Black Hole Analogues and The Universality of the Hawking Effect

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Wave Equation

1+1 dimensional Painlevé-Gullstrand-Lemaître metric

$$ds^2 = dt^2 - [dx - v(x)\, dt]^2 = [1 - v^2]\, dt^2 + 2v\, dt\, dx - dx^2$$

Generalised Klein-Fock-Gordon equation

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x}\, v(x)\right) \left(\frac{\partial}{\partial t} + v(x)\, \frac{\partial}{\partial x}\right) \phi = \left(\frac{\partial^2}{\partial x^2} + F\left[\frac{\partial^2}{\partial x^2}\right]\right) \phi$$

Taylor expansion around horizon → surface gravity

$$v(x) = -1 + \kappa x + \mathcal{O}(\kappa^2 x^2)$$

- stationary modes with frequencies ω
- large cut-off $k_{\text{cutoff}} \gg \kappa, \omega$
- close to the horizon $\kappa |x| \ll 1$
- but still many cut-off lengths away $|x| k_{\text{cutoff}} \gg 1$
- spatial Laplace transformation

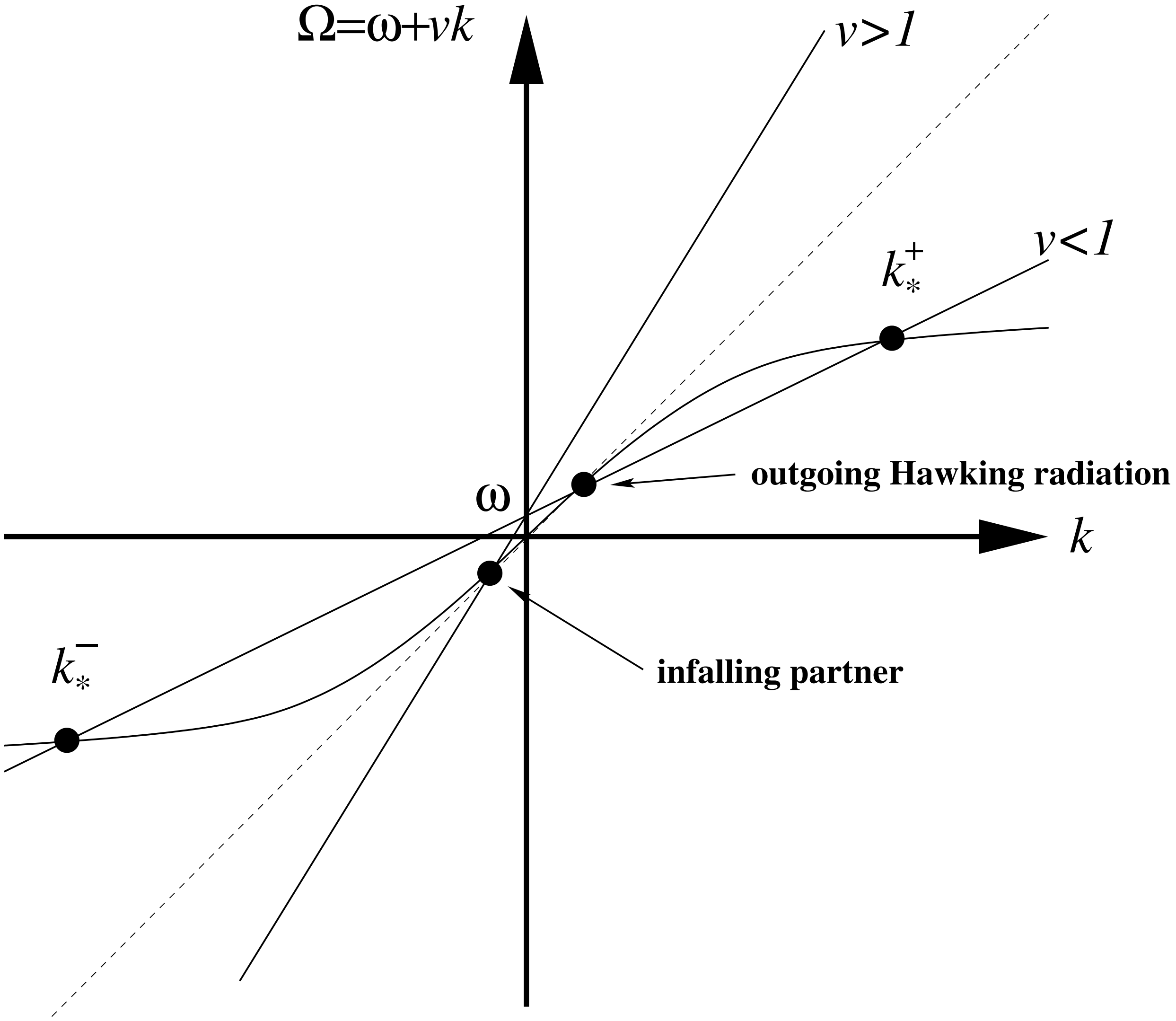
$$\left(F[s^2] - 2\kappa\, \frac{\partial}{\partial s}\, s^2 - 2(i\omega - \kappa)s\right) \widetilde{\phi}_\omega = 0$$

[Idea from Corley and Jacobson]

Dispersion Relation

$$(\omega + vk)^2 = k^2 - F[-k^2]$$

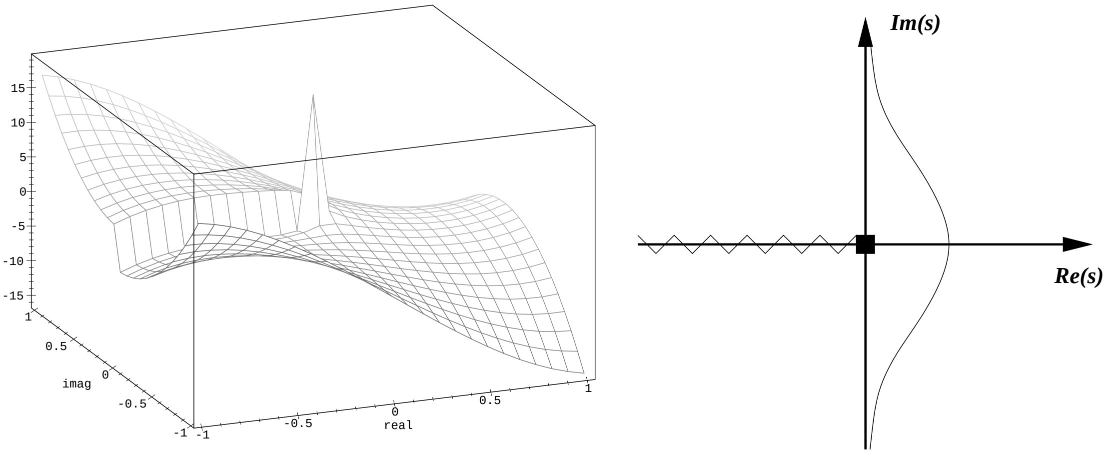
Sub-luminal case



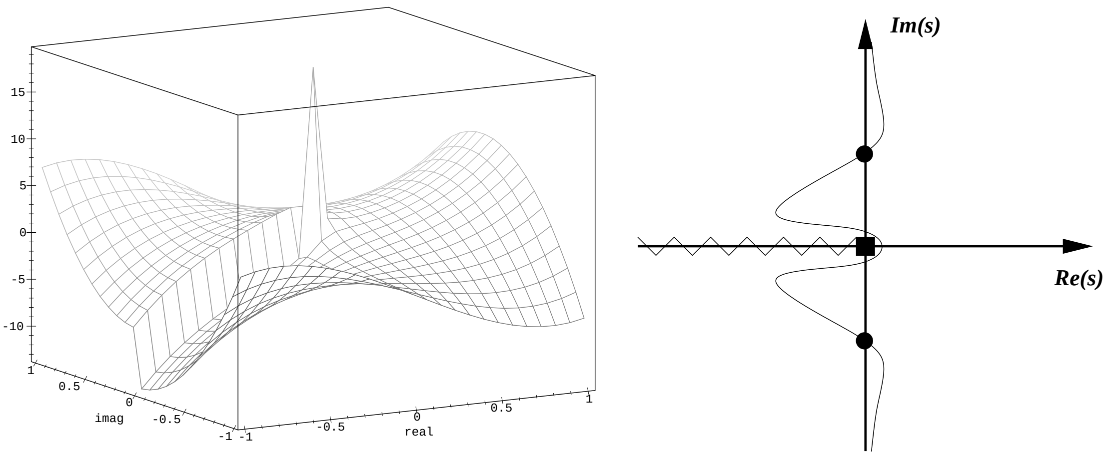
- dispersion $F[s^2]$ analytic $\rightarrow$ Laurent/Taylor
- coefficients and radius of convergence of order one
- asymptotically well separated from $\omega \propto k$

Saddle-Point Method

$$\widetilde{\phi}_\omega(s) = \frac{s^{-i\omega/\kappa}}{s}\ \exp\left\{\int ds\, \frac{F[s^2]}{2\kappa\, s^2}\right\}$$



Beyond horizon $x < 0$ exponential suppression



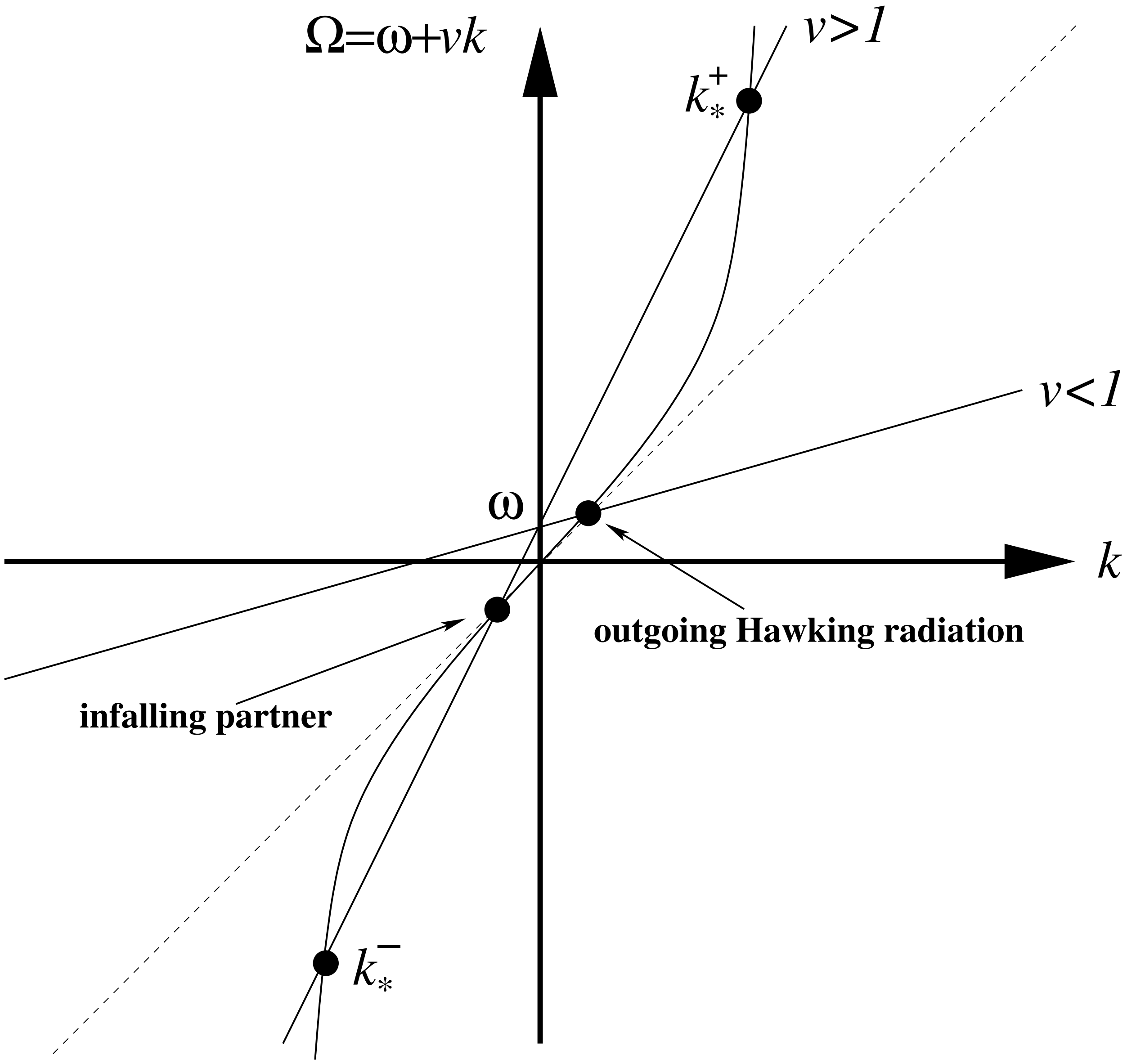
Decomposition of Hawking radiation for $x > 0$

$$\phi_\omega^{\text{out}} = \alpha_\omega \phi_+^{\text{in}} + \beta_\omega \phi_-^{\text{in}}\ , \qquad \boldsymbol{\alpha}\cdot\boldsymbol{\alpha}^\dagger - \boldsymbol{\beta}\cdot\boldsymbol{\beta}^\dagger = \mathbf{1}$$

Branch cut $\to$ Bogoliubov coefficients

$$|\beta_\omega| = e^{-\pi\omega/\kappa}|\alpha_\omega| \ \leadsto\ |\beta_\omega|^2 = \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

Superluminal Dispersion Relation

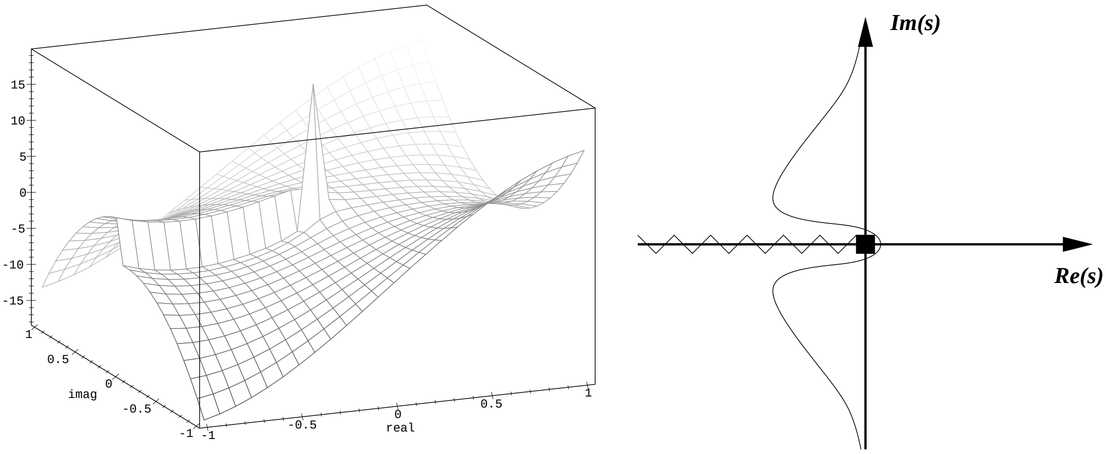


Origin of Hawking radiation

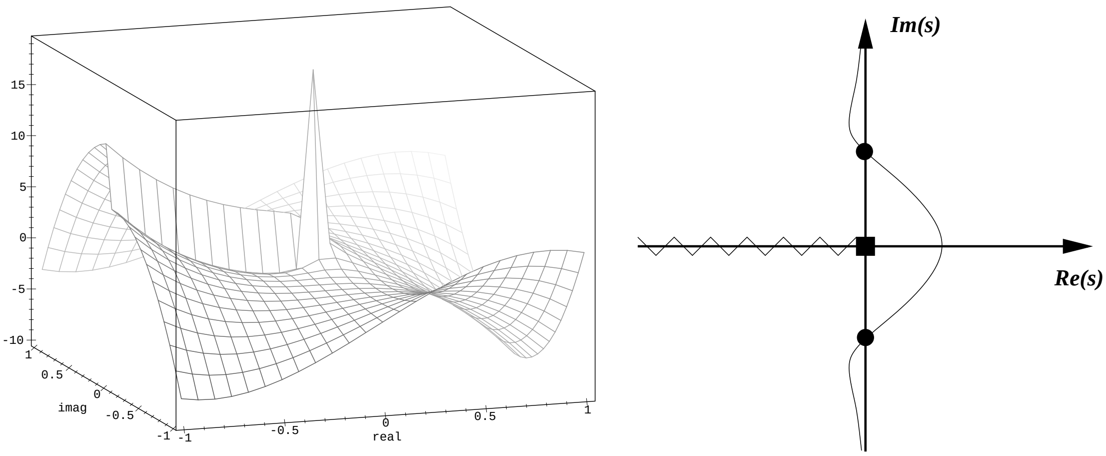
- sub-luminal: outside the black hole
- super-liminal: inside the black hole

Same assumptions otherwise

Again Saddle-Point Method



$\rightarrow$ Hawking radiation for $x > 0$



Beyond horizon $x < 0$ Planckian in-modes

$$|\beta_\omega| = e^{-\pi\omega/\kappa}|\alpha_\omega| \;\leadsto\; |\beta_\omega|^2 = \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

$\rightarrow$ again Hawking temperature

Entanglement

Opposite branch cut → infalling partner particles

$$\phi_\omega^{\rm partner} = \beta_\omega^{\rm partner}\phi_+^{\rm in} + \alpha_\omega^{\rm partner}\phi_-^{\rm in}$$

Complete set of out-modes (Hawking plus partner)

$$\phi_-^{\rm in} = \alpha_\omega^{\rm inv}\phi_\omega^{\rm partner} + \beta_\omega^{\rm inv}\phi_\omega^{\rm Hawking}$$

Initial (e.g., infalling) vacuum state

$$\left[\hat{a}_\omega^{\rm partner} + e^{-\pi\omega/\kappa}(\hat{a}_\omega^{\rm Hawking})^\dagger\right]|0\rangle_{\rm in} = \hat{a}_{\rm in}\,|0\rangle_{\rm in} = 0$$

→ Entanglement (pure state) inside ↔ outside

$$|0\rangle_{\rm in}^\omega \propto \exp\left\{e^{-\beta\omega/2}(\hat{a}_\omega^{\rm partner})^\dagger(\hat{a}_\omega^{\rm Hawking})^\dagger\right\}|0\rangle_{\rm out}^\omega$$

→ Effective thermal density matrix for outside

→ Black hole entropy/information paradox?

Sub-luminal versus Super-luminal?

Universality of Hawking Radiation

Trajectory of freely falling observer/detector

$$r_*(t \uparrow \infty) \simeq -t - A\,\exp\left\{-\frac{t}{2M}\right\} + B$$

Proper time $d\tau^2 = ds^2[t, r_*(t)]$ and modes

$$\tau \sim \exp\left\{-\frac{t}{2M}\right\} \ \rightsquigarrow \ F_\omega^{\rm in}(U) = \frac{1}{\sqrt{4\pi\omega}}\, e^{-i\omega U}$$

Doubly exponential behaviour $U = -4M\, e^{-[t - r_*]/(4M)}$

$\rightarrow$ Hawking radiation if modes leave Planck regime in their ground state with respect to freely falling observers

- freely falling frame
- ground state
- adiabatic evolution

Example: wave equation considered before

Counter-Examples

A: not freely falling frame (= fluid frame)

$$(\omega + v_{\rm fl} k)^2 = k^2 v_{\rm ph}^2(k) - 2i\omega\gamma(k)$$

Coupling to Planckian excitations in rest frame $2i\omega\gamma(k)$

$$\omega_+ \approx \frac{k^2}{2} \, \frac{v_{\rm ph}^2(k) - v_{\rm fl}^2}{v_{\rm fl} k + i\gamma(k)}$$

Miles instability $\Im(\omega) > 0$ → excitations

B: breakdown of adiabaticity

$$\omega^2 = \sin^2 k + m^2$$

Weakly time-dependent metric far away from black hole

$$ds^2 = a^2(t)[dt^2 - dx^2]$$

Normal mode expansion

$$\ddot{\phi}_k + \left[\sin^2 k + a^2(t)\, m^2\right] \phi_k = 0$$

create Planckian particles at $k \approx \pi\mathbb{N}$

C: black hole laser [Corley and Jacobson]

Summary

- black hole analogues reproduce major features
- underlying physics is understood (in principle)
- investigate influence of cut-off
- Hawking effect for large class of models

  ground state/freely falling frame/adiabaticity
- but also exceptions conceivable
- black hole information paradox?
- kinematics, but not dynamics: no Einstein Eq.

Outlook

New physics from the cosmic microwave background?

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