Quantum Error Correction and the Low Energy Problem in Quantum Gravity

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- The Low-Energy Problem in Quantum Gravity: find propagating modes in a background-independent quantum theory of gravity. Necessary, in order to make contact with physics.
- Quantum Information Theory: the low-energy problem is analogous to error-correction. The propagating modes are "noiseless subsystems".
- Example: the Schwarzchild black hole in LQG
- Implications for background-independence and locality.
- The relation between error correction and the renormalization group?

- Noiseless subsystems in quantum gravity:
  work in progress with David Poulin (IQC/PI)
- The Schwarzchild black hole as a noiseless subsystem:
  O.Dreyer, FM and L.Smolin, hep-th/0409056
- Noiseless subsystems and the renormalization group:
  work in progress with O.Dreyer, L.Smolin and P.Zanardi

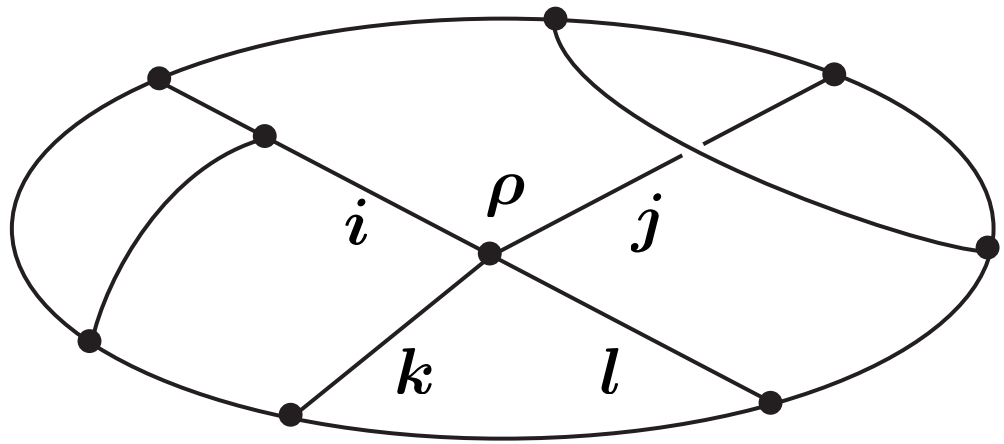
Quantum gravity must contain general relativity and quantum theory.

GR: Only events are physical. Any coordinates we may use to describe them have no physical meaning. Coordinate distances are not physical quantities.
In general relativity the metric is a dynamical variable. There is no background space and time.
GR is a background-independent theory.

Quantum gravity must be background-independent.

Spin Networks

Background independent quantum space



$$i, j, k, .. \in SU(2)$$

$$\rho : i \otimes j \otimes k \otimes l \longrightarrow 1$$

Important results about spin networks:

1. In the large spin limit, abstract (not embedded) graphs reproduce directions in Euclidean 3-space

(Penrose)

2. They are the basis states for background independent spatial geometry (Loop Quantum Gravity)

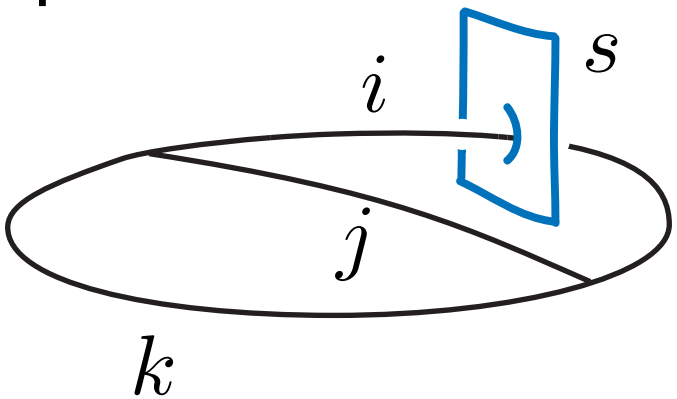
$$\langle S|S'\rangle = \delta_{SS'}$$

(Rovelli Smolin Ashtekar Lewandowski)

3. Area and volume operators have discrete spectra

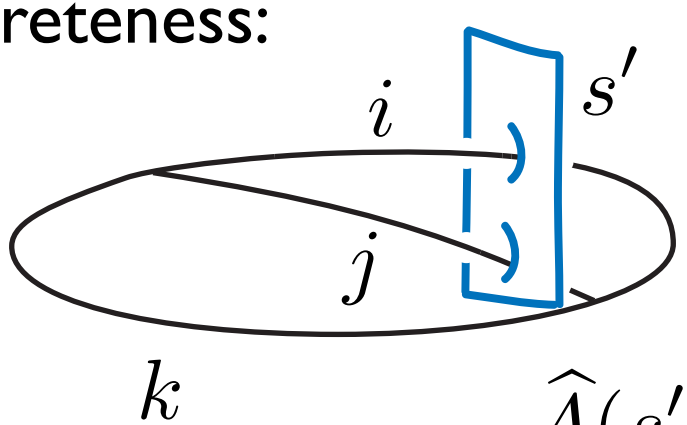
(Rovelli Smolin)

Area operator:



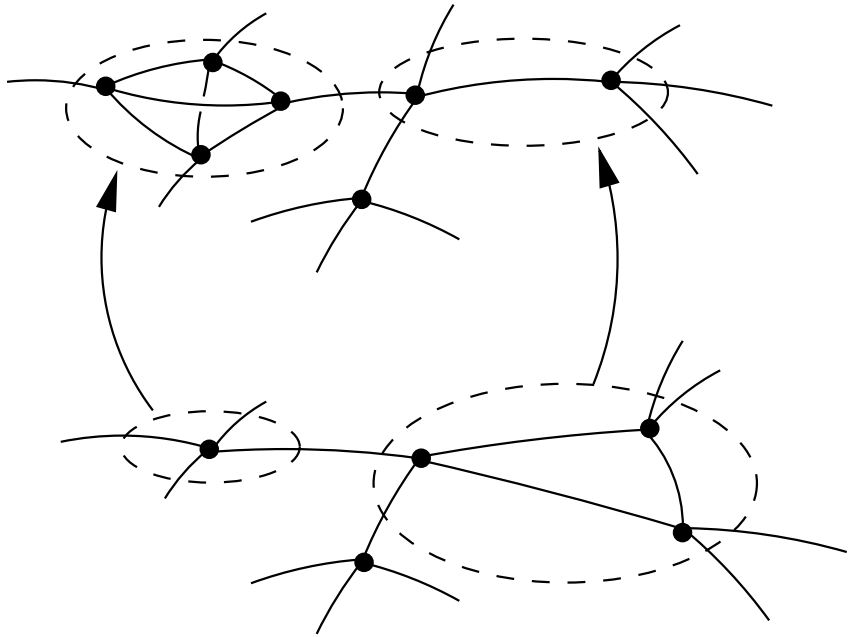
$$\widehat{A}(s) = l_{\mathrm{Pl}}^2 \sqrt{i(i+1)}$$

Discreteness:

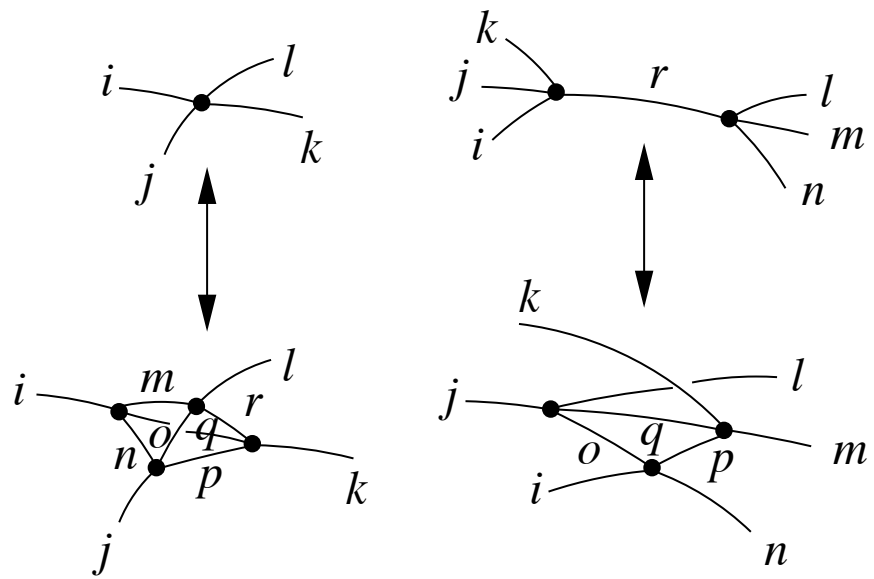


$$\widehat{A}(s') = l_{\mathrm{Pl}}^2 \left(\sqrt{i(i+1)} + \sqrt{j(j+1)} \right)$$

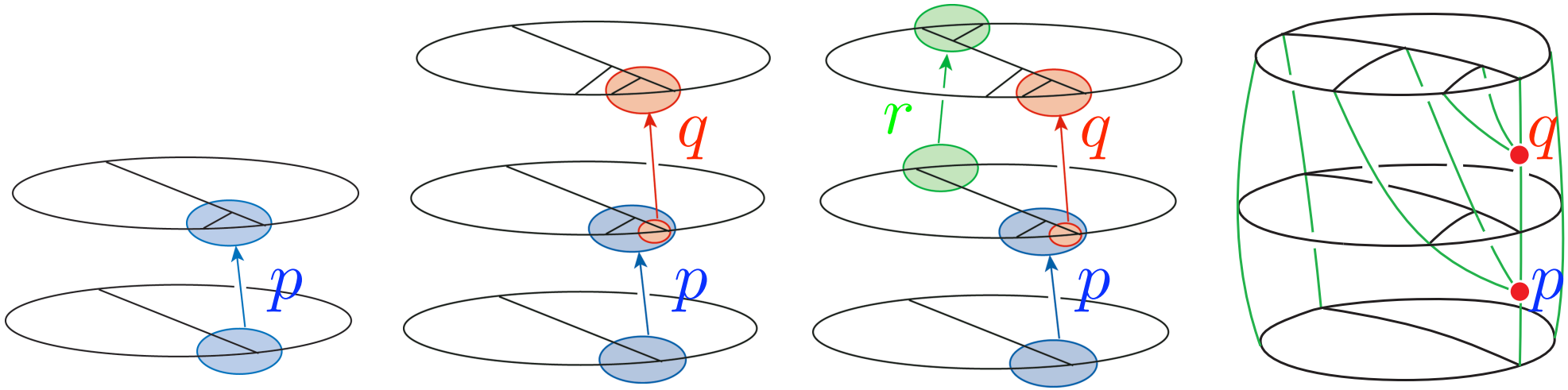
Local evolution of spin network graphs is by local moves:



There is a small number
of generating moves.
(cf quantum gates)

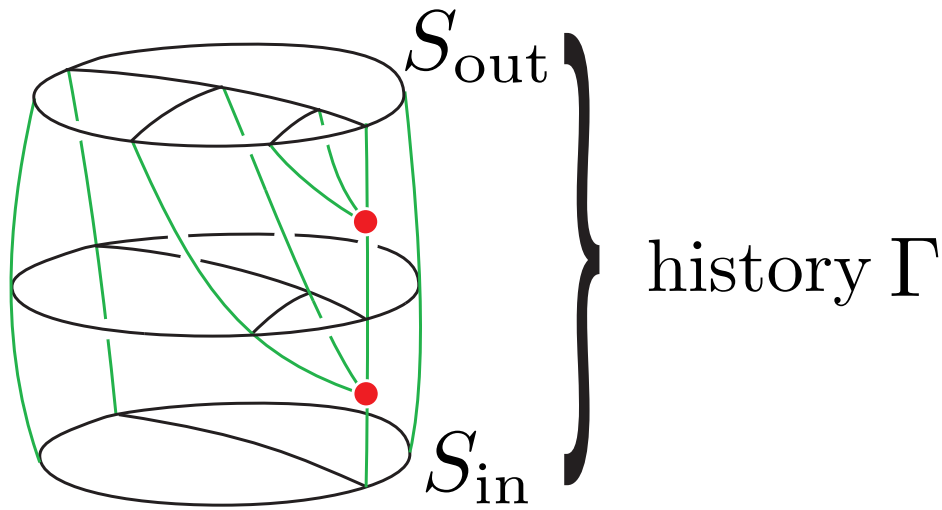


Evolution of spin networks generates a "spacetime network", or Spin Foam.



A spin foam Γ is a 2-dimensional complex with faces labeled by irreducible representations of the group and edges labeled by intertwiners.
Spacelike cuts through spin foams are spin networks.

Background independent models of quantum gravity:

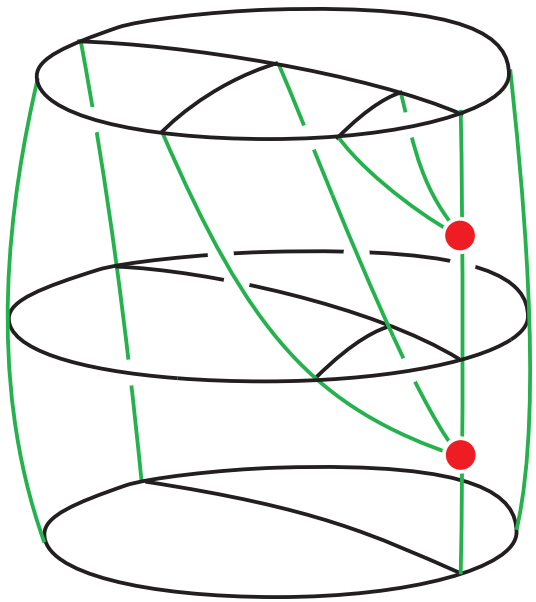


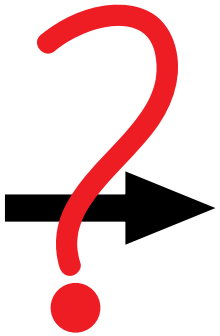
$$A(S_{\text{in}}, S_{\text{out}}) = \sum_{\Gamma} \sum_{\text{labels on } \Gamma} \prod_{f \in \Gamma} \dim j_f \prod_{v \in \Gamma} A_v(\{j\})$$

$$= \sum_{\text{histories}} \sum_{\text{labels}} \prod_{\text{events}} A(\text{event})$$

A choice of 2-complexes, group and amplitudes gives a model.

The low-energy problem





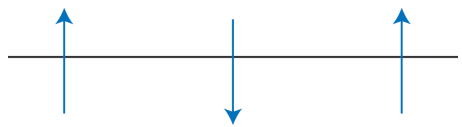
Planck scale events, causal structure, “spacetime”

Low energy ordinary spacetime

I need to be able to identify effective propagating degrees of freedom.

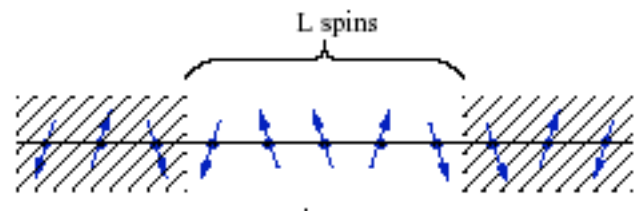
$$A(S_{\text{in}}, S_{\text{out}}) = \sum_{\Gamma} \sum_{\text{labels on } \Gamma} \prod_{f \in \Gamma} \dim j_f \prod_{v \in \Gamma} A_v(\{j\})$$

Cf. condensed matter theory:



$$H = -g \sum_{\langle ij \rangle} \sigma_i \cdot \sigma_j$$

microscopic



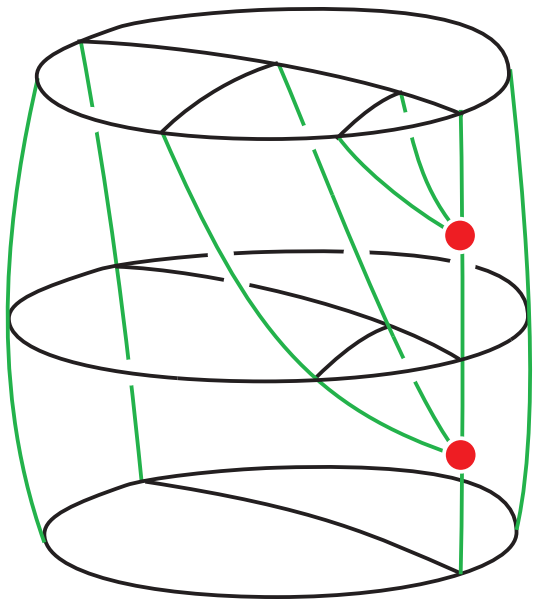
$$|k\rangle = \sum_k f(k)|k\rangle$$

macroscopic

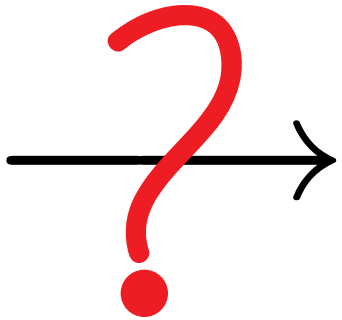
How can we obtain similar results for a background independent theory?

Note: To recover QFT in the low energy limit, we need to coarse-grain $\times 10^{20}$ and be left with a unitary theory.

The low-energy problem



Background-independent quantum gravity theory

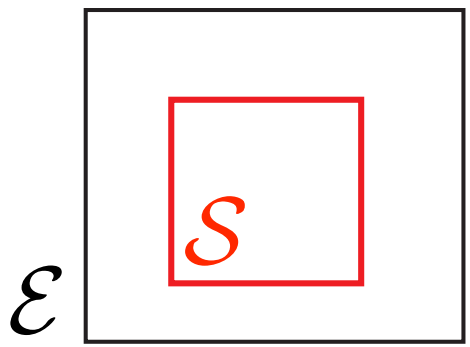


LAB
(particles on a fixed background)

This is the main open problem of background-independent approaches to quantum gravity (loop quantum gravity, spinfoams, etc).

The Idea:

What we see (QFT&GR) is what nature encoded.
(Almost) noiseless with respect to the Planck scale fluctuations.



$$\mathcal{H}^{\text{Total}} = \mathcal{H}^{\mathcal{S}} \otimes \mathcal{H}^{\mathcal{E}}$$

$$H = H_{\mathcal{S}} \otimes \mathbf{1}_{\mathcal{E}} + \mathbf{1}_{\mathcal{S}} \otimes H_{\mathcal{E}} + H_I$$

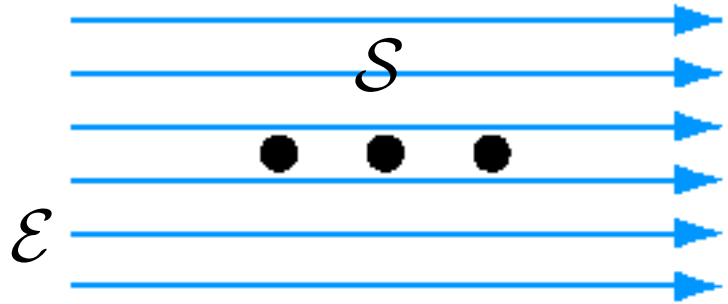
$$\phi : \mathcal{H}^{\mathcal{S}} \to \mathcal{H}^{\mathcal{S}'}$$

Propagating mode: A subsystem of $\mathcal{H}_{\mathcal{S}}$ that is conserved under ϕ, i.e., evolves unitarily: <u>Noiseless Subsystem</u>

When symmetries are present, protected subsystems (free particles) exist.

Noiseless Subsystems in Quantum Error Correction

Example:



$$\mathcal{H}_{\mathcal{S}} = \mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^2$$

$$S_\alpha = \sum_{i=1}^{3} \frac{\sigma_\alpha^i}{2}$$

$\alpha = x, y, z$

$su(2)$ rotations

Symmetric under S_3

$$H^{\text{int}} = \sum_\alpha S_\alpha \otimes E_\alpha$$

$$\mathcal{H}_{\mathcal{S}} \simeq \mathcal{H}_{\frac{3}{2}} \oplus \underbrace{\mathcal{H}_{\frac{1}{2}} \oplus \mathcal{H}_{\frac{1}{2}}}_{|\lambda, s_z\rangle \simeq |\lambda\rangle_{\text{NS}} \otimes |s_z\rangle}$$

$\lambda = 0, 1$

$s_z = \pm\frac{1}{2}$

protected noisy

$$\boxed{\mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \simeq \mathcal{H}_{\text{NS}} \otimes \mathcal{H}_{\frac{1}{2}}}$$

Noiseless subsystem

On

$$\mathcal{H}_{\frac{1}{2}} \otimes \mathcal{H}_{\frac{1}{2}} \simeq \mathcal{H}_{\mathrm{NS}} \otimes \mathcal{H}_{\frac{1}{2}}$$

the evolution takes the form

$$S_\alpha \simeq \mathbf{1}_{\mathcal{H}_{\mathrm{NS}}} \otimes \sigma_{(\alpha)}$$

That is, $\mathcal{H}_{\mathrm{NS}}$ behaves as a "free particle" (coherent degree of freedom).

Note: $\mathcal{H}_{\mathrm{NS}}$ is not a subspace of $\mathcal{H}_{\mathcal{S}}$.

Noiseless subsystems

Knill, Laflamme & Viola, Zanardi & Rasetti

- Interacting system $\mathcal{H}_{\mathcal{S}}$ and environment $\mathcal{H}_{\mathcal{E}}$. $\mathcal{H}_{\mathcal{S}} = \mathbf{C}^{2^{\otimes 3}}$

- Interaction algebra $\mathcal{A}^{\mathrm{int}} \subseteq \mathcal{B}\left(\mathcal{H}^{\mathcal{S}}\right)$ decomposes as $su(2)$

$$\mathcal{A}^{\mathrm{int}} \simeq \bigoplus_j \mathbf{1}_{\mu_j} \otimes \mathcal{B}\left(\mathbf{C}^{d_j}\right)$$

$S_\alpha \simeq \mathbf{1}_{\mathcal{H}_{\mathrm{NS}}} \otimes \sigma_\alpha$

- Corresponding interaction commutant $\mathcal{A}^{\mathrm{int}\prime} \subseteq \mathcal{B}\left(\mathcal{H}^{\mathcal{S}}\right)$ is

$$\mathcal{A}^{\mathrm{int}\prime} \simeq \bigoplus_j \mathcal{B}\left(\mathbf{C}^{\mu_j}\right) \otimes \mathbf{1}_{d_j}$$

- Symmetries in $\mathcal{A}^{\mathrm{int}} \iff$ non-trivial $\mathcal{A}^{\mathrm{int}\prime}$.

- $\mathcal{H}_{\mathcal{S}}$ decomposes as

$$\mathcal{H}_{\mathcal{S}} \simeq \bigoplus_j \boxed{\mathbf{C}^{\mu_j}} \otimes \mathbf{C}^{d_j}$$

$\mathcal{H}_{\frac{3}{2}} \oplus \mathcal{H}_{\mathrm{NS}} \otimes \mathcal{H}_{\frac{1}{2}}$

Noiseless Subsystems in Quantum Gravity

- Noiseless subsystems are useful for describing the long-term behavior of the system because they are conserved.

- If we divide the quantum gravitational field into subsystems, those properties that are conserved under interactions between the subsystems will characterize the low-energy limit spacetime geometry.

- The commutant $\mathcal{A}^{\mathrm{int}'}$ should include the symmetries of a classical spacetime (e.g. Poincare, deSitter).

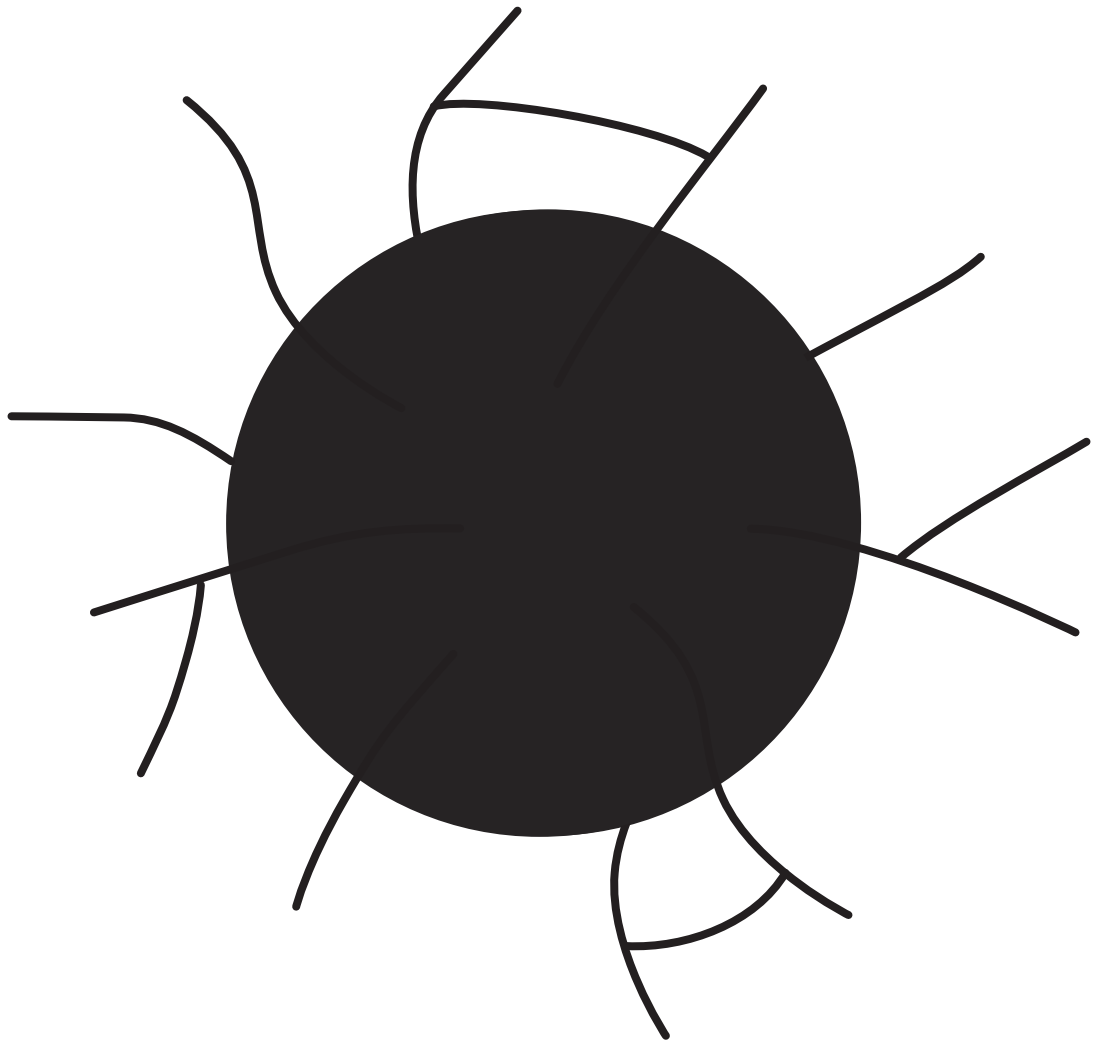
Example....

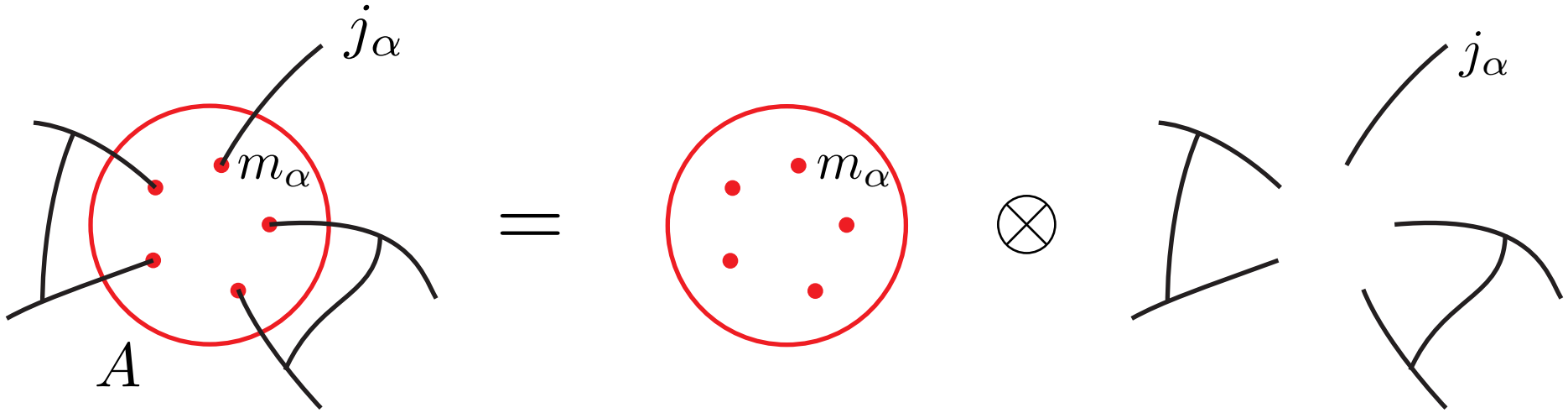
Example of a NS:

The Schwarzchild black hole in Loop Quantum Gravity

O.Dreyer, FM and L.Smolin,hep-th/0409056

We can use the spin networks to count the states of a black hole.





System

Environment

$|m_\alpha| \leq j_\alpha$

$\sum_\alpha m_\alpha = 0$

$\mathcal{H}_A^{\mathcal{S}} =$ invariant states of $U(1)$ Chern-Simons on punctured S^2

$\mathcal{S}^{\mathcal{E}}_{\{j_\alpha\}} =$ spinnets ending on the horizon

Ashtekar, Krasnov & Baez

$$\boxed{\mathcal{H}^{\text{total}} = \sum_{\{j_\alpha\}} \mathcal{H}_A^{\mathcal{S}} \otimes \mathcal{S}^{\mathcal{E}}_{\{j_\alpha\}}}$$

Q: Which states in $\mathcal{H}_A^{\mathcal{S}}$ correspond to the quantum analogue of a Schwarzchild black hole of area A?

Use symmetries in the system-environment interaction:

A classical SBH has $SO(3)$ symmetry.

➡ If there is a microscopic analogue of a SBH, there must be a microscopic analogue, G_q, of $SO(3)$ in the dynamics, that becomes $SO(3)$ in the classical limit.

➡ This corresponds to a non-trivial $\mathcal{A}^{\mathrm{int}'}$ that identifies the QSBH as a noiseless subsystem, i.e.,

$$\boxed{G_q \subseteq \mathcal{A}^{\mathrm{int}'}}$$

We do not know $\mathcal{B}\left(\mathcal{H}_A^{\mathcal{S}}\right)$ or $\mathcal{A}^{\mathrm{int}}$, but it is possible to restrict it sufficiently to find the noiseless subsystem corresponding to G_q.

G_q : Microscopic $SO(3)$

- G_q is a discrete group

- $\mathcal{A}^{\text{int}}$ must act simultaneously and locally on $\mathcal{H}^{\mathcal{S}}$ and $\mathcal{H}^{\mathcal{E}}$

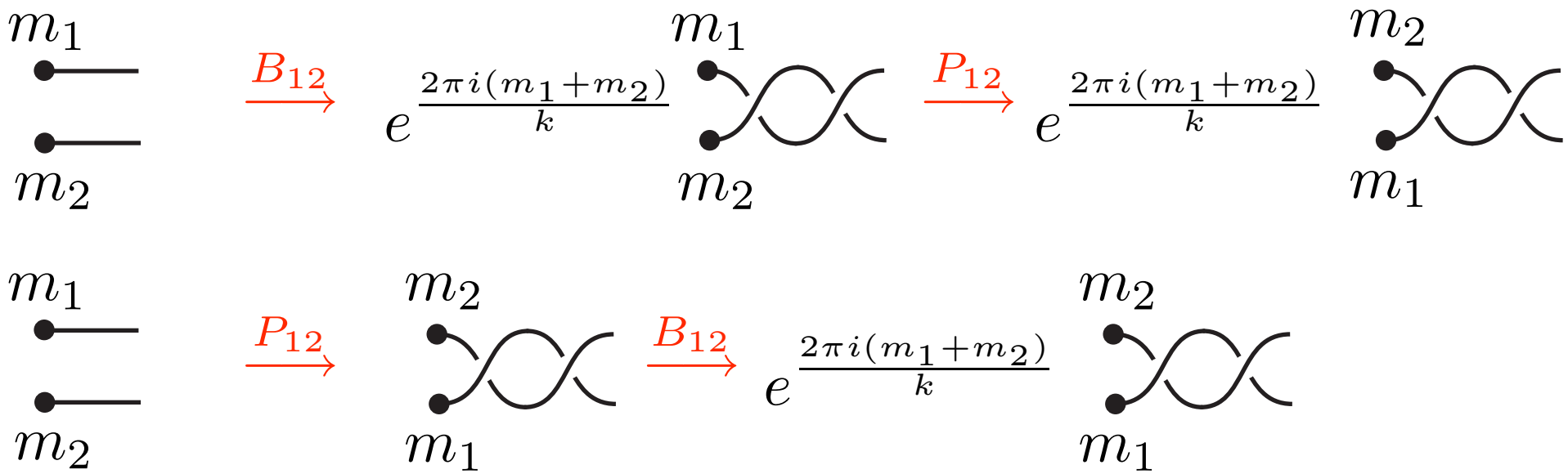
- A sufficient set of generators for $\mathcal{A}^{\text{int}}$ is:
  - Add/remove punctures
  - Braid (symmetric)

$$B = \sum_{\langle i,j \rangle} B_{ij}$$

- m_1 —— j_1 $\xrightarrow{B_{12}}$ $e^{\frac{2\pi i(m_1+m_2)}{k}}$ m_1  j_1
  m_2 —— j_2 m_2 j_2

- $G_q : \mathcal{H}_A^{\mathcal{S}} \longrightarrow \mathcal{H}_A^{\mathcal{S}} \Rightarrow$ braidings only

$\longrightarrow$



$$\Rightarrow \quad \boxed{G_q \subset \prod_j P_{K_j}}$$

$K_j =$ no. of punctures with same j

- G_q must be transitive $\Rightarrow K_j =$ number of punctures

  All j are the same in a quantum Schwarzchild black hole

$\longrightarrow$

The state space of the quantum Schwarzchild black hole is the noiseless subsystem corresponding to the G_q symmetry of the dynamics.

$$\boxed{\mathcal{H}_A^{\mathrm{QSBH}} = \mathcal{H}_j^{\mathrm{QBH}}}$$

Quantum Schwarzschild Black Hole

$$\begin{aligned} S_A^{\mathrm{QSBH}} &\approx S_{j_{\min}}^{\mathrm{QBH}} \\ &= \ln \dim \mathcal{H}_{j_{\min}}^{\mathrm{QBH}} \end{aligned}$$

Quasinormal modes: excited states of the NS.

Rotating black holes: no transitivity.

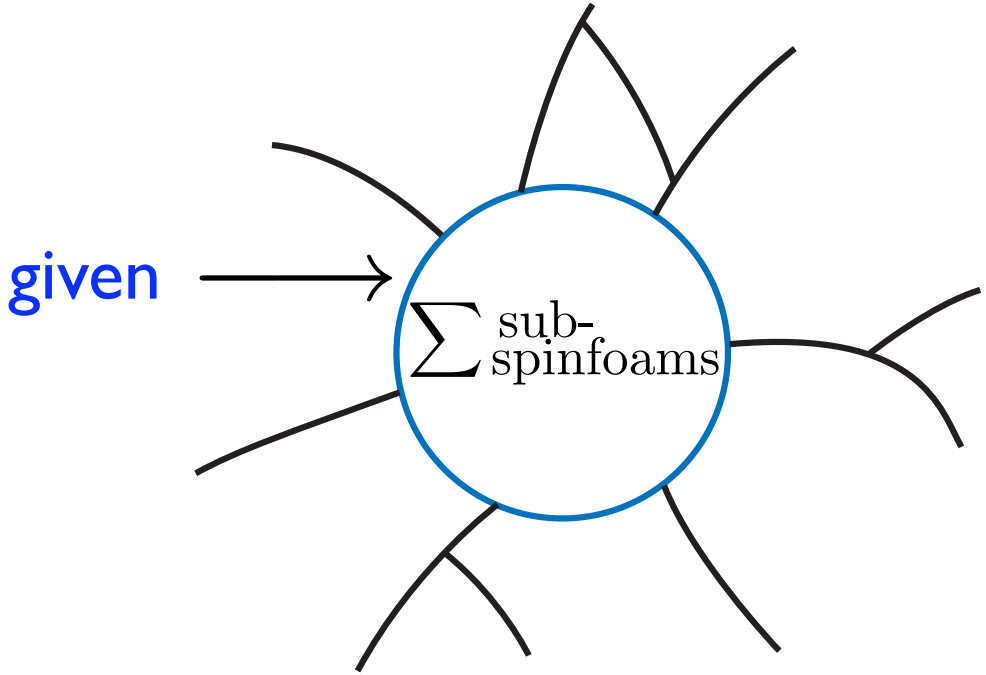
Lessons

- The NS is a dynamical construction.
  NB: Symmetries at the kinematical level gives $\mathcal{H}_m^{\mathrm{QBH}}$.
- For the low-energy behavior, we only need the interaction dynamics, and from that, we only need the symmetries.
- $\mathcal{A}^{\mathrm{int}}$ depends on the energy level/approximation we are interested in.
- Conversely, in model-building, we can use the low-energy symmetries to constrain the microscopic dynamics.

Propagating modes vs background independence

The NS method is background-independent in the sense that it does not rely on a particular graph/state and is defined via the dynamics.

BUT: The NS is a subsystem of a subsystem: $\mathcal{H}_j^{\mathrm{QBH}}$ in $\mathcal{H}_A^{\mathrm{QSBH}}$.
It finds the first if you give the second.



There are very interesting situations in QG when the boundary is given.

The boundary does not need to be a physical one:

$$\mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^2$$

$$\bullet \quad \bullet \quad \bullet$$

$$\downarrow$$

$$\mathbf{C}^2$$

$$\bullet$$

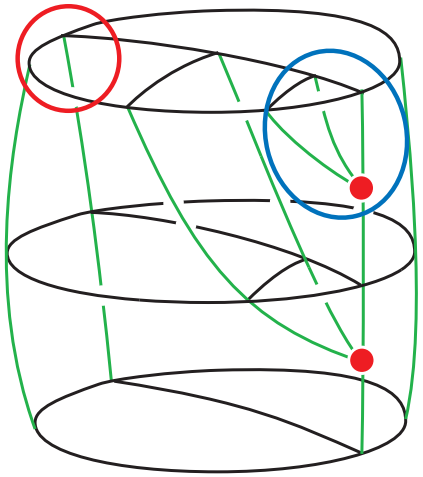
high energy modes environment

$$\downarrow$$ boundary

low energy modes

Relate NS to renormalization group for quantum systems.
NS as quasiparticles.
In progress with O.Dreyer, L.Smolin and P.Zanardi.

NS & locality,
or,
Microscopic vs Macroscopic locality



One can try to assign geometric/local/causal properties to the subfoams/subgraphs.
But they are not eigenstates of the dynamics. One has to take into account the sum over foams.

Note that in the $\mathbf{C}^{2^{\otimes 3}}$ example, the preserved qubit is none of the originals.

NS method suggests:
Assign locality/geometric/causal properties to the effective NSs.

Outlook:

Black hole case: Quasinormal modes as excitations in
Rotating black holes
BH in spin foams/other models
$G_q \to SO(3)$

Propagating modes in quantum gravity with a boundary

Separate scales: environment $\leftrightarrow$ small-scale fluctuations
system $\leftrightarrow$ coarse-grained fluctuations

NS theory: Approximate/emergent NS definition
Poincare symmetry