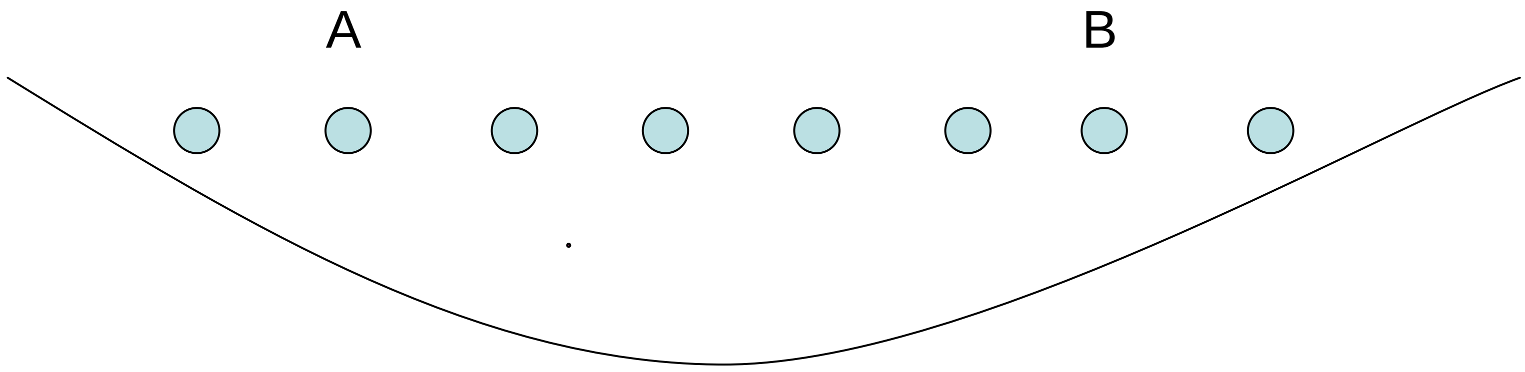
"Swapping" spatial $\rightarrow$ internal states

$$|vac\rangle|\Downarrow\Downarrow\rangle \ \overset{U}{\rightarrow}\ |\chi\rangle(|\Uparrow\Uparrow\rangle + e^{-\beta}|\Downarrow\Downarrow\rangle)$$

$$U=(e^{i\alpha\, x\, \sigma_x} \otimes e^{i\beta\, p\, \sigma_y})\otimes\ldots$$

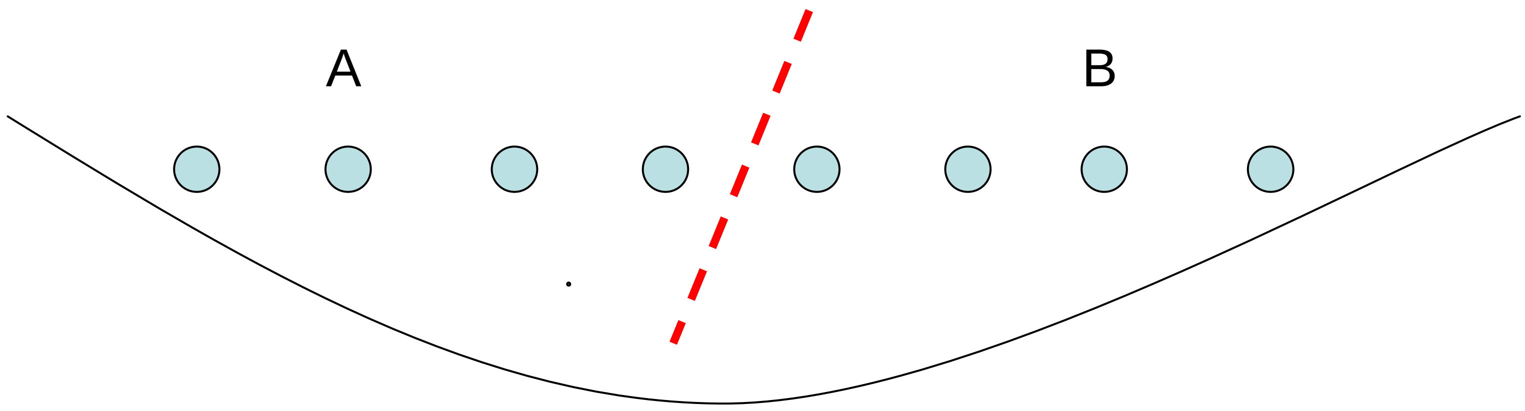
$E_{formation}(\rho_{final})$
accounts for 97% of the calculated
Entantlement: $E(|vac\rangle)$=0.136 e-bits.

Long Ion Chain



But how do we check that ent. is not due to “non-local” interaction?

Long Ion Chain



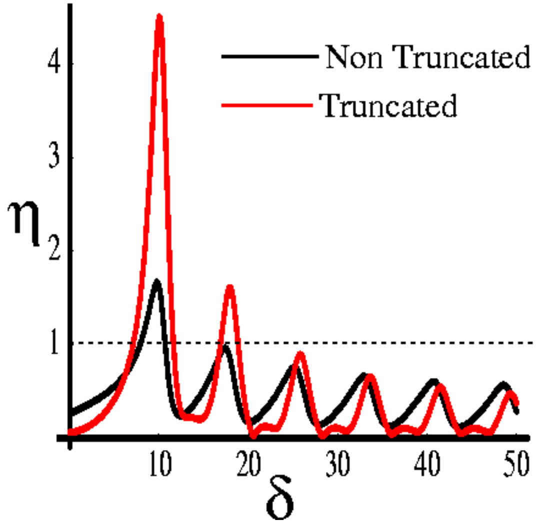
But how do we check that ent. is not due to “non-local” interaction?

$$H_{AB} \rightarrow \quad H_{truncated}=H_A \oplus H_B$$

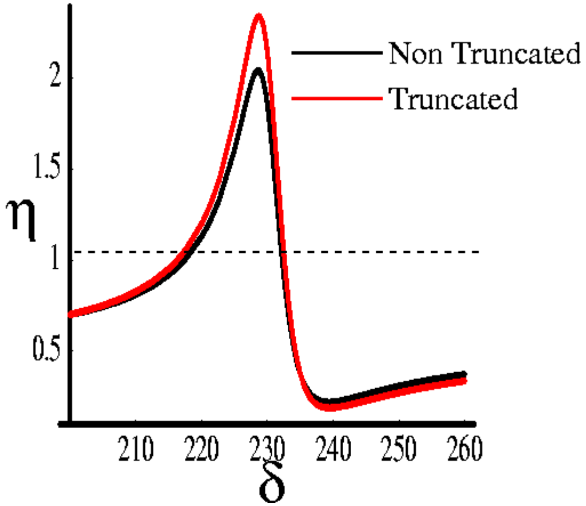
We compare the cases with a truncated and free Hamiltonians

Long Ion Chain

L=6,15, N=20



L=10,11   N=20



η=exchange/emission >1 , signifies entanglement.

δ denotes the detuning, L the locations of A and B.

Summary

Atom Probes:

Vacuum Entanglement can be “swapped” to detectors.

Bell’s inequalities are violated (“hidden” non-locality).

Ent. reduces exponentially with the separation.

High probe frequencies are needed for large separation.

Harmonic Chain:

Persistence of ent. for large separation is linked with localization of the interior modes. This seems to provide a mechanism for “shielding” entanglement from the exterior regions.

Linear ion trap:

-A proof of principle of the general idea is experimentally feasible for two ions.

-One can entangle internal levels of two ions without performing gate operations.