Philosophy of Financial Modelling

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OUTLINE

- The purpose of modelling is to make improvements in the operational aspects of decision making.

- The current state of affairs in decision making has built in models in place, with known deficiencies.

- We seek through our models an improved set of decision aids that help us probe the consequences of particular actions in worlds that we perceive as relevant to our future.

- There are many activities that end up requiring models and we can at best provide a partial list.
  - Understanding the risk exposures of current positions.
  - Altering these exposures in an efficient and economical manner.
  - Discovering trading opportunities.
  - Designing trades to exploit these opportunities.
  - Creating financial products that meet customer needs.
  - Delivering desired products at economical terms.

Risk Management Models

- Foremost in risk exposure assessment is the construction of the spot slide. It is well understood that a book of derivative products is sensitive to movements in the underlying and the spot slide values the book for different levels of the underlier. For each product on the books, a model must be run to determine the change in the resulting value. This will require a generation of a space of paths for the prices of the underlying assets and an accompanying valuation of the financial product. Each product has a default model assigned for this activity.

- Other risk exposures of interest include movements in volatilities, skews and vol spreads on the vanilla option surface. These activities create a need for reasonable path spaces consistent with a variety of relevant configurations of the option surface.

- Option positions can be taken to control the overall level of exposure to movements in volatility, skew and the vol spread.

- A first order analysis indicates that some products are exposed in a very nonlinear way to movements in the volatility or the skew. In this case an assessment is sought of the sensitivity of the product value to the volatility of the volatility or the volatility of the skew or the vol spread for that matter.

- By calibrating a stochastic vol or stochastic skew model to the option surface one may seek to define option positions that immunize or manage the product exposure to vol of vol and vol of skew.

Trading Models

- Unlike risk management models, trading models seek to depart realistically from market prices with a view to detecting possibly mispriced assets leading to trading opportunities.

- These models typically specify a set of latent or unobservable factors that are postulated to follow a Markov stochastic evolution and the prices of a set traded assets are then expressed in terms of these factors via a discounted expected cash flow calculation based on a set of paths consistent with the factors. One then takes the time series of traded asset prices to simultaneously infer the paths of the factors by an application of non-linear filtering and to estimate by maximum likelihood the parameters of the dynamics in the factor stochastic evoloution.

- The parameters once estimated on a past time series are then never reestimated for years unless one observes a bias developing in the residuals with a consequent accumulation of trading losses, upon which a reestimation is triggered with a need on occasion to increase the number of factors.

- The factors may represent movements in the underlying physical reality as in stochastic volatility or they may represent movements in risk premia as in stochastic skews. The trader is singularly uninterested in what they represent, as long as they continue to provide trading opportunities.

- Models may indicate a significant departure of market prices from model prices on the basis of which a trade is set up. However, one has then to determine the exit strategy or the stop loss/collect gain rule.

- One typically waits till the convergence is restablished subject to a fixed time exposure and a preset loss exposure.

- The assets traded using models include swap rates, swaptions, vanilla options, and credit default swaps using capital structure arbitrage models.

Models and Product Design

- Models play a critical role in the development of financial products. We understand that unlike many other commercial products financial products are produced after they are sold. They are not necessarily hedged to replication as this is typically not possible, but they are hedged to risk exposures that are tolerable.

- Hence a proposed product coming from customer requests must be analysed for the risks associated with its issue. One may then determine strategies associated with controlling this risk exposure and the likely costs of doing so. The latter can then be built into the price.

- The product once issued will go into a default mark to market system with exposure to movements in the spot and the option surface. This will impose costs that are to some extent foreseeable and need to be charged for in the price.

Models and Products

- A basic principle in short horizon derivative product design is the recognition that one may hold any function $c(S)$ of the stock price, accessing curvature by option positioning.

- For an optimal design $c^*(S)$ it is easily seen that the marginal utility per initial dollar spent must be equated across states or that for a physical density $p(S)$ and a risk neutral or pricing density $q(S)$ for utility function $U(C)$ we must have that

$$\frac{U'(c^*(S))p(S)}{q(S)} = \lambda$$

- It follows that

$$c^*(S) = \left(U'\right)^{-1}\left(\lambda\frac{q(S)}{p(S)}\right)$$

- The general shape of the desired product then follows the structure of

$$\frac{p(S)}{q(S)}.$$

- Given some temporal stability of the densities p, q and their relative positioning one gets the demand for the payment through time of structured non-linear functions of the stock price.

- These functions create the need for the analysis of their risk structures and the costs of their delivery.

Gaussian Products

- The basic model is to consider both p, q to be Gaussian with $S(0) = 1$ and respective means μ, r and common volatilities σ.

- The product structure is then

$$S^{\frac{\mu - r}{\sigma^2}}$$

- Limiting the structure to a compact interval in the return, we get a cliquet for $\mu > r$ and a reverse compounding cliquet for $\mu < r$.

- Now consider differences in volatilities, with statistical volatility below its risk neutral counterpart, the log structure switches to quadratic in the log of the spot with a negative coefficient and hence we a reverse swing cliquet that pays more for small absolute returns and a declining payout as the absolute return rises.

- In the opposite case with statistical volatilities dominating, as may be relevant today, we get the swing cliquet, that has payouts increasing in the absolute return.

Estimated Products

- One may estimate both densities in the parametric class of say the $CGMY$ model, using time series data for the statistical density and options data for the risk neutral.

- The results for the year 2002 on the SPX, DAX, $FTSE$, $IBEX$, and $NIKKEI$ are

Statistical Estimation

	SPX	DAX	FTSE	IBEX	NIKKEI
vols	0.1679	0.2569	0.1718	0.2222	0.2445
C	13.02	23.04	.2927	2.79	5.11
G	94.64	65.24	51.99	63.08	68.57
M	100.2	78.10	56.37	75.60	66.16
Y	0.5348	0.4925	1.21	0.8963	0.7982

- The corresponding risk neutral results are, using averages for 12 mid month estimations,

Risk Neutral Parameter Estimates

	SPX	DAX	FTSE	IBEX	NIKKE
vols	0.3332	0.5088	0.3246	0.3372	0.6182
C	0.8689	1.2594	0.2902	0.8757	3.6502
G	6.9420	5.7464	5.1308	8.2975	10.2088
M	31.1907	27.9887	41.7202	40.9873	28.5528
Y	0.8801	0.9914	1.1902	0.9625	0.9228

- For a one month horizon we present graphs for the statistical and risk neutral densities and the optimal product structure for a zero mean and interest rate.

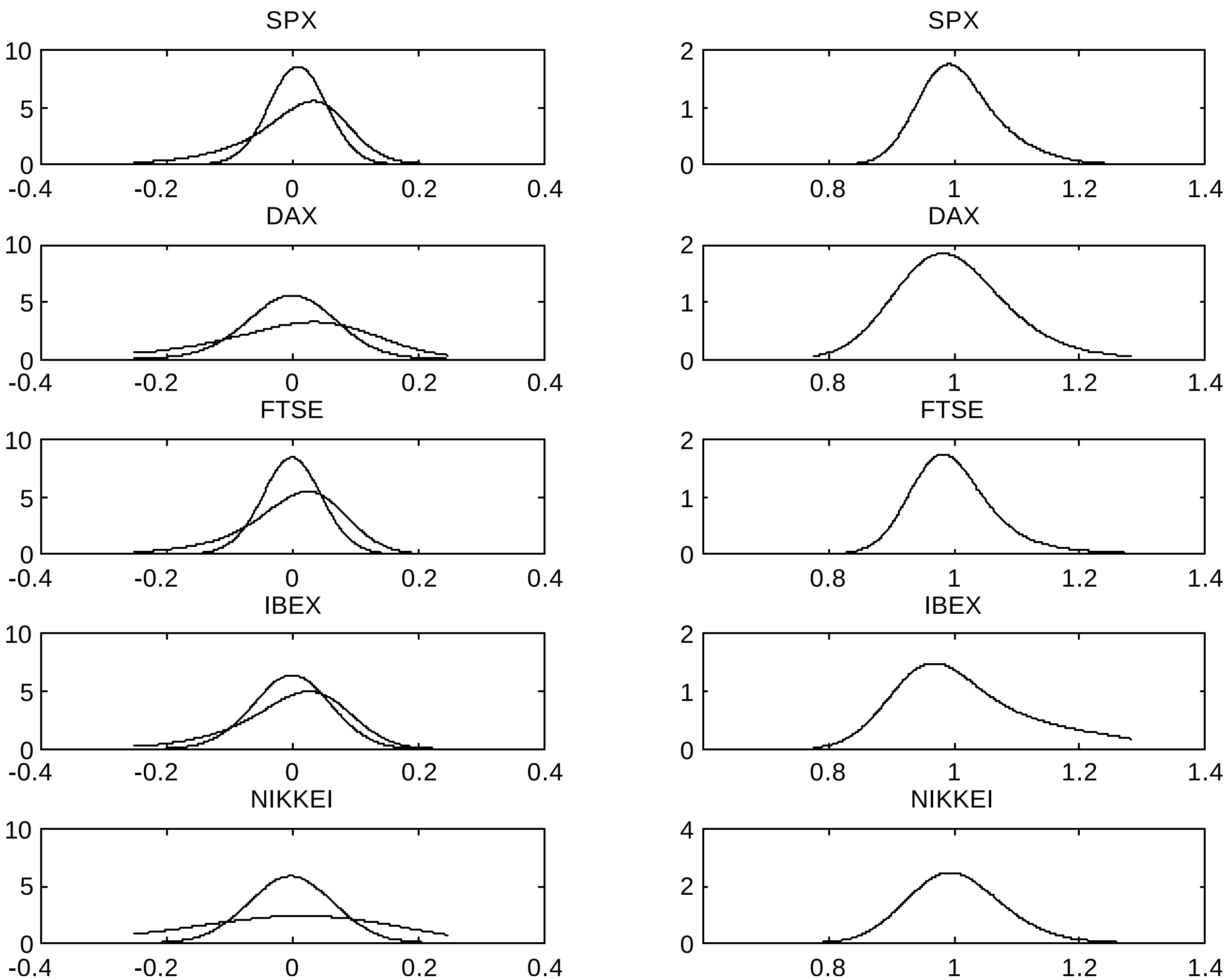

SPX
10
5
0
-0.4
-0.2
0
0.2
0.4
SPX
2
1
0
0.8
1
1.2
1.4
DAX
10
5
0
-0.4
-0.2
0
0.2
0.4
DAX
2
1
0
0.8
1
1.2
1.4
FTSE
10
5
0
-0.4
-0.2
0
0.2
0.4
FTSE
2
1
0
0.8
1
1.2
1.4
IBEX
10
5
0
-0.4
-0.2
0
0.2
0.4
IBEX
2
1
0
0.8
1
1.2
1.4
NIKKEI
10
5
0
-0.4
-0.2
0
0.2
0.4
NIKKEI
4
2
0
0.8
1
1.2
1.4

Figure 1:

Down and In Trigger Reedemable Notes

- Yet another interesting product is the 50% down and in Trigger reedemable Note that pays 100 in the absence of the down event, and otherwise pays the minimum of the final value and 100.

- Using the above risk neutral and statistical parameters for $CGMY$ and methods for exact analytical pricing of equity default swaps for Lévy processes with two sided jumps developed here at Isaac Newton in collaboration with Soren Asmussen and Martin Pistorius we derive the five year statistical and risk neutral probabilities of first passage to the 50% down barrier at 2% statistically and 14% risk neutrally.

- The EDS quote at 50% recovery is statistically $14bp$ and risk neutrally $141bp$.

- The product sells a high price low probability event and in recognition of marginal utility going to infinity at zero maintains exposure to a nonnegative cash flow.

Surface Exposures of Products

- These products have in addition to the exposure to movements in the spot asset on which they are contingent, an exposure to movements in the option surface when it comes to a mark to market analysis of the products.

- We present with a view to illustrating surface deltas and gammas the cliquet and reverse cliquet volgamma and the ATM swing cliquet skewgamma.

Marked to Market Risks of the Variance Swap

- The variance swap contract trades quite popularly and options on volatility are beginning to trade. The risk exposures of these contracts are of interest and there is then interest in models that address them.

- The well known robust hedge for the variance swap is the short position in the log contract.

- This may easily be priced by any model fitting the option smile at a single maturity with a known characteristic function for the log price relative.

- We may also extract from the market the ATM volatility, the 95-105 3 month skew and the 95-100-105 3 month convexity.

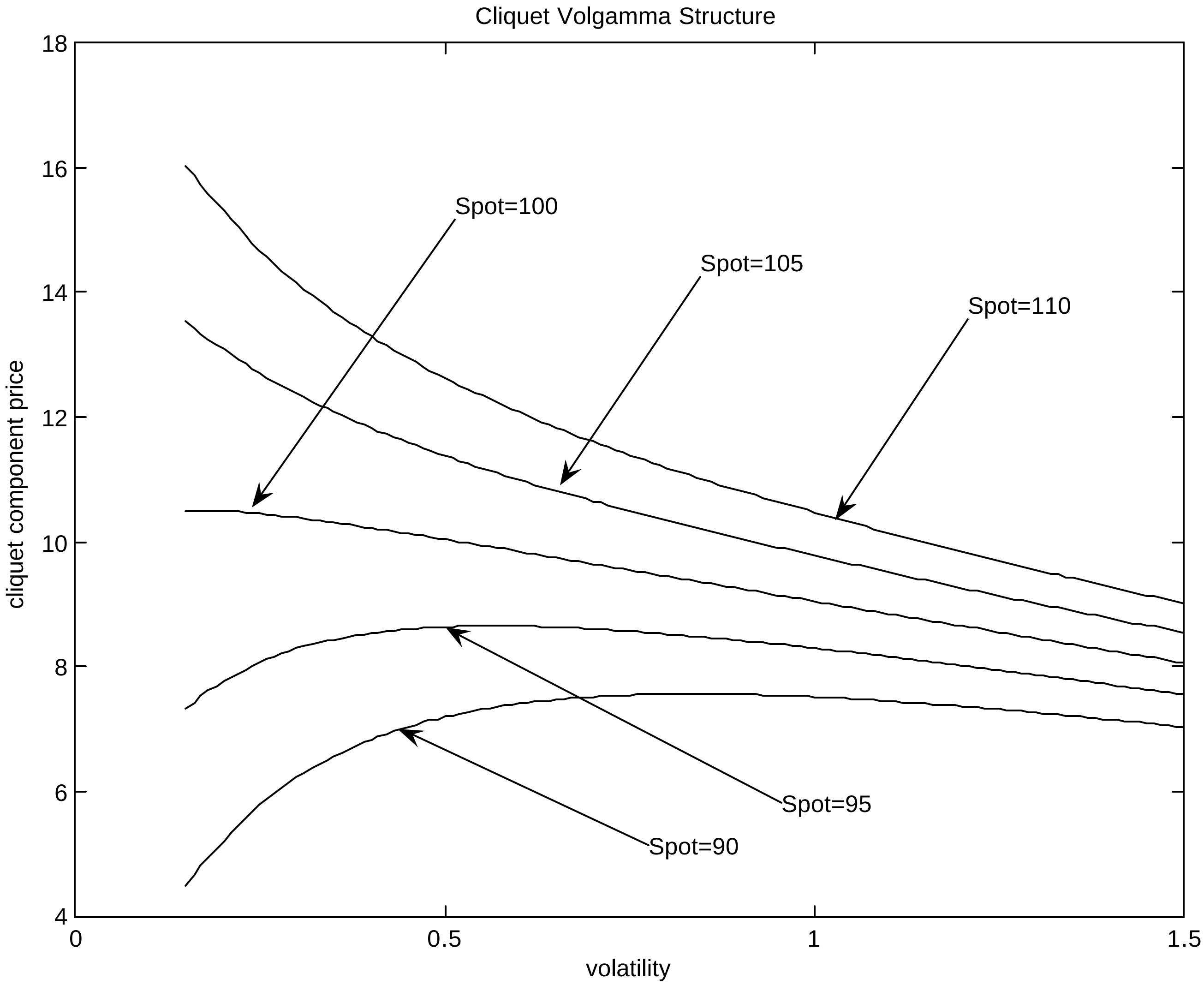
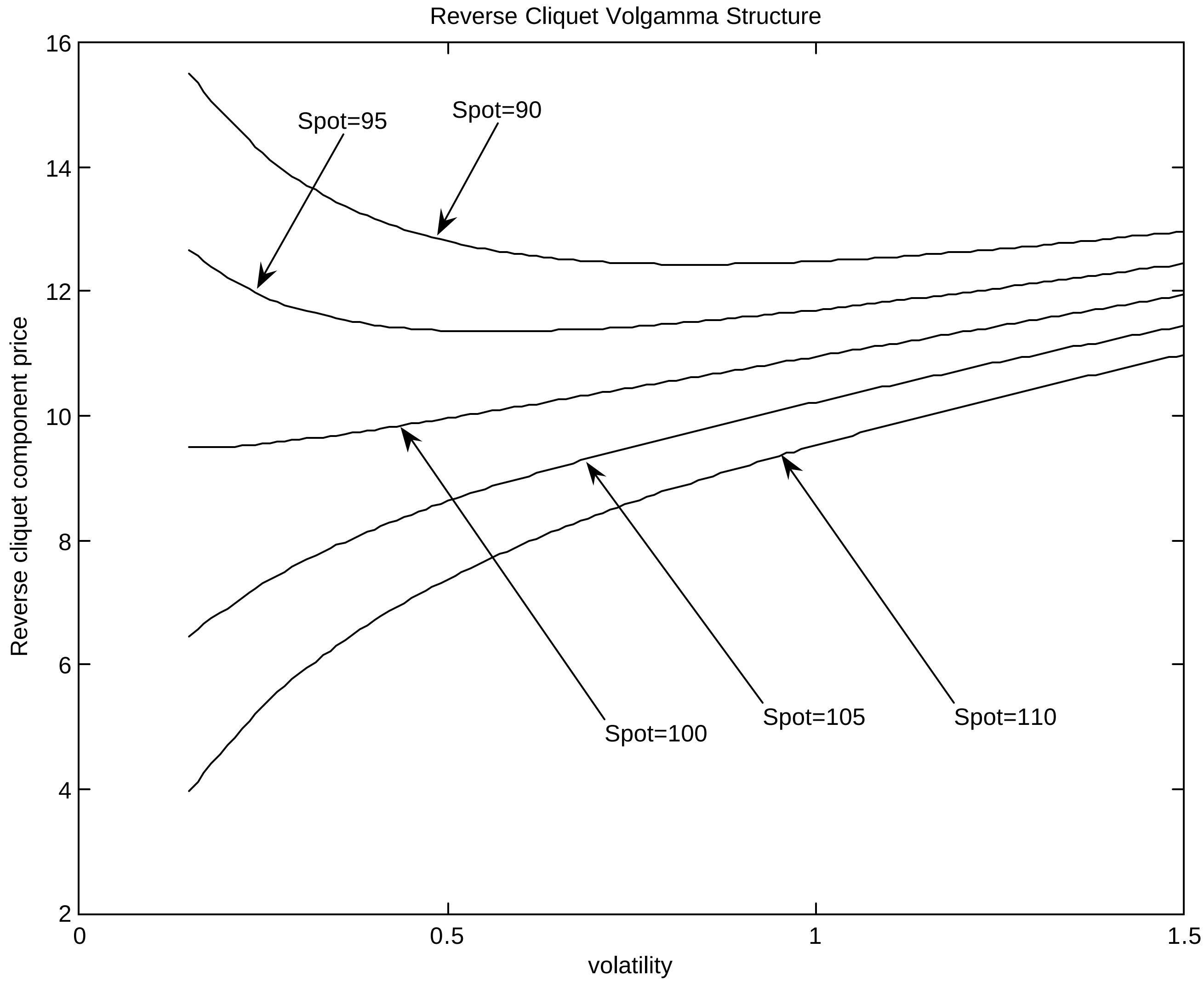

Cliquet Volgamma Structure
18
16
14
12
10
8
6
4
0
0.5
1
1.5
cliquet component price
volatility
Spot=100
Spot=105
Spot=110
Spot=95
Spot=90

Figure 2:



Reverse Cliquet Volgamma Structure
Reverse cliquet component price
volatility
16
14
12
10
8
6
4
2
0
0.5
1
1.5
Spot=95
Spot=90
Spot=100
Spot=105
Spot=110

Figure 3:

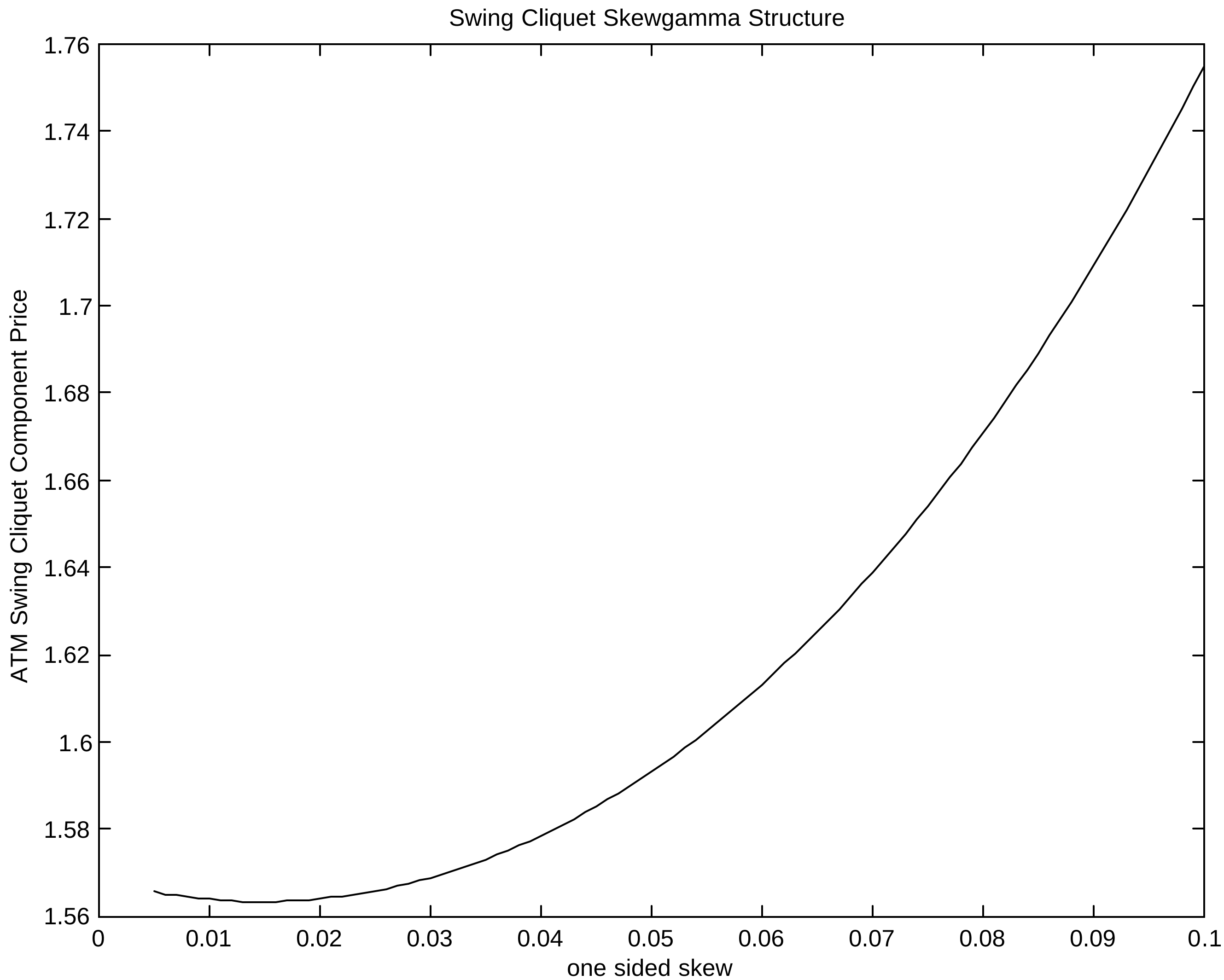

Swing Cliquet Skewgamma Structure
ATM Swing Cliquet Component Price
one sided skew
1.76
1.74
1.72
1.7
1.68
1.66
1.64
1.62
1.6
1.58
1.56
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

Figure 4:

Demand for Models

- The realization of these sensitivities creates a need for option calidrated models that may be used for structured product marking with features for realistic evolution of volatilities, skews, and volatility spreads, and exposure for products with significant surface gammas to stochastic volatility, stochastic skews convexity and stochastic volatility spreads.

- The base model widely used in risk analysis for structured equity products is the Dupire local volatility model.

- Well known problems are the collapse in forward volatilities and skews. I give an example on six month forward implied volatility curves going out some years for a calibration to SPX on July 6 2004.

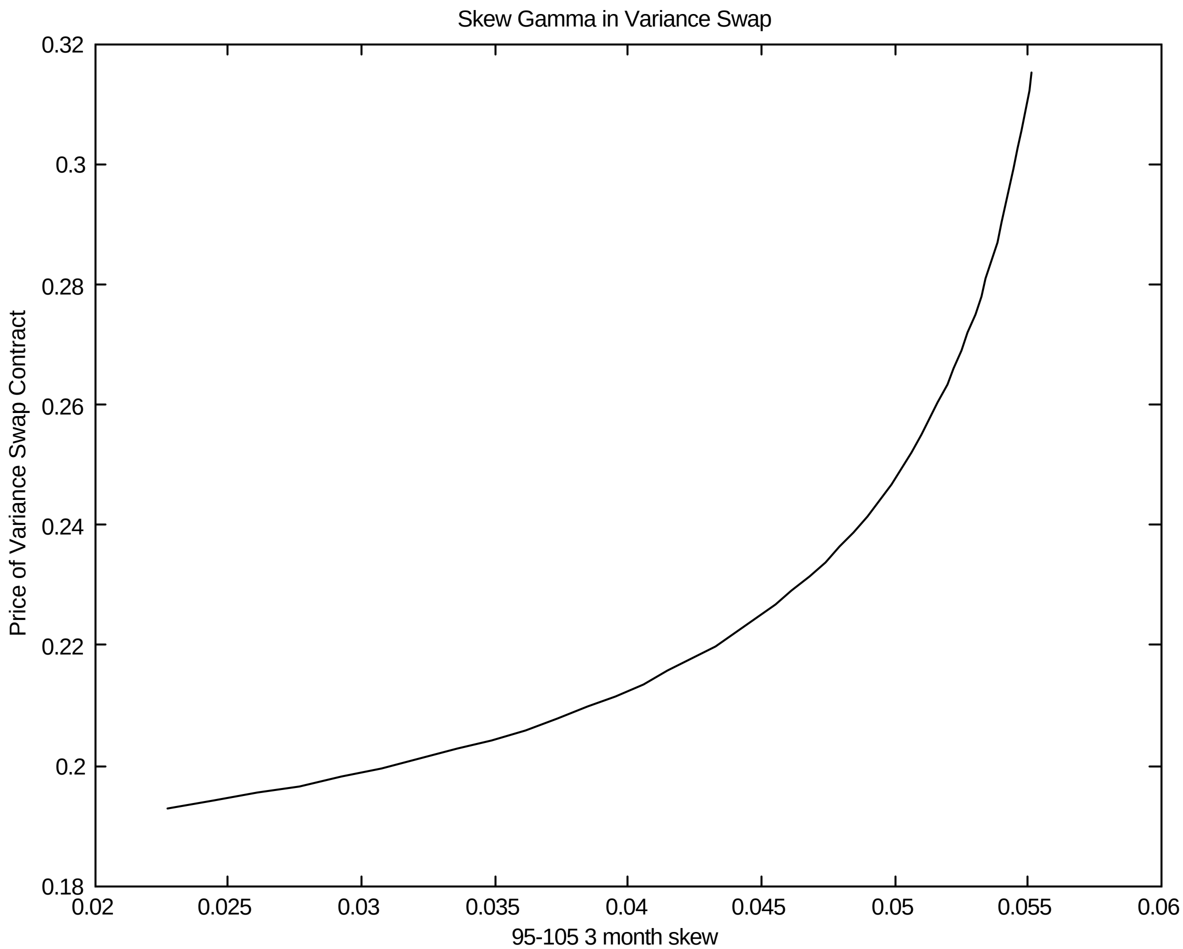
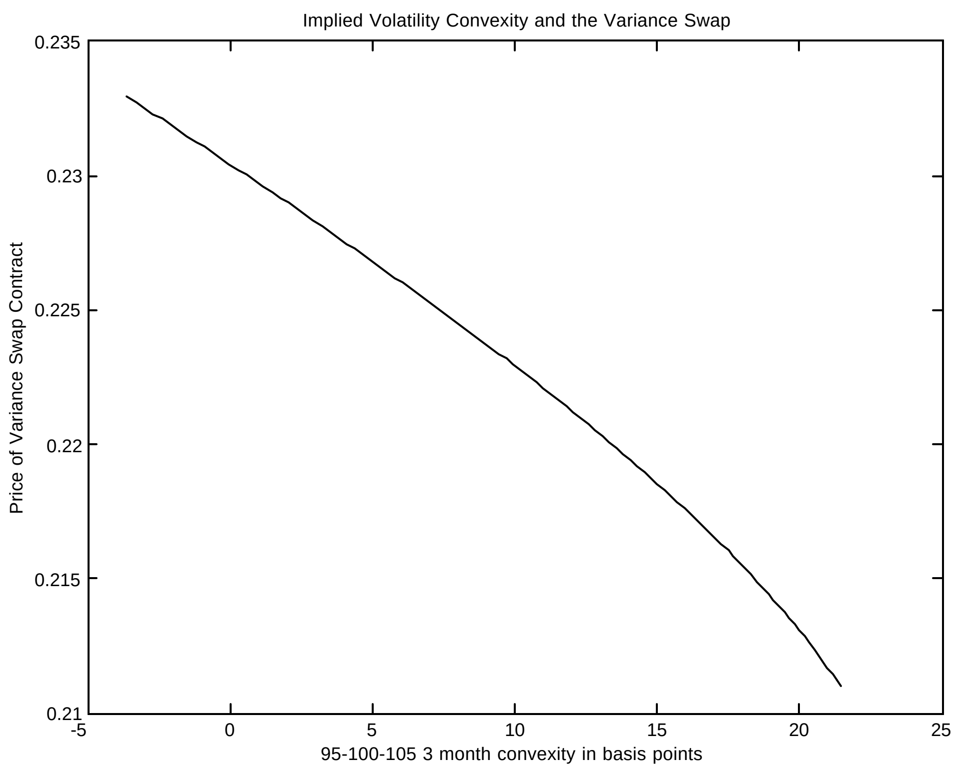
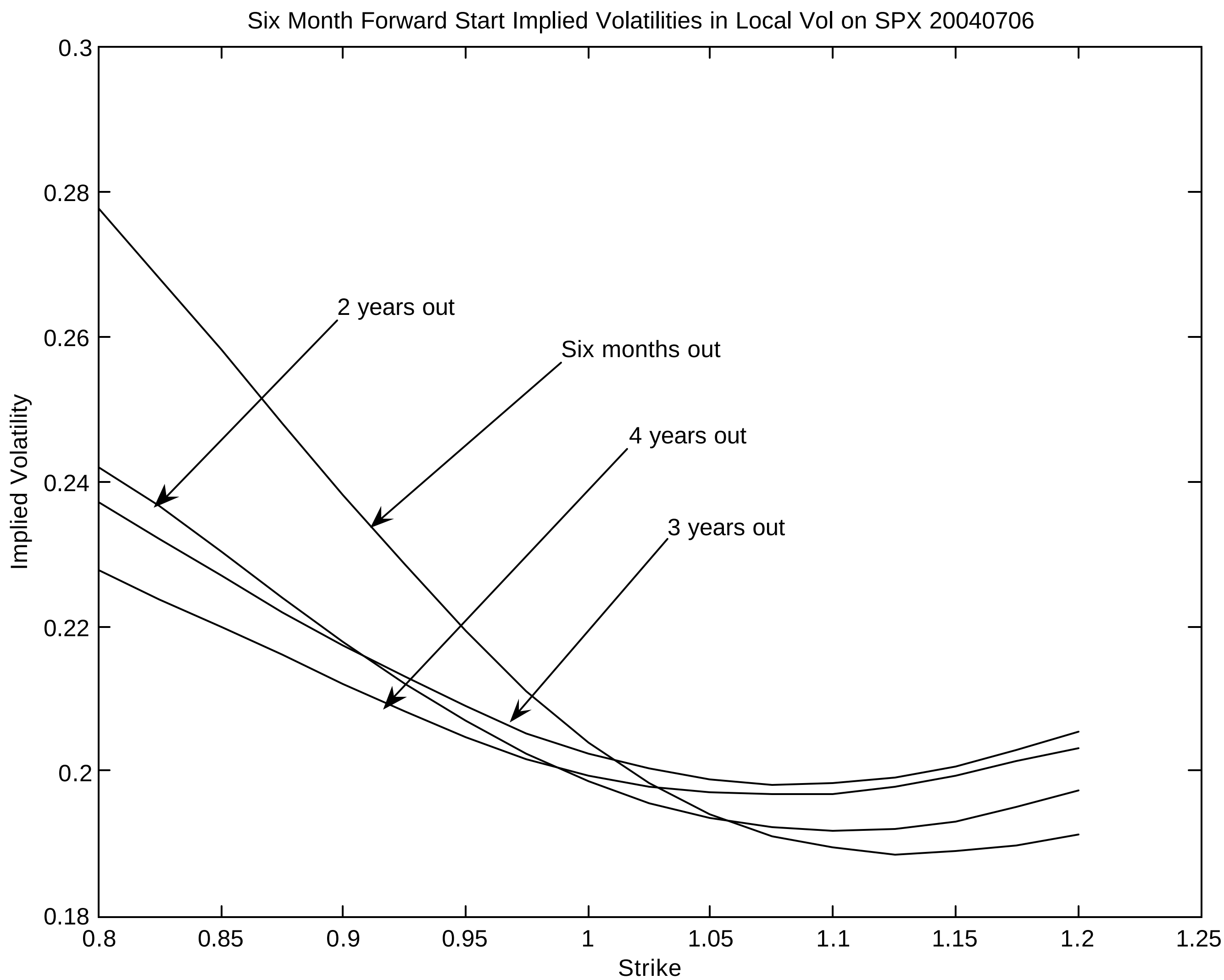

Skew Gamma in Variance Swap
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
Price of Variance Swap Contract
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.055
0.06
95-105 3 month skew

Figure 5:



Implied Volatility Convexity and the Variance Swap
Price of Variance Swap Contract
0.235
0.23
0.225
0.22
0.215
0.21
-5
0
5
10
15
20
25
95-100-105 3 month convexity in basis points

Figure 6:



Six Month Forward Start Implied Volatilities in Local Vol on SPX 20040706
Implied Volatility
Strike
2 years out
Six months out
4 years out
3 years out
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
1.25

Figure 7:

Preserving Skews

- Local Volatility is for these reasons not well adapted to pricing cliquets that have significant exposures to the structure of forward skews.

- Alternatively, one has to build in some ad-hoc adjustment to the local volatility price when marking such skew sensitive products.

- Our construction of local Lévy models was motivated by these considerations and the basic idea was to hard wire some skew into the price dynamics by making the local motion that of a pure jump Lévy process with a higher rate of negative jumps relative to comparably sized positive ones.

- We therefore replace the local volatility dynamics of

$$dS = (r - \eta)Sdt + \sigma(S,t)dW(t)$$

where we equivalently view $\sigma(S,t)dW(t)$ as $W(\sigma^2(S,t)dt)$

and now write

$$dS = (r - \eta)Sdt + \int_{-\infty}^{\infty} S(t_{-})\left(e^x - 1\right)\left(m(dx,ds) - k(x)a(S(t_{-}),t)dxdt\right)$$

- The Lévy process with Lévy measure

$$k(x)dx$$

is now running at a space time dependent speed function given by

$$a(S,t).$$

Recovering Speed Functions

- For local volatility the basic recovery result from call option prices $C(K,T)$ for strikes K and maturity T is

$$\sigma^2(K,T) = 2\frac{C_T + \eta C + (r-\eta)KC_K}{K^2 C_{KK}}$$

- For local Lévy the speed function

$$a(K,T) = \frac{b(\ln(K),T)}{K^2 C_{KK}}$$

where we name the function $b(\ln(K),T)$ as the log speed function.

Recovering Log Speed Functions

- For the log speed function we work with prices in log strike k and write

$$c(k,T) = C(e^k, T)$$

and note that

$$\int_{-\infty}^{\infty} b(y,T)\psi_e(k-y)dy = c_T + \eta c + (r-\eta)c_k$$

where $\psi_e(x)$ is the double exponential tail of the Lévy measure defined for $x > 0$ by

$$\begin{aligned}\psi_e(x) &= \int_x^{\infty} dz e^z \int_z^{\infty} k(u)du \\ &= \int_x^{\infty} \left(e^z - e^x\right) k(x)dx\end{aligned}$$

A similar expression holds for $x < 0$.

Derivation of Basic Recovery Result

- We begin with the Meyer-Tanaka formula for semimartingales $X = (X_t, 0 \le t \le T)$ that generalizes Ito's lemma for the case of convex functions $f(X_t)$.

- From Meyer-Tanaka we have that

$$f(X_t) = f(X_0) + \int_0^t f'(X(u_-))dX(u)$$
$$+\frac{1}{2}\int_0^t \frac{\partial^2}{\partial x^2} f(X(u_-))\frac{\partial}{\partial t}L_t(X(u_-))$$
$$+\int_0^t \int_{-\infty}^{\infty} \left[\begin{array}{c} f(X(u_-)+x) \\ -(f(X(u_-)) + f'(X(u_-))x) \end{array} \right] \mu(dx, du$$

- $L_t(a)$ is local time of the continuous martingale component at the level a and is globally defined by

$$\int_{-\infty}^{\infty} \phi(a) L_t(a) da = \int_0^t \phi(X(u_-)) d\langle X^c \rangle_u$$

where X^c is the continuous martingale component of the semimartingale X. Furthermore the second partial of the function f is in the sense of distributions.

- The final integral term is worth a comment and represents the gap risk of the function relative to the tangent.

- We now apply the Meyer-Tanaka formula to the convex function given by the call option payoff $(S_T - K)^+$.

- On computation we get the result

$$
\begin{aligned}
&(S_T - K)^+ = (S_0 - K)^+ + \int_0^T \mathbf{1}_{S(u_-)>K} dS(u) \\
&+\frac{1}{2}\int_0^T \mathbf{1}_{S(u_-)=K}\sigma^2(S(u_-),u)S^2(u_-)du \\
&+\sum_{u\leq T} \mathbf{1}_{S(u_-)>K}(K - S(u))^+ \\
&+\sum_{u\leq T} \mathbf{1}_{S(u_-)<K}\,(S(u) - K)^+
\end{aligned}
$$

- That the gap risk on a call is the sum of crossover terms receiving the call on the up move and the put on the down move is easily seen on constructing the graph.

- Computing expectations we have in terms of the marginal densities $q_t(S)$

$$
\begin{aligned}
& e^{rT} C(K,T) = (S_0 - K)^+ \\
& + \int_0^T \int_K^\infty dY q_u(Y) Y (r-\eta) du \\
& + \frac{1}{2} \int_0^T q_u(K) \sigma^2(K,u) K^2 du \\
& + \int_0^T \int_K^\infty dY q_u(Y) \int_{-\infty}^{\ln\left(\frac{K}{Y}\right)} (K - Ye^x)\, \nu(dx, du) \\
& + \int_0^T \int_0^K dY q_u(Y) \int_{\ln\left(\frac{K}{Y}\right)}^\infty (Ye^x - K)\, \nu(dx, du)
\end{aligned}
$$

- Differentiating with respect to T and specializing the jump compensator yields the result

$$
\begin{aligned}
&re^{rT}C + e^{rT}C_T = \\
&(r-\eta)\int_K^\infty dY\ Yq_T(Y) \\
&+\frac{1}{2}q_T(K)\sigma^2(K,T)K^2 \\
&+\int_K^\infty dY\ Yq_T(Y)a(Y,T)\times \\
&\int_{-\infty}^{\ln\left(\frac{K}{Y}\right)}\left(e^{\ln\left(\frac{K}{Y}\right)} - e^x\right)k(x)dx \\
&+\int_0^K dY\ Yq_T(Y)a(Y,T)\times \\
&\int_{\ln\left(\frac{K}{Y}\right)}^{\infty}\left(e^x - e^{\ln\left(\frac{K}{Y}\right)}\right)k(x)dx
\end{aligned}
$$

- We now solve for C_T using some facts relating option prices and the risk neutral density

$$\begin{aligned} e^{-rT}\int_K^\infty Y q_T(Y)dY &= C - KC_K \\ e^{-rT} q_T(K) &= C_{KK} \end{aligned}$$

to get the result

$$C_T = -\eta C - (r-\eta)KC_K + \frac{\sigma^2(K,T)}{2}K^2 C_{KK}$$
$$+\int_K^\infty dY\ Y\ C_{YY} a(Y,T) \times$$
$$\int_{-\infty}^{\ln\left(\frac{K}{Y}\right)} \left(e^{\ln\left(\frac{K}{Y}\right)} - e^x\right) k(x)dx$$
$$+\int_0^K dY\ Y\ C_{YY} a(Y,T)\ \times$$
$$\int_{\ln\left(\frac{K}{Y}\right)}^\infty \left(e^x - e^{\ln\left(\frac{K}{Y}\right)}\right) k(x)dx$$

- We now recognize the double exponential tail evaluated at $\ln(K) - \ln(Y)$ and switch to log space with

$$\begin{aligned} k &= \ln(K) \\ y &= \ln(Y) \\ c &= C(e^k, t) \end{aligned}$$

and write in terms of these variables and functions the result

$$\begin{aligned} b(y,t) &= e^{2y} C_{YY}(e^y, t) a(e^y, t) \\ a(Y,t) &= \frac{b(\ln(Y), t)}{Y^2 C_{YY}} \end{aligned}$$

and

$$b * \psi_e = c_t + \left(r - \eta + \frac{\sigma^2(e^k, t)}{2} \right) c_k - \frac{\sigma^2(e^k, t)}{2} c_{kk}$$

- The speed function recovery strategy is to invert the convolution operator for the log speed function $b(y,t)$ and then to define $a(Y,t)$ as described.

On Implementing Log Speed Recovery

- The convolution of log speed with the double exponential tail is the adjusted calendar spread observed in local volatility.

- We define by $h(k)$ the adjusted calendar spread

$$h(k) = c_T(k) + \eta c(k) + (r - \eta)c_k(k)$$

and observe that after exponential dampening for $\alpha > 0$ we may construct the Fourier transform

$$\begin{aligned} \widehat{h}(u - i\alpha) &= \int_{-\infty}^{\infty} e^{iuk+\alpha k} h(k) dk \\ &= \widehat{b}(u - i\alpha)\widehat{\psi}_e(u - i\alpha) \end{aligned}$$

- The Fourier transform of the double exponential tails is known in closed form for the $CGMY$ models and is given by

$$\widehat{\psi}_e(\xi) = \int_{-\infty}^{\infty} e^{i\xi z}\psi_e(z)dz$$

$$= \frac{\Gamma(-Y)}{i\xi(1+i\xi)}\left[\begin{array}{c} (M-(1+i\xi))^Y - M^Y \\ -(1+i\xi)\left((M-1)^Y - M^Y\right) \\ +(G+1+i\xi)^Y - G^Y \\ -(1+i\xi)\left((G+1)^Y - G^Y\right) \end{array}\right]$$

- We may therefore recover

$$\widehat{b}(u-i\alpha) = \frac{\widehat{h}(u-i\alpha)}{\widehat{\psi}_e(u-i\alpha)}$$

We then get log speed by Fourier inversion

$$b(k,t) = \frac{e^{-\alpha k}}{2\pi}\int_{-\infty}^{\infty} e^{-iuk}\widehat{b}(u-i\alpha)du$$

On the Fourier Transform of the adjusted Calendar Spread

- The Fourier transform of the adjusted calendar spread is given by

$$
\begin{aligned}
&\widehat{h}(u - i\alpha) \\
= \; & \int_{-\infty}^{\infty} e^{iuk} e^{\alpha k} \left[\begin{array}{c} c_t(k,t) + \eta c(k,t) \\ +(r-\eta) c_k(k,t) \end{array} \right] dk \\
= \; & \eta A(u,t) + B(u,t) + (r-\eta) C(u,t)
\end{aligned}
$$

For this purpose we need efficient computations for

$$
\begin{aligned}
A(u,t) &= \int_{-\infty}^{\infty} e^{iuk} e^{\alpha k} c(k,t) dk \\
B(u,t) &= \int_{-\infty}^{\infty} e^{iuk} e^{\alpha k} c_t(k,t) dk \\
C(u,t) &= \int_{-\infty}^{\infty} e^{iuk} e^{\alpha k} c_k(k,t) dk
\end{aligned}
$$

For C we note that

$$\begin{aligned} C(u,t) &= -\frac{1}{\alpha+iu}\int_{-\infty}^{\infty} e^{iuk+\alpha k} c(k,t)dk \\ &= -\frac{A(u,t)}{\alpha+iu} \end{aligned}$$

- We also note that the term A is available in closed form for most models calibrated by Fourier transform as

$$A(u,t) = \frac{e^{-rt}\phi(u-i(\alpha+1),t)}{(\alpha+iu)(\alpha+1+iu)}$$

- For B we differentiate A with respect to t.

On Simulating Local CGMY

- Once the speed function has been recovered, to price forward starts, cliquets, options on volatility and other structured products we need to simulate the Lévy process involved.

- We have written the CGMY Lévy process as a time changed Brownian motion where

$$
\begin{aligned}
X_{CGMY}(t) &= \theta T(t) + W(T(t)) \\
\theta &= \frac{G-M}{2}
\end{aligned}
$$

- The Lévy measure of the $CGMY$ time change is absolutely continuous with respect to the one-sided stable subordinator with Lévy measure

$$
\begin{aligned}
\nu_0(dy) &= \frac{K}{y^{\left(1+\frac{Y}{2}\right)}} dy \\
K &= \frac{C\pi^{.5}}{2^{\frac{Y}{2}}\Gamma\left(\left(\frac{Y+1}{2}\right)\right)}
\end{aligned}
$$

- The Lévy measure of the $CGMY$ time change has the form

$$\nu_1(dy) = \nu_0(dy)E[e^{-yZ}]$$

where Z is the random variable

$$\begin{aligned} Z &= \frac{B^2 - A^2}{2} + \frac{B^2}{2}\frac{\gamma_{\frac{Y}{2}}}{\gamma_{\frac{1}{2}}} \\ B &= \frac{G+M}{2} \\ A &= \frac{G-M}{2} \end{aligned}$$

where $\gamma_{Y/2}, \gamma_{1/2}$ are independent gamma variates with unit scale and shape parameters $y/2, 1/2$.

Simulating the CGMY Time Change

- We may simulate the CGMY time change by simulating the one-sided $Y/2$ stable subordinator and throwing away jumps y for which

$$\exp\left(-\frac{B^2-A^2}{2}y\right) E\left[\exp\left(-\frac{B^2}{2}y\right)\frac{\gamma_{\frac{Y}{2}}}{\gamma_{\frac{1}{2}}}\right] < u_2$$

for an independent uniform variate u_2.

- The expectation of the ratio of the gamma variates may be explicitly evaluated in terms of the Hermite functions that is known in terms of ${}_1F_1$.

$$
\begin{aligned}
E\left[e^{-\frac{B^2}{2}y}\frac{\gamma_{Y/2}}{\gamma_{1/2}}\right] &= 2^{\frac{Y}{2}}B^Y\frac{\Gamma\left(\frac{Y}{2}+\frac{1}{2}\right)}{\Gamma(Y)\Gamma\left(\frac{1}{2}\right)}y^{\frac{Y}{2}}I\left(Y, B^2y, \frac{B^2y}{2}\right) \\
I(\nu, a, \lambda) &= \int_0^\infty x^{\nu-1}e^{-ax-\lambda x^2}dx \\
&= (2\lambda)^{-\frac{\nu}{2}}\Gamma(\nu)h_{-\nu}\left(\frac{a}{(2\lambda)^{\frac{1}{2}}}\right)
\end{aligned}
$$

Choice of Localizing CGMY Lévy Process

- We estimate for 20040706 the $CGMY$ model on option data for the first maturity exceeding one month. The estimated parameters values for G, M, Y were

$$G = 5; M = 13; Y = .5$$

- We used these values for the localizing Lévy process and obtained the log speed and speed functions.

- I present a graph of the resulting speed and log speed functions

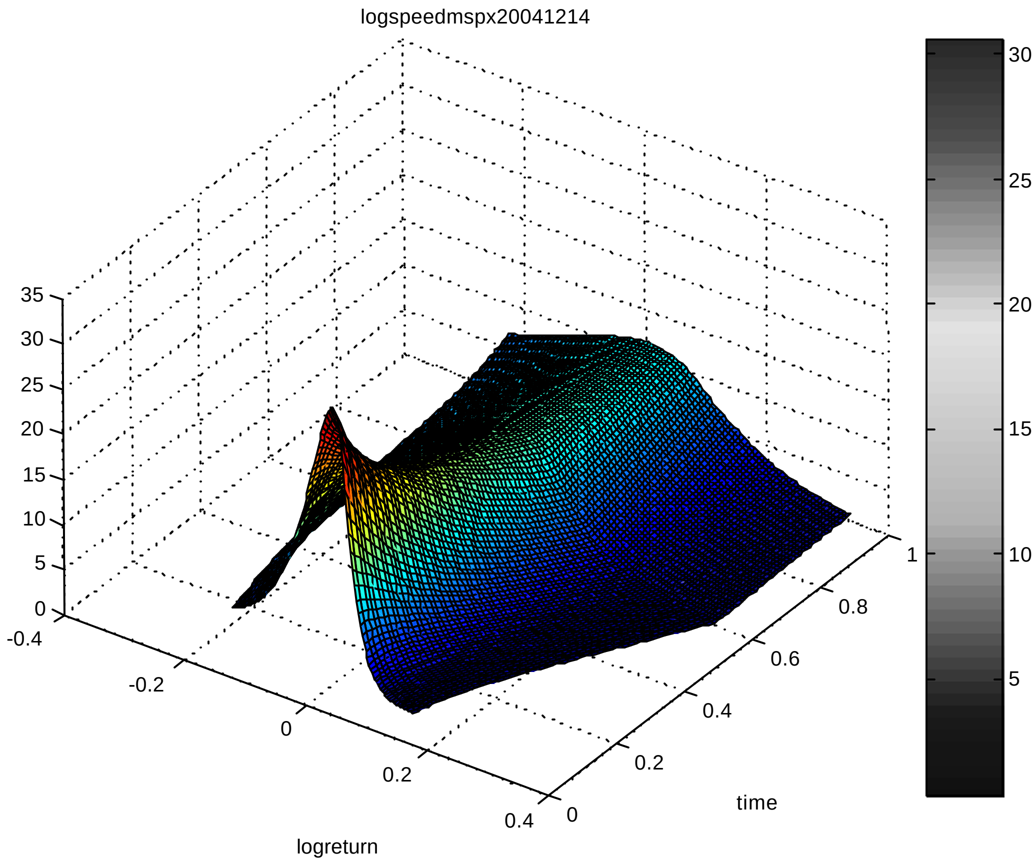

logspeedmspx20041214
35
30
25
20
15
10
5
0
-0.4
-0.2
0
0.2
0.4
logreturn
0
0.2
0.4
0.6
0.8
1
time
30
25
20
15
10
5

Figure 8:

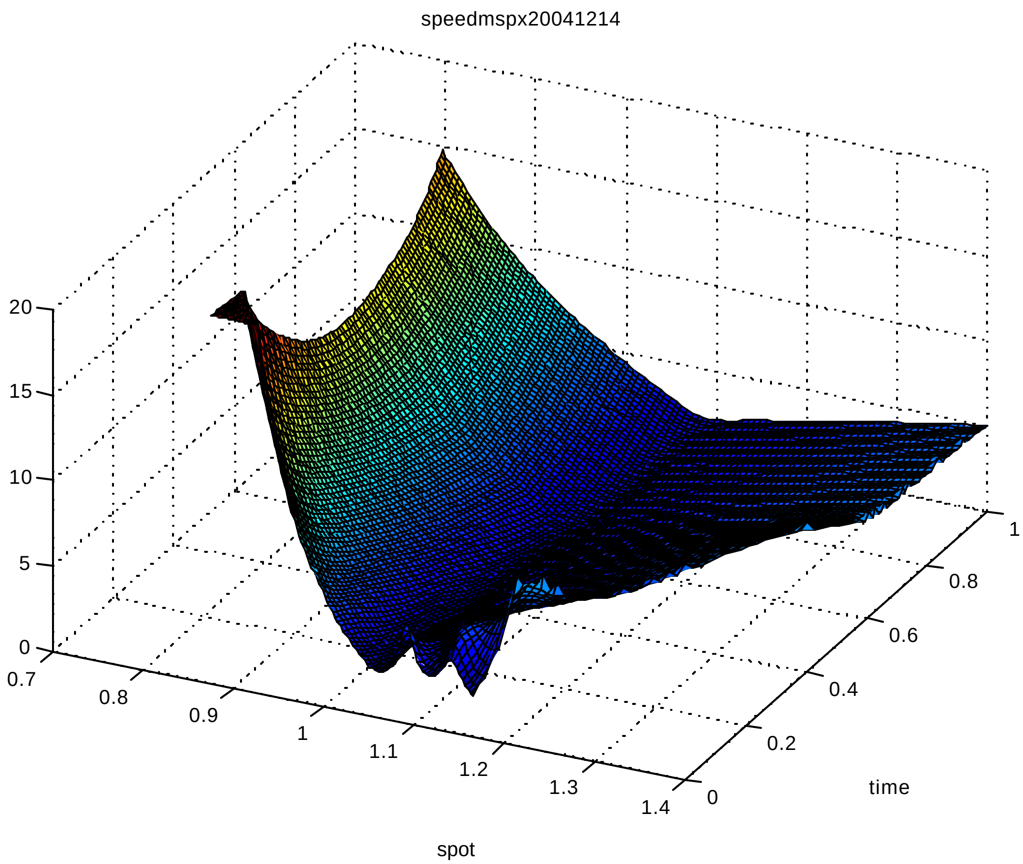


Figure 9:

Local Lévy Six Month Forward Starts

- We now graph the implied volatilities on forward starting options of a six month maturity using the Local CGMY Lévy model on SPX for 20040706 for which we observed the collapse of volatilities and skews in local volatility.

We observe that both the volatilities and skews maintain their levels and shape.

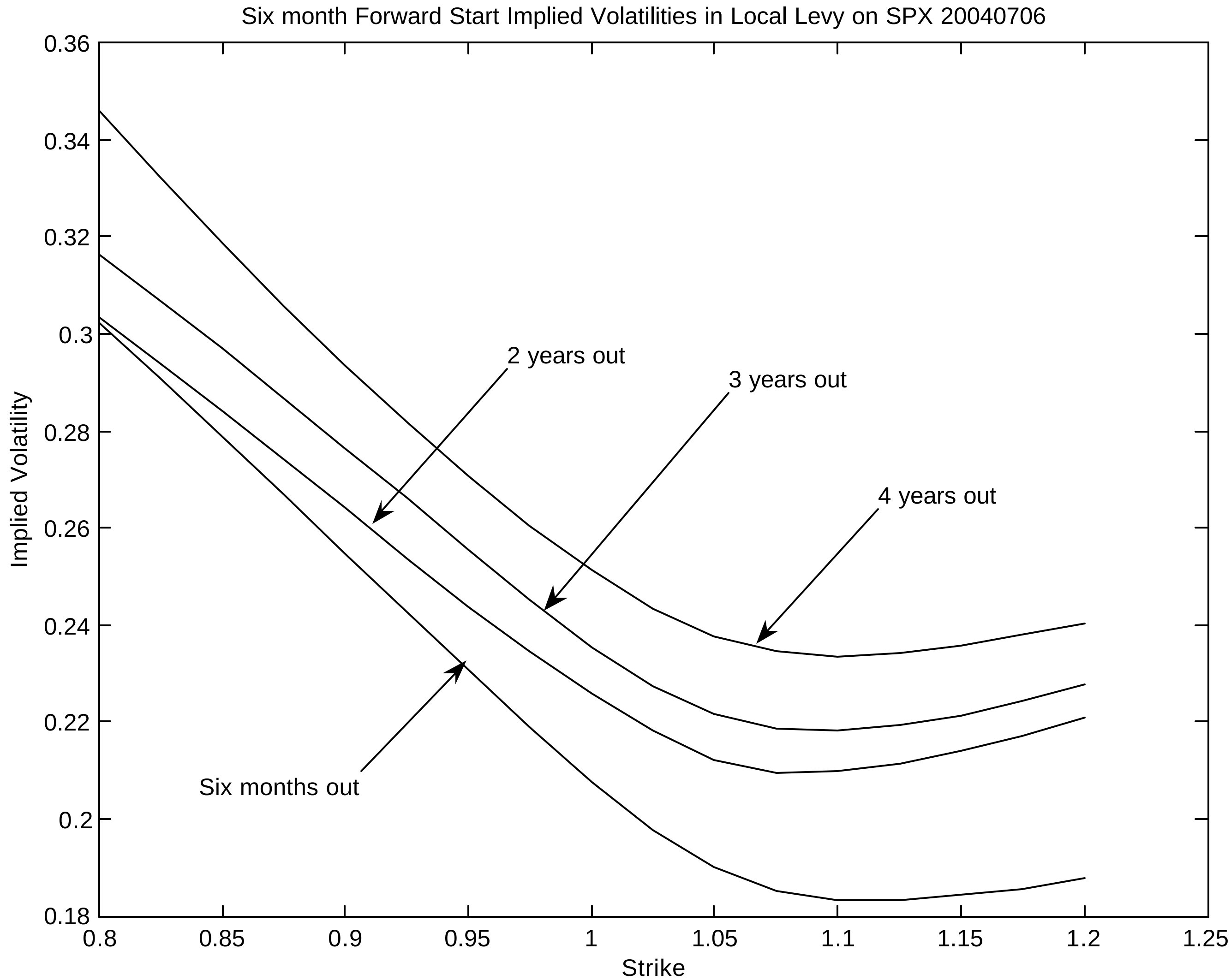

Six month Forward Start Implied Volatilities in Local Levy on SPX 20040706
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
1.25
Strike
Implied Volatility
2 years out
3 years out
4 years out
Six months out

Figure 10: