Quantum non-demolition measurements:

One path to truly scalable quantum computation

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Why should optical quantum computation be considered?

- Optics realization has a number of advantages:
  - Flexibility in geometrical structure and computational structure.
    - Naturally flying qubits
    - Free to choose qubits to qunats.
  - High speed computational time.
  - Easy interface to communication devices.

- However, the lack of optical nonlinear operation prevents two qubits to interact.
  - No conditional operation can be done.
  - Even no computation can be performed.
- Conditional operation can in principle give us a scalable quantum computational schemes.
  - Linear optics quantum computation (cf. KLM scheme)
- Non-deterministic operations are powerful, but it is very difficult for the entire scheme to be scalable in a real system and sense.

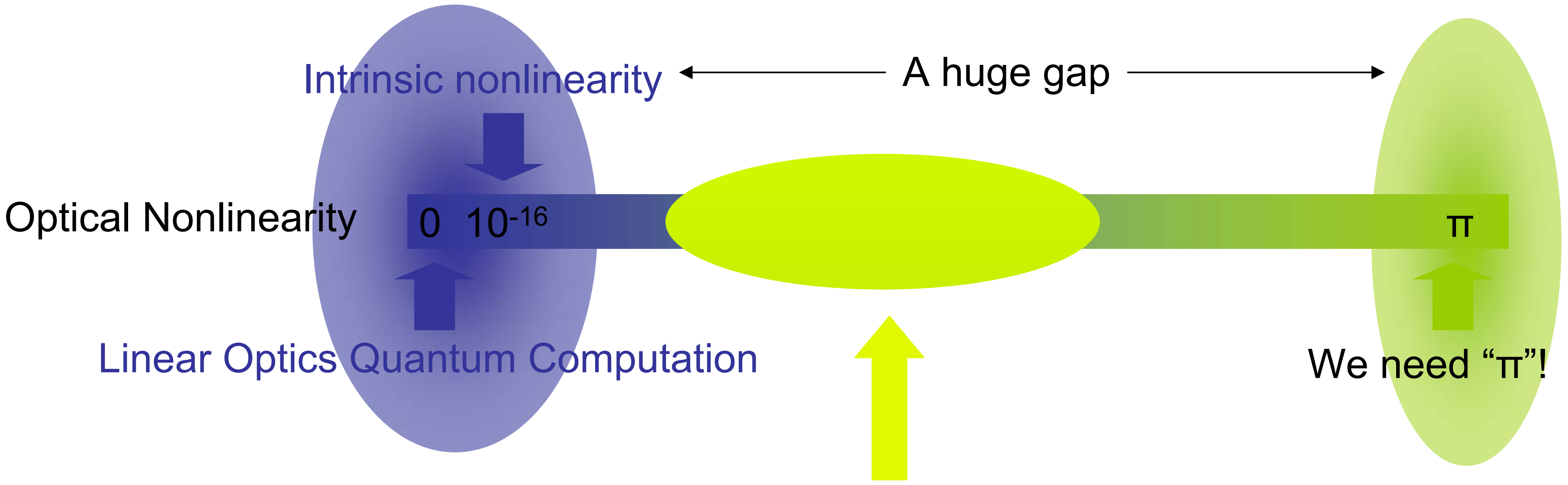
Success Probability and Scalability

- If all elements are perfect,
  - the upper bound for the success probabilities for non-linear gates are[1]:
    - NS gate ½ success probability.
    - CNOT gate ¾ success probability.
  - Existing non-linear gate models with linear optics have even lower success probabilities even with perfect operations and detections.
    - The best success probability[2]
      - NS gate ¼
      - CNOT 2/27
    - Franson's CNOT gate: ¼ with entangled source[3].
- On top of this, single-photon source and high-efficient photon counting are experimentally demanding tasks.
- Low success probabilities of nonlinear gates make linear optics quantum computation difficult to scale.
- Can small nonlinearity help optics quantum computation to be scalable?

[1] E. Knill, Phys. Rev. A 68 064303 (2003). [2] E.Knill, *et. al*., Nature 409, 46 (2001), S. Scheel, *et al*., Phys. Rev. A 68, 032310 (2003)
[3] T.B.Pittman, *et. al.*, Phys. Rev. A 64, 062311 (2001)

In the huge gap

- Linear optics quantum computation is hard to scale.
- Intrinsic nonlinearity is too small for nonlinear gates.

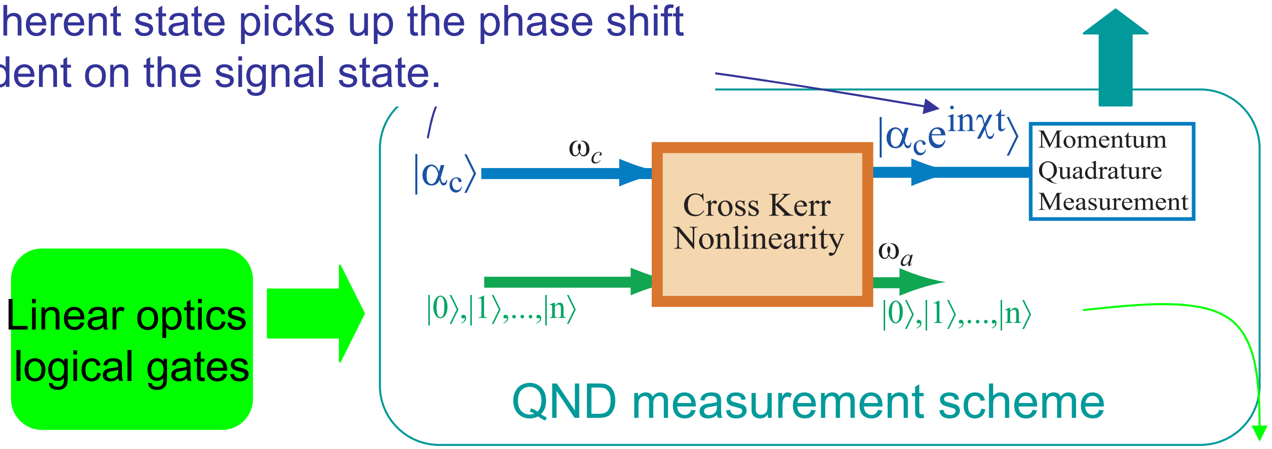


- No quantum computation schemes considered in between.
- Can we use small nonlinearity to make optical quantum computation more scalable?
- A new regime of optical quantum computation?

Evaluate all nonlinearities in linear optics quantum computation

- In linear optics quantum computation schemes, nonlinearity is hidden in qubit sources and detections.
- How nonlinear are these elements?
- Non-linearity is necessary to perform high-efficiency photon via homodyne detector.
- It is known that QND measurement can be achieved by cross-Kerr non-linearity.*

The photon number resolving detector:



Measurement result

The coherent state picks up the phase shift dependent on the signal state.

$|\alpha_c\rangle$ ω_c

Cross Kerr Nonlinearity

$|\alpha_c e^{in\chi t}\rangle$

Momentum Quadrature Measurement

ω_a

$|0\rangle,|1\rangle,...,|n\rangle$ $|0\rangle,|1\rangle,...,|n\rangle$

Linear optics logical gates

QND measurement scheme

Output state

The photon-number state generator:

Photon-number state

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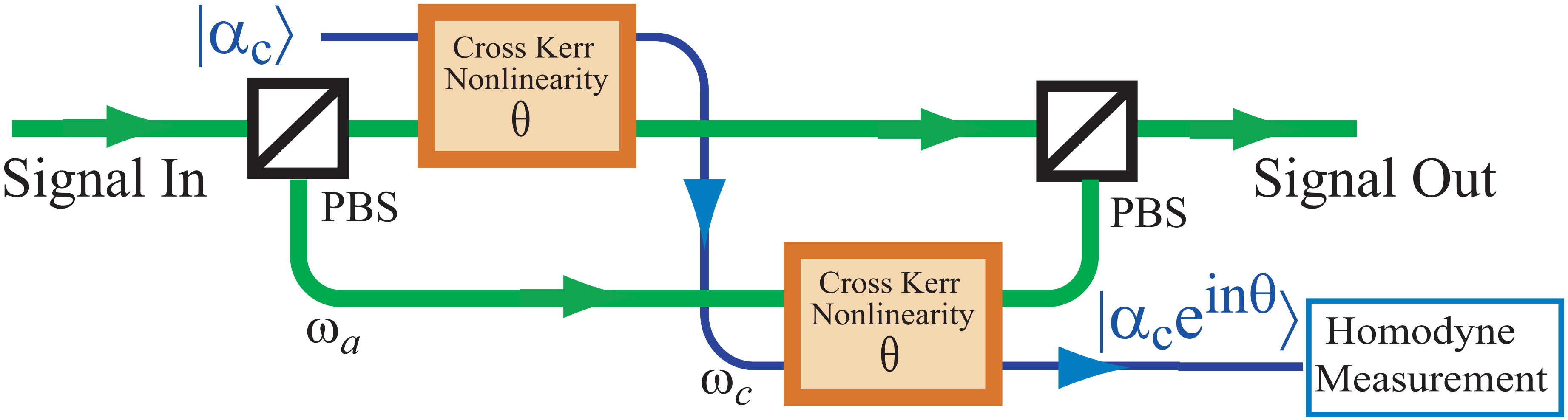
* G. J. Milburn and D. F. Walls, Phys. Rev. A 30, 56 (1984), N Imoto, et. al., Phys. Rev. A 32, 3287 (1985)

Polarisation Preserving QND detector

- It is likely that our optical qubit will be encoded in polarisation
- If we want to determine if the photon is there or not, we will not want to destroy the polarisation information
- The previous QND can be simply adapted to do this*

The polarisation preserving photon number resolving detector:



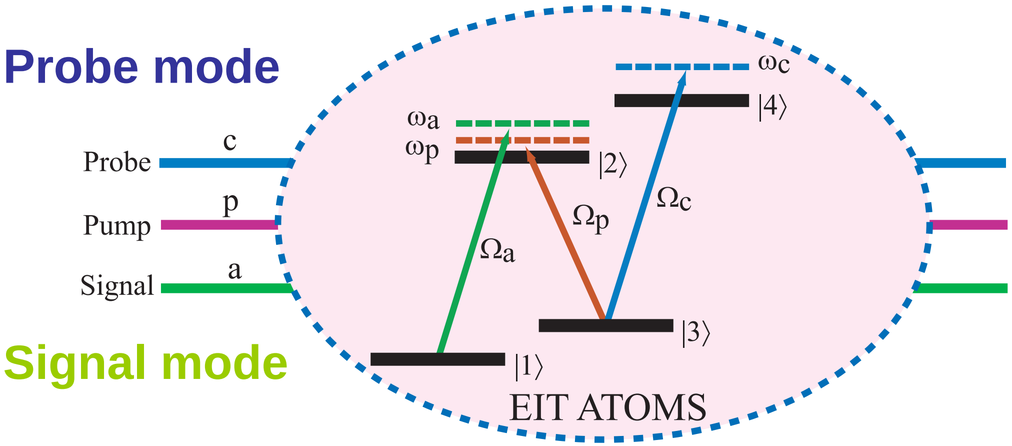
How big (or small) a nonlinearity θ is needed? $\alpha_c \theta \sim 4$

* W.J Munro, Kae Nemoto, R. Beausoleil and T. P. Spiller, quant-ph 0310066

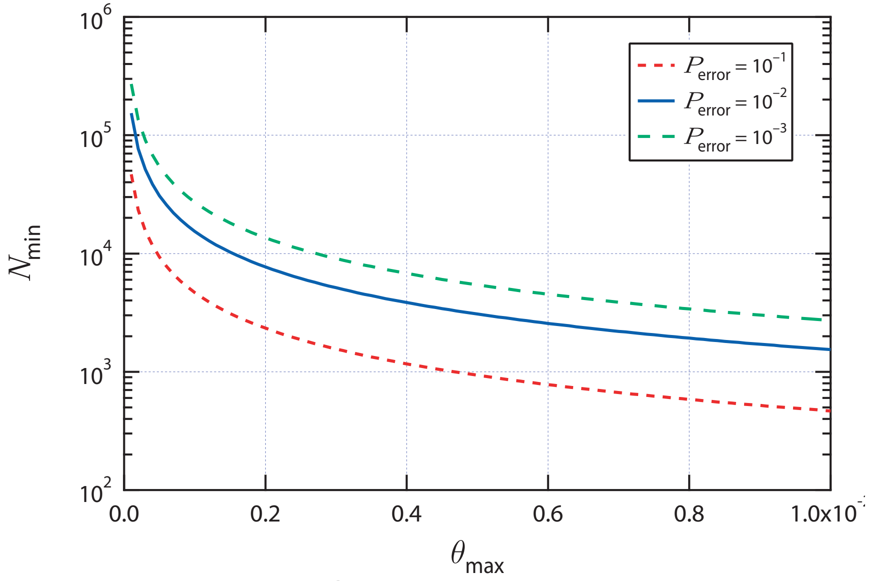
EIT (Electromagnetically Induced Transparency) system

- How do we generate the cross Kerr nonlinear effect?
- One way is to use an EIT system to generate the nonlinearity we need for the QND measurement.

W.J.Munro, *et. al.*, quant-ph/0310066



Interaction between a four-level atom.



Number of atoms needed to get a certain θ for a fixed discrimination error

- There are other ways to generate this small cross-Kerr effects
  - Nonlinear effects in fibers, Cavity QED, micro cavity resonators …

QND logic gates

- The QND measurement scheme
  - Single photon source and photon-number detection

- The QND measurement scheme + Linear optics elements is universal.
  - There is no hidden nonlinearity.
  - Nonlinearity is enhanced in the QND measurement scheme.
    - Small nonlinearity can be exponentially enhanced by the intensity of probe mode.

- However the low success probabilities stays the same.
  - The scalability stays the same as linear optics quantum computation.
  - The QND measurement can be used as nonlinearity amplifier. Can we use the property to construct more scalable logic gates?

Entangling gate

1. Prepare the initial state on the second qubit to be $|\Psi_{in}\rangle_2 = |H\rangle + |V\rangle$.
2. The output state before the measurement is

Ir

$$|\Psi_T\rangle = c_0|\mathrm{HH}\rangle|\alpha\rangle + c_1|\mathrm{VV}\rangle|\alpha\rangle + c_0|\mathrm{HV}\rangle|\alpha e^{i\theta}\rangle + c_1|\mathrm{VH}\rangle|\alpha e^{-i\theta}\rangle$$

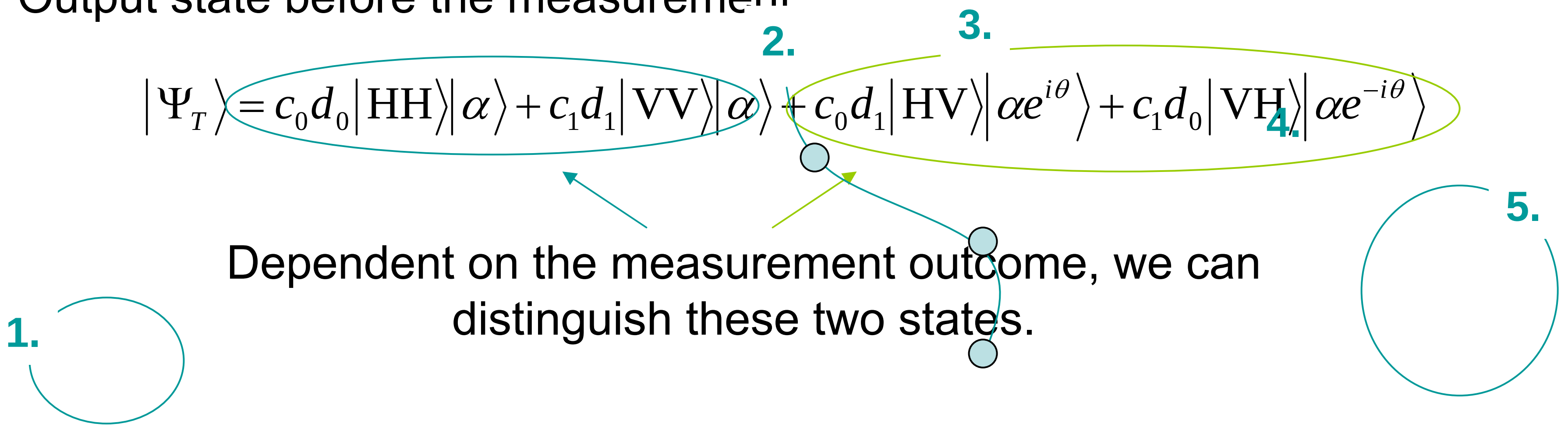
3. After the measurement, the state for the two qubits is

ıe

$$\begin{cases} |\Psi\rangle = c_0|HH\rangle + c_1|VV\rangle & \text{for } X > X_0 \\ |\Psi\rangle = c_0 e^{i\phi(x)}|HV\rangle + c_1 e^{-i\phi(x)}|VH\rangle & \text{for } X \le X_0\,,\ X_0 = \alpha(1+\cos\theta) \end{cases}$$

4. The classical feed forward can correct the phase *Φ*(*x*).
5. The output state is $|\Psi_{out}\rangle = c_0|HH\rangle + c_1|VV\rangle$.

Output state before the measurement:



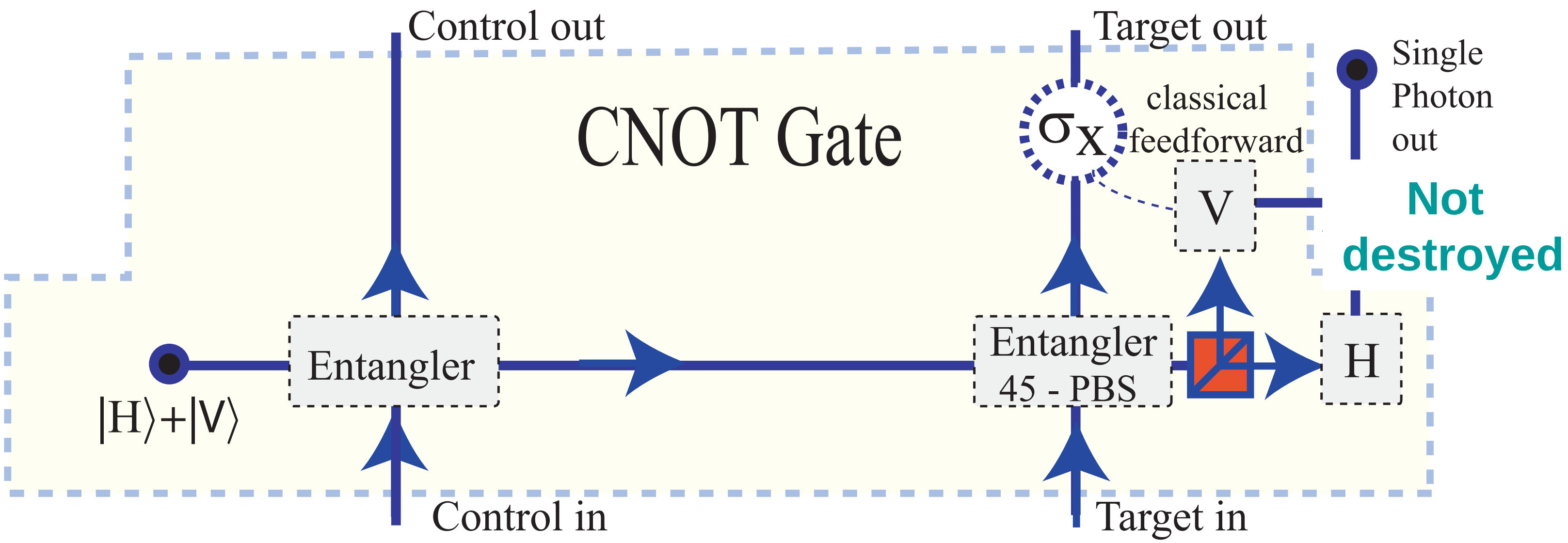
$$|\Psi_T\rangle = c_0 d_0|\mathrm{HH}\rangle|\alpha\rangle + c_1 d_1|\mathrm{VV}\rangle|\alpha\rangle + c_0 d_1|\mathrm{HV}\rangle|\alpha e^{i\theta}\rangle + c_1 d_0|\mathrm{VH}\rangle|\alpha e^{-i\theta}\rangle$$

Dependent on the measurement outcome, we can distinguish these two states.

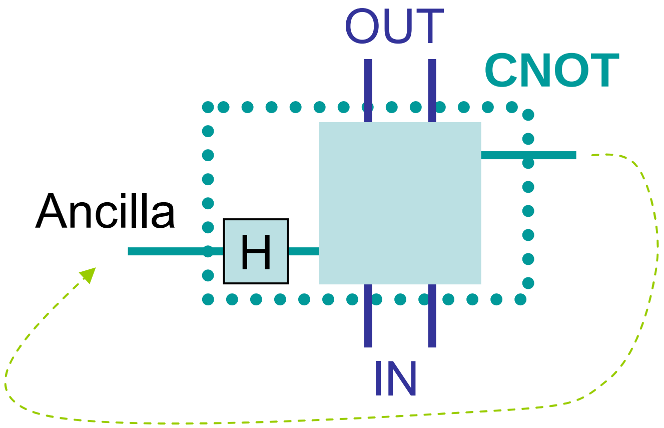
- the homodyne measurement is not critical: real need is phase sign erasure

CNOT gate

- The entangling gate can be used to construct CNOT gate.
- Based on the Franson CNOT gate*, we use the entangling gate and PBS and 45-PBS gates.



- This CNOT gate is near-deterministic.
  - Requires five small nonlinear gates.
  - Computational circuit requires n+1 qubits!!

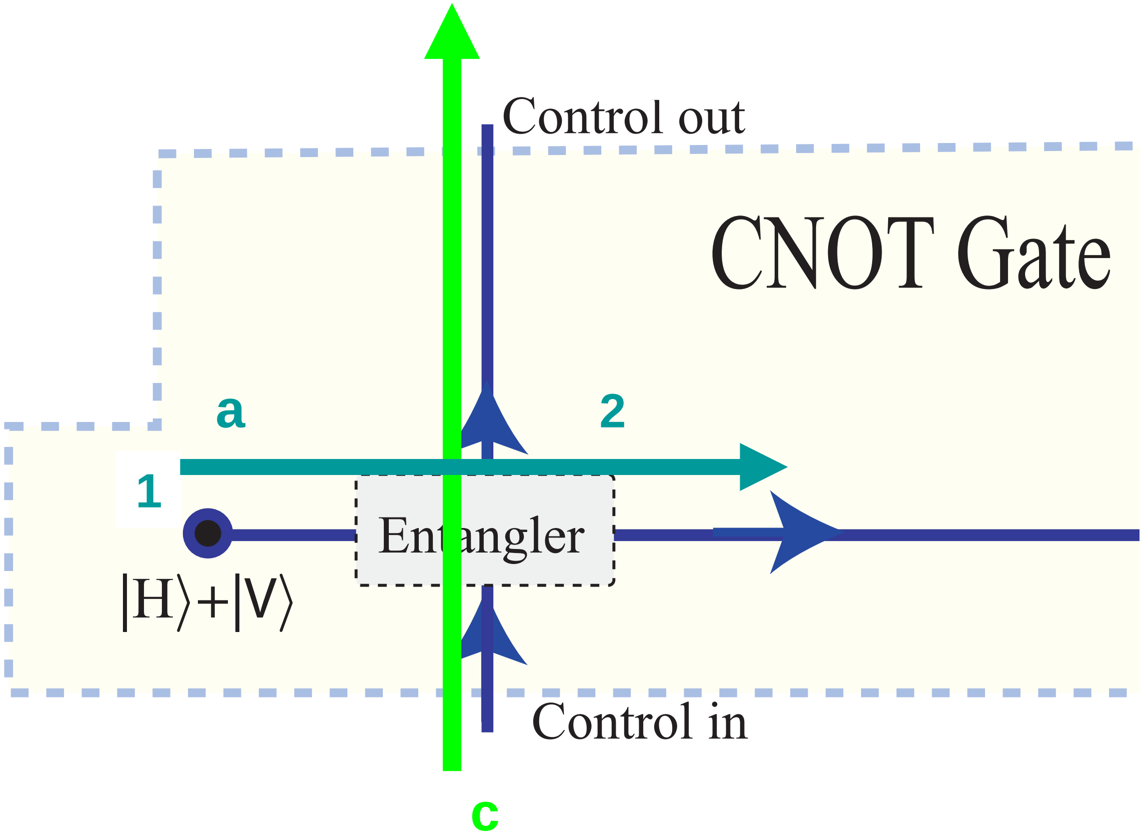


* T.B.Pittman, *et. al.*, Phys. Rev. A 64, 062311 (2001)

Control qubit in operation

- Consider an initial state

$$\underbrace{\left(c_0|H\rangle_c + c_1|V\rangle_c\right)}_{\text{Qubit c}} \otimes \underbrace{\left(|H\rangle + |V\rangle\right)}_{\text{Qubit a}} \otimes \underbrace{\left(d_0|H\rangle_t + d_1|V\rangle_t\right)}_{\text{Qubit t}}$$



1. Initial ancilla state is

$$|H\rangle_a + |V\rangle_a$$

2. Qubit-*c* and Qubit-*a* are entangled.

$$c_0|HH\rangle_{ca} + c_1|VV\rangle_{ca}$$

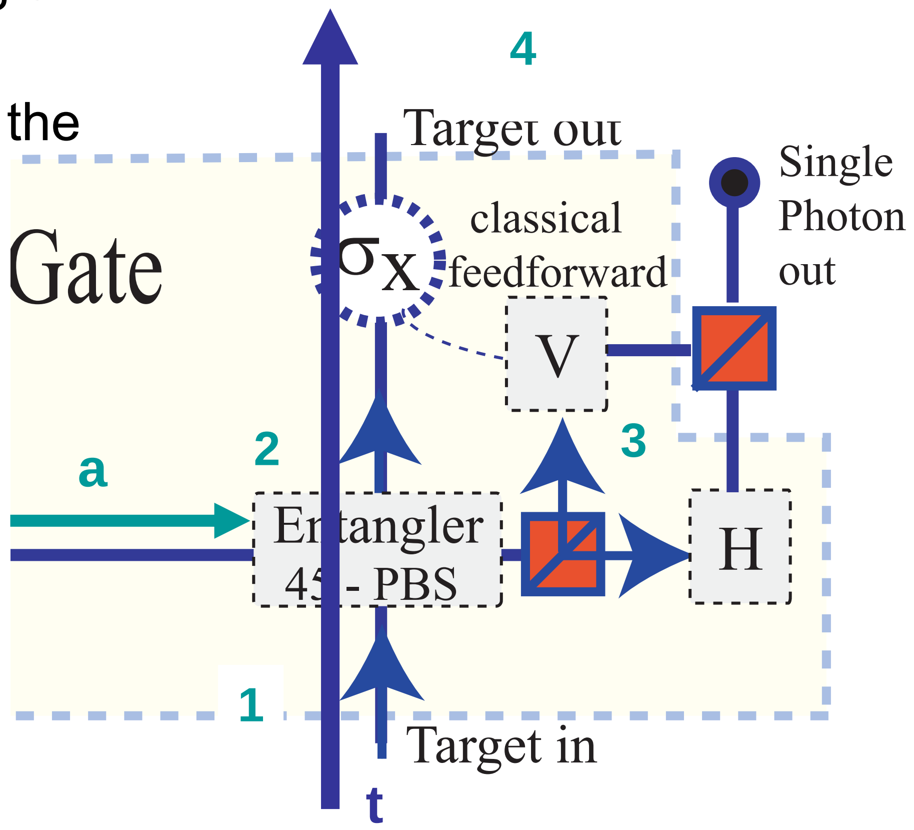
The target qubit

1. The initial state for Qubit-*b* and Qubit-*t* : $\underbrace{\left(c_0|HH\rangle_{ca}+c_1|VV\rangle_{ca}\right)}_{\text{Qubits c,a}}\otimes\underbrace{\left(d_0|H\rangle_t+d_1|V\rangle_t\right)}_{\text{Qubit t}}$

2. This 45-entangling gate is the entangler with 45-PBS instead of PBS.

   Qubit a and Qubit a are entangled in the basis of |DD> and $|\bar{D}\bar{D}\rangle$.

3. The classical feed forward correct the sign in the phase factor. (If V clicks, then do bit-flip on the target mode.)



4. The final state:

$$|\psi_{out}\rangle_{ct}=c_0d_0|HH\rangle_{ct}+c_0d_1|HV\rangle_{ct}+c_1d_0|VV\rangle_{ct}+c_1d_1|VH\rangle_{ct}$$

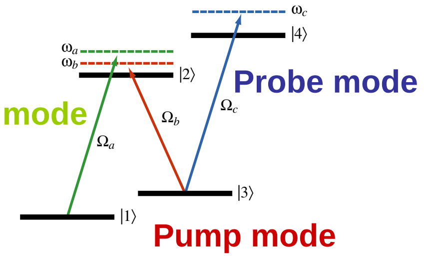
QND Logic for Scalability

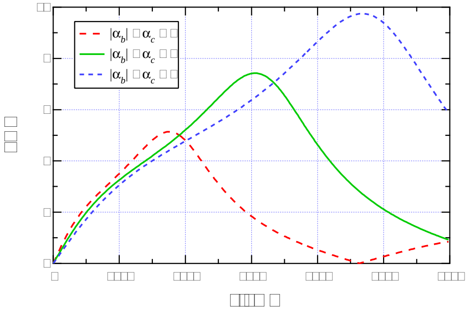
- Small nonlinearity pushes the upper bound of success probability to one.
  - How does the success probability approach to one?
    - the nonlinear amplification is exponential to the fixed amount of nonlinearity.
  - How feasible is this?
    - The intensity of coherent probe beam is 10^9 for the CNOT gate to suppress error rate to **$\sim 10^{-5}$.**
    - The intrinsic failure probability in QND logic can be in principle suppressed smaller than imperfection of each element.
    - This can be further improved by strategic state discrimination schemes.

A new regime of optical quantum computation

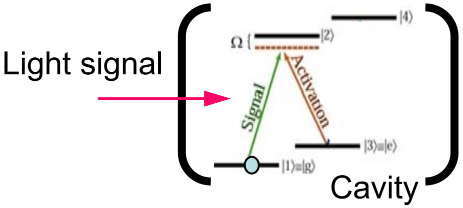
- **EIT system**

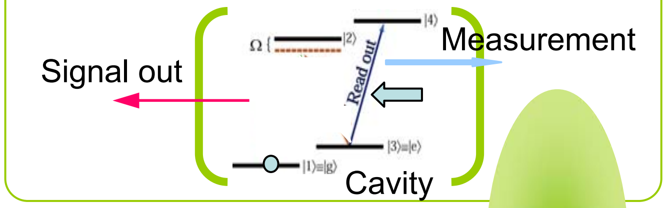
(Electromagnetically Induced Transparency)



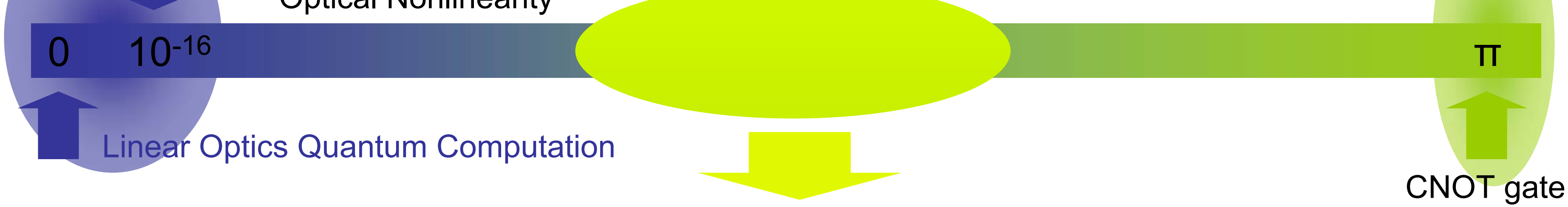


- **Optical Faber**
- **Cavity QED**





New physics to generate nonlinearity



New regime of quantum information processing

Quantum communication

Bell state analyser
Multi-partite entangled states (GHZ state)
Entanglement distribution protocols
Interface protocols

Scalable optical quantum computation

Gate distribution
Quantum wiring
Application to other physical realizations
New architectures based on the new technology

Summary: QND Logic for Scalability

- The QND logic allows us to exceed the Knill bounds on success probability of non-linear gates in linear optics quantum computation.
  - Our QND-based gates perform a near-deterministic CNOT operation.
    - Three single photon sources (can be made from weak coherent states)
    - **Five weak cross-Kerr nonlinearities ~10^{-5}**
    - Coherent light laser probe beams and homodyne detectors
    - Basic linear optics elements
    - Classical feed-forward.
  - No overhead resources are required to implement a universal set of gates in optical quantum computation.
- This suggests that the QND-based non-linear gates can be far more efficient to construct scalable linear optics quantum computation.