

Decomposing swap spreads

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Before we begin...

Interest rate swap with maturity date T :

- Party A pays to B a fixed amount (the *swap rate*) c_T at every date $t = 1, \dots, T$
- B pays to A the 'floating' amount

$$\rho_{t-1} = \frac{1}{p(t-1, t)} - 1 \quad (1)$$

at dates $t = 1, \dots, T$

Here, $p(s, t)$ is the price at date s of a zero coupon bond maturing at date t where $s \leq t$.

- The swap rate is the rate which gives the contract initial value 0
- In reality, the floating payment is **not** linked to Treasuries
- But if it were, we would get

$$c_T = \frac{1 - p(0, T)}{\sum_{i=1}^T p(0, i)} \quad (2)$$

- c_T in this case is a par bond yield, since we have

$$1 = c_T \sum_{i=1}^T p(0, i) + p(0, T) \quad (3)$$

- In reality, the floating rate payment is linked to LIBOR.
This rate is higher than the Treasury rate, due to – among other things – credit risk
- Also, the Treasury rate is lower than the riskless rate, due to a convenience yield to owning Treasuries
- We therefore have a **swap spread**, i.e. a difference between the fixed rate on a swap and the yield on a corresponding Treasury bond
- We want to understand the term structure and the dynamics of this spread!

Main goals/questions

- Decomposing swap spreads into credit and liquidity components based on joint pricing model for Treasuries, swaps and corporates
- Which is closer to 'the riskless rate': Treasury yields or swap rates?
- Is LIBOR - General Collateral (GC) repo rates a good measure of short term AA credit risk?

Most directly related literature

- Collin-Dufresne and Solnik (JF, 2001)
- Duffee (RFS, 1999), Duffie and Singleton (JF, 1997)
- Liu, Longstaff and Mandell (WP, 2003), He (WP, 2001)
- Grinblatt (IRF, 2001)
- Reinhart and Sack (2002)
- Lando (1998)

Model specification: The latent factors

State vector X consists of 6 independent diffusion processes with an affine drift and volatility structure with P and Q evolution

$$\begin{aligned} X_t &= (X_{1t}, \dots, X_{6t})' \\ dX_{it} &= k_i(X_{it} - \theta_i)dt + \sqrt{\alpha_i + \beta_i X_{it}} dW_i^P, i = 1, \dots, 6, \\ dX_{it} &= k_i^* X_{it} dt + \sqrt{\alpha_i + \beta_i X_{it}} dW_i^Q, i = 1, \dots, 6, \end{aligned}$$

For identification purposes, we normalize the Q -means to be zero.

Affine technology allows us to price the different securities in closed form

The riskless rate and the Treasury securities

- $r^g(X) = a + X_1 + X_2$ (The government short rate)
- $r(X) = r^g(X) + (e + X_5)$ (The riskless rate)
- $e + X_5$ is the convenience yield associated with holding treasuries.
- The price of the treasuries depends on two factors and has the form

$$P^g(t, T) = \exp(A^g(T - t) + \mathbf{B}^g(T - t)'X_t)$$

The corporate bonds

We model simultaneously the yield curves for four different rating classes in banks and financials.

The price at time t of zero-coupon bond rated i at t and maturing at T is

$$v^i(t, T) = E_t^Q \exp \left(- \int_t^T (r(X_u) + \lambda(X_u, \eta_u) du) \right)$$

The corporate bonds

The pricing formula

$$v^i(t, T) = E_t^Q \exp \left(- \int_t^T (r(X_u) + \lambda(X_u, \eta_u) du) \right)$$

requires us to specify the default intensity for each state and the migration between non-default states:

$$\begin{aligned} \lambda(X, i) &= \nu_i \mu(X_s) && \text{(loss-adjusted default rate)} \\ a_{ij}(X_t) &= \lambda_{ij} \mu(X_t) && \text{(migration)} \\ \mu(X) &= b + X_3 + X_4 + c(X_1 + X_2) \end{aligned}$$

Interpret μ as a common random factor controlling migration intensities and default-rates

The corporate bonds

$$a_{ij}(X_t) = \lambda_{ij}\mu(X_t) \quad (\text{migration})$$

requires the input of a baseline generator matrix

\tilde{A}	AAA	AA	A	BBB	SG
AAA	-0.0976	0.0847	0.0122	0.0007	0
AA	0.0157	-0.1286	0.1090	0.0028	0.0011
A	0.0010	0.0267	-0.1012	0.0678	0.0057
BBB	0.0009	0.0024	0.0669	-0.1426	0.0723
SG	0	0.0004	0.0066	0.1220	-0.1291

The 'baseline' intensities after collapsing spec grades into one category

The corporate bonds

The price of a zero coupon corporate bond in rating class i at time t is of the form:

$$v^i(t, T) = \sum_{j=1}^{K-1} c_{ij} E_t \left(\exp \left(\int_t^T d_j \mu(X_u) - r(X_u) du \right) \right)$$

where the constants c_{ij} and d_j can be computed explicitly

The swap rates

The short rate on the swap as set on date t and paid at date $t + 0.25$ is modelled as

$$L(t, t + 0.25) = \frac{1}{v^{LIB}(t, t + 0.25)} - 1$$

where

$$v^{LIB}(t, t + 0.25) = E_t^Q \exp \left(- \int_t^{t+0.25} \lambda^{LIB}(X_s) ds \right)$$

$$\lambda^{LIB}(X_s) = r(X_s) + \nu_{AA} \mu(X_s) + S(X_s)$$

$$S(X) = d + X_6$$

$S(X) = 0$ would correspond to an assumption of homogeneous LIBOR-swap market credit quality, i.e. that the short AA corporate rate and LIBOR were the same.

The swap rates

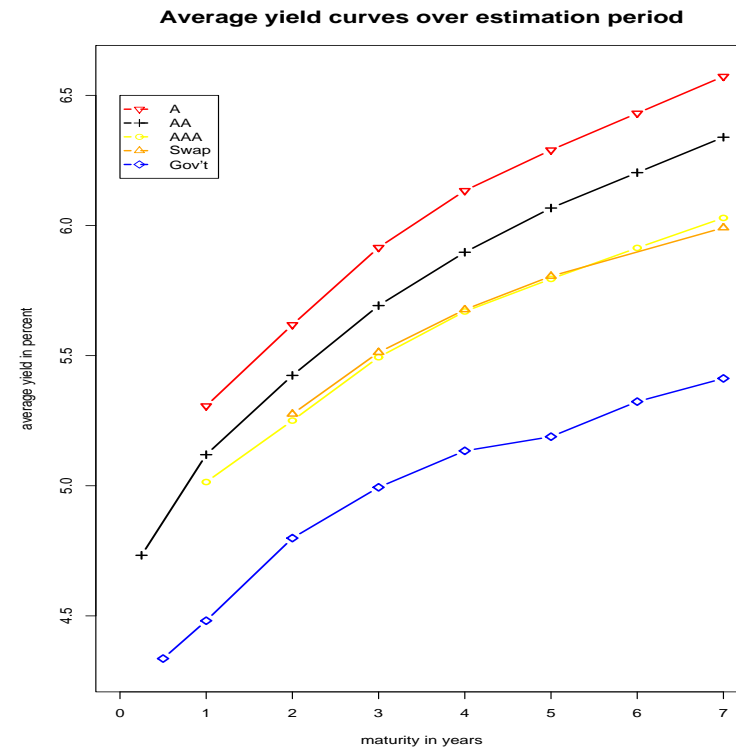
Assume that the swap rate can be found by discounting both sides of the swap using the riskless rate.

This corresponds to ignoring counterparty risk (cf. Duffie and Huang (1996))

We get closed form solution for swap rates as well.

Data

- US government yields, maturities 6 months; 1,2,3,4,5,6,7 years
- AAA, AA A, BBB financials (banks also in AAA/AA) 2,3,4,5,6,7 yrs
- US\$ swap rates, 2,3,4,5,7 yrs
- 3-month LIBOR



Average curves

Estimation

- We use a Kalman filter technique, i.e. use approximations to represent the system as

$$\begin{aligned}y_t &= A_t + B_t X_t + \epsilon_t & \epsilon_t &\sim N(0, H_t) \\X_t &= C_t + D_t X_{t-1} + \eta_t & \eta_t &\sim N(0, Q_t)\end{aligned}$$

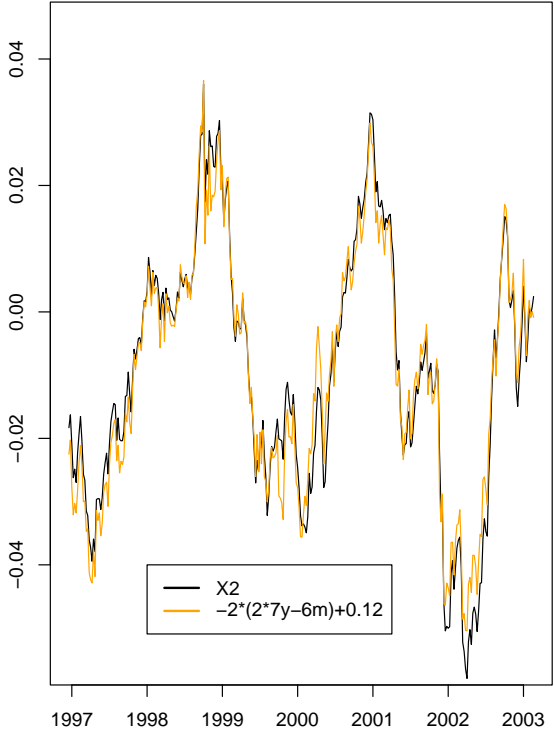
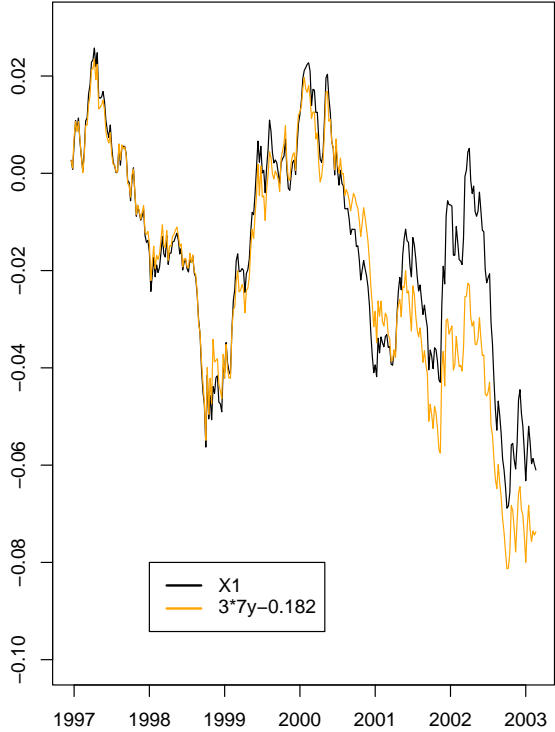
- C_t, D_t are chosen to match conditional means and variances (which are linear in X_{t-1})
- The yields y_t are only linear for zero-coupon bonds. We use linear approximation of $y_t = f(X_t)$ around forecast $X_{t|t-h}$

- The Kalman filter recursion computes - for a given set of parameters - the estimates of the latent variables and the value of the likelihood function
- The maximum likelihood estimator (in the approximating model) is found by varying the parameters (not an easy exercise)

	€0.25	€0.5	€1	€2	€3	€4	€5	€6	€7	average
Govt										
Mean		1.72	-2.18	1.68	1.38	0.94	-4.87	1.05	3.82	0.442
St. dev.		9.24	8.83	9.48	7.06	5.45	6.58	6.9	8.03	7.7
ρ		0.861	0.926	0.933	0.922	0.91	0.907	0.898	0.869	0.903
AAA										
Mean			-0.9	-2.96	0.8	0.85	-1.84	-2.71	-1.55	-1.187
St. dev.			7.68	6.11	8.69	8.01	7.29	6.95	8.01	7.53
ρ			0.861	0.81	0.912	0.911	0.873	0.892	0.851	0.873
AA										
Mean			2.44	2.34	3.52	2.35	1.29	-0.06	1.78	1.951
St. dev.			9.69	7.75	6.27	7.01	6.45	5.42	6.3	6.98
ρ			0.91	0.876	0.805	0.875	0.828	0.838	0.765	0.843
A										
Mean			0.96	-1.82	0.41	0.08	-2.16	-1.71	2.75	-0.2129
St. dev.			6.81	8.22	4.91	5.08	5.63	5.2	6.37	6.03
ρ			0.819	0.85	0.708	0.787	0.828	0.848	0.779	0.803
BBB										
Mean			-1.29	-0.49	1.28	3.04	0.74	1.27	3.52	1.153
St. dev.			10.04	7.01	9.56	11.8	14.34	14.9	16.37	12

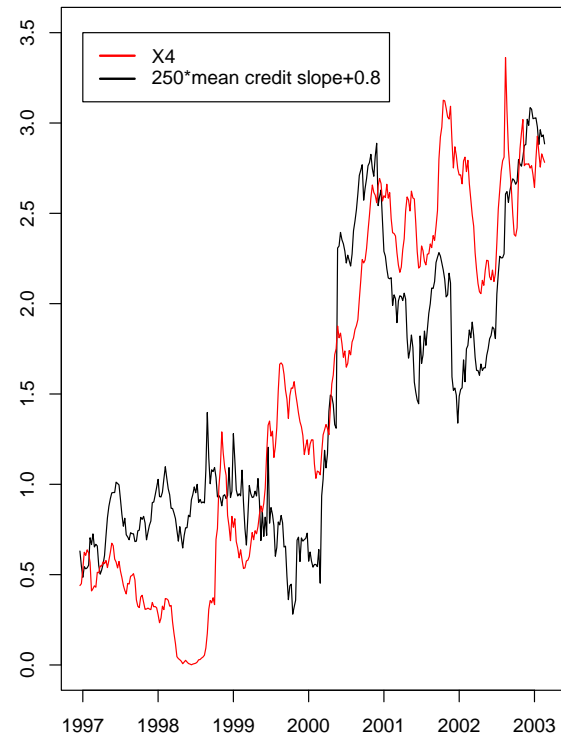
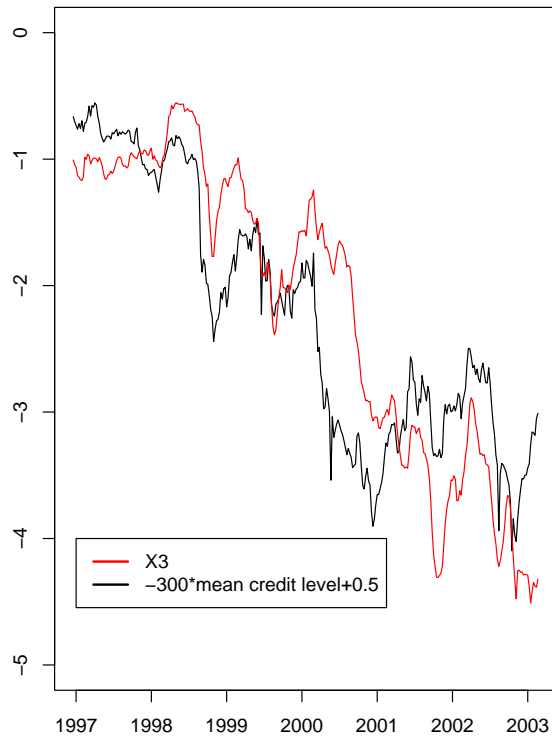
ρ		0.755	0.67	0.632	0.605	0.655	0.707	0.741	0.681
LIBOR									
Mean	1.23								1.23
St. dev.	15.06								15.06
ρ	0.869								0.869
Swap									
Mean		-2.05	0.97	1.16	0.87			-0.6	0.07
St. dev.		8.53	4.53	4.21	4.45			6.12	5.57
ρ		0.933	0.811	0.755	0.705			0.811	0.803

Interpretation of government factors



The treasury factors

Interpretation of credit risk factors

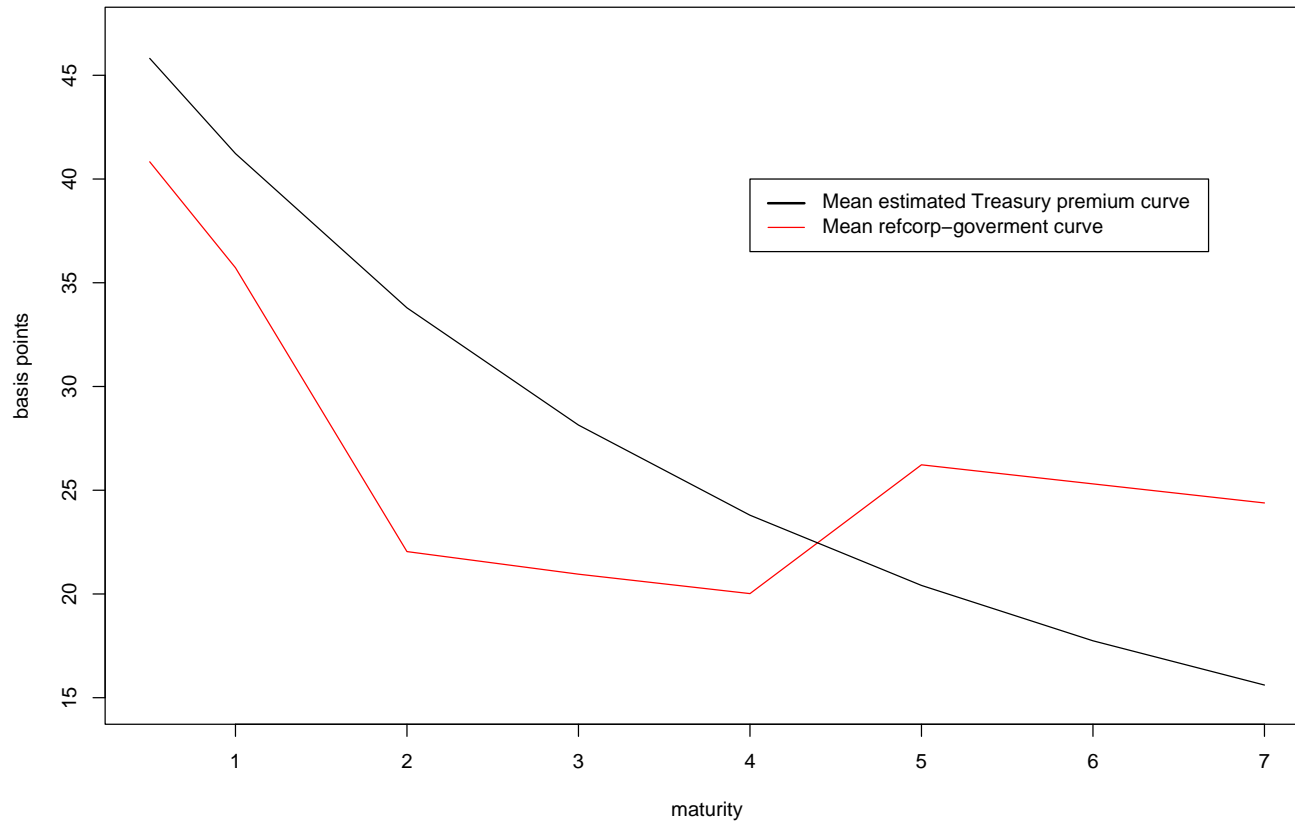


The credit curve factors

The treasury factor

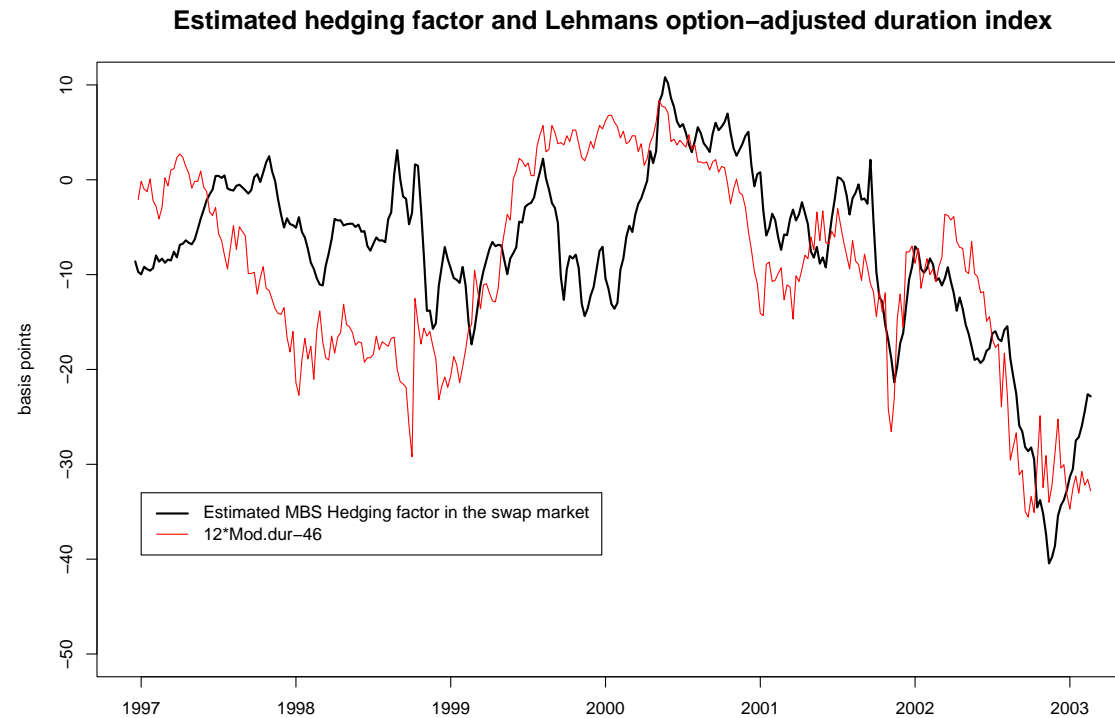
- We were unable to fit the treasuries and corporate bonds and swaps simultaneously without this factor
- The convenience yield has a term structure
- We compare short end to GC repo - Treasury bill spread and the entire term structure of convenience yields to refcorp - government spreads
- Highest correlation of refcorp-govt spread and liquidity factor is in 5 yr segment (0.64)

Mean estimated Treasury premia and the proxy Refcorp – government

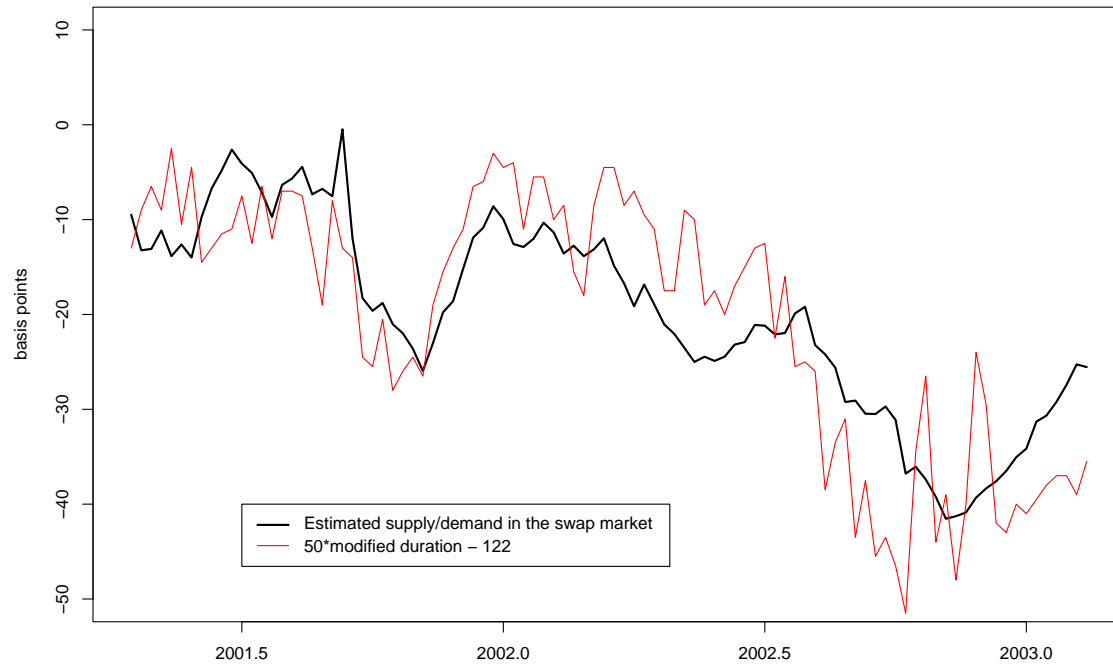


MBS duration and the swap factor

- We need separate factor for swap yields
- Separates LIBOR and short corporate curve
- Hedging of agency MBS portfolios seems to be the driving factor
- Example: Interest rates down \Rightarrow duration down
agencies enter as fixed receivers \Rightarrow swap rate down



The swap factor and an affine function of Lehman Modified Duration MBS index

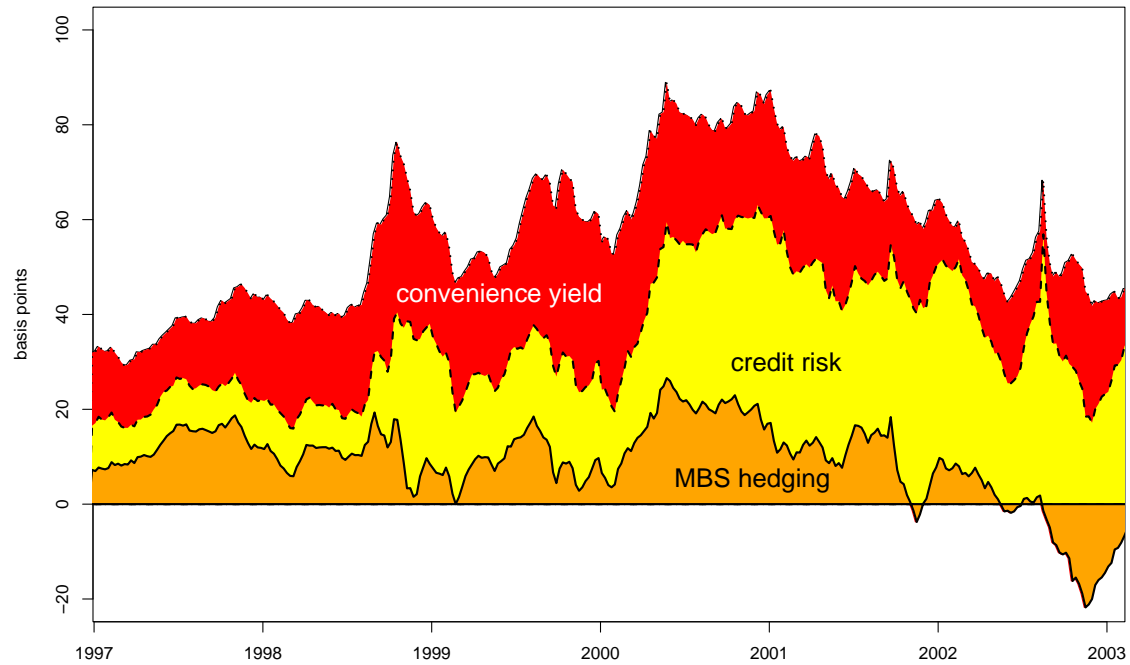


The swap factor and an affine function of Lehman Modified Duration MBS index

Decomposing swap spreads

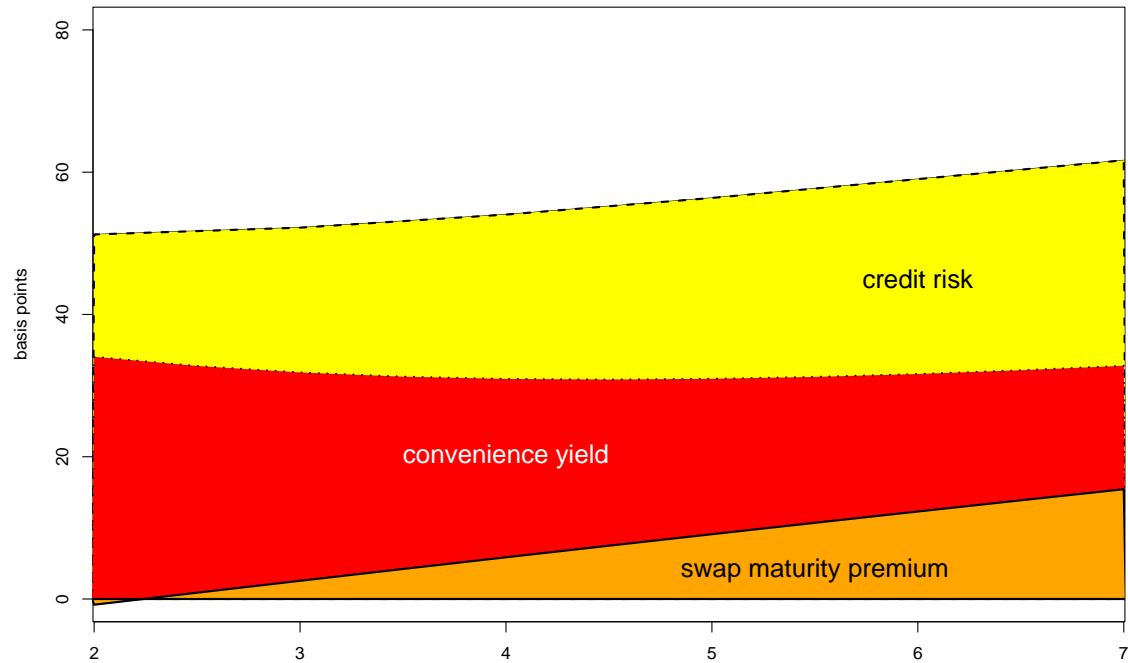
- We decompose the spread down to treasuries into contributions from swap factor, treasury factor, and from the credit risk component in the underlying LIBOR rate

Decomposition of the 5-year swap spread



Decomposing the swap spreads - dynamic evolution

Decomposing the term structure of swap spreads



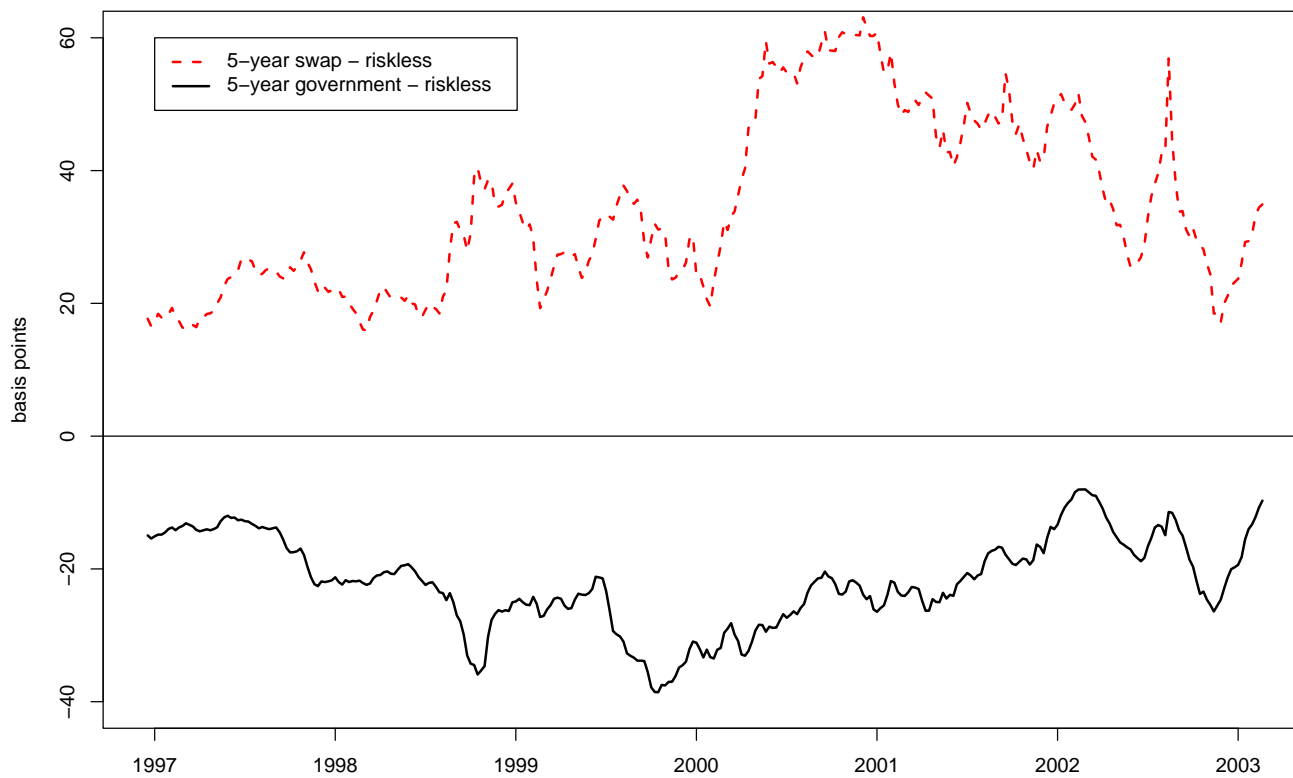
Decomposing the swap spreads - average curves

The term structure of the swap factor

maturity	2	3	4	5	7
average effect	-0.8	2.6	5.9	9.1	15.4

The average effect in basis points of the swap factor on swap rates across maturities. The effect of the swap factor on maturity T at time t is calculated as $-\frac{1}{T} \log(E_t(\exp(-\int_t^{t+T} S(u)du)))$.

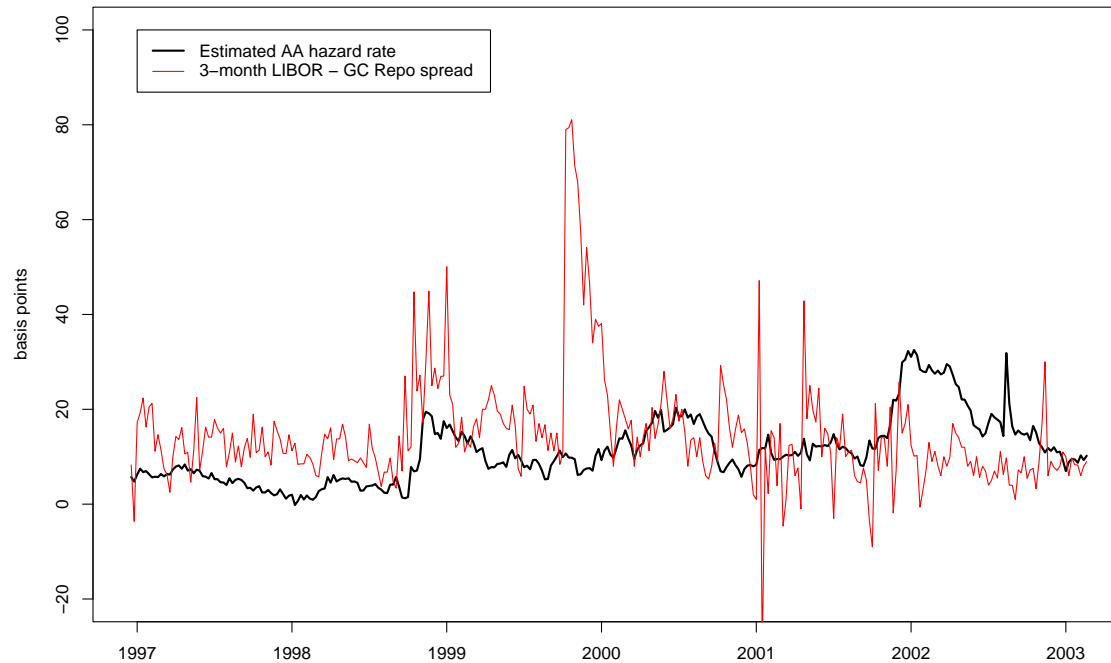
Distance from the 5-year riskless rate to the government and swap rate



The AA short term credit spread

- We compare the short term LIBOR-GC repo spread and find the latter to be too volatile to serve as proxy for short term credit spreads
- Our inclusion of corporate bonds keeps the spread 'in check' making it less volatile and less mean reverting
- This is important for the presence of a credit risk component in long term swap spreads

Estimated 3-month AA hazard rate and the proxy LIBOR – GC repo

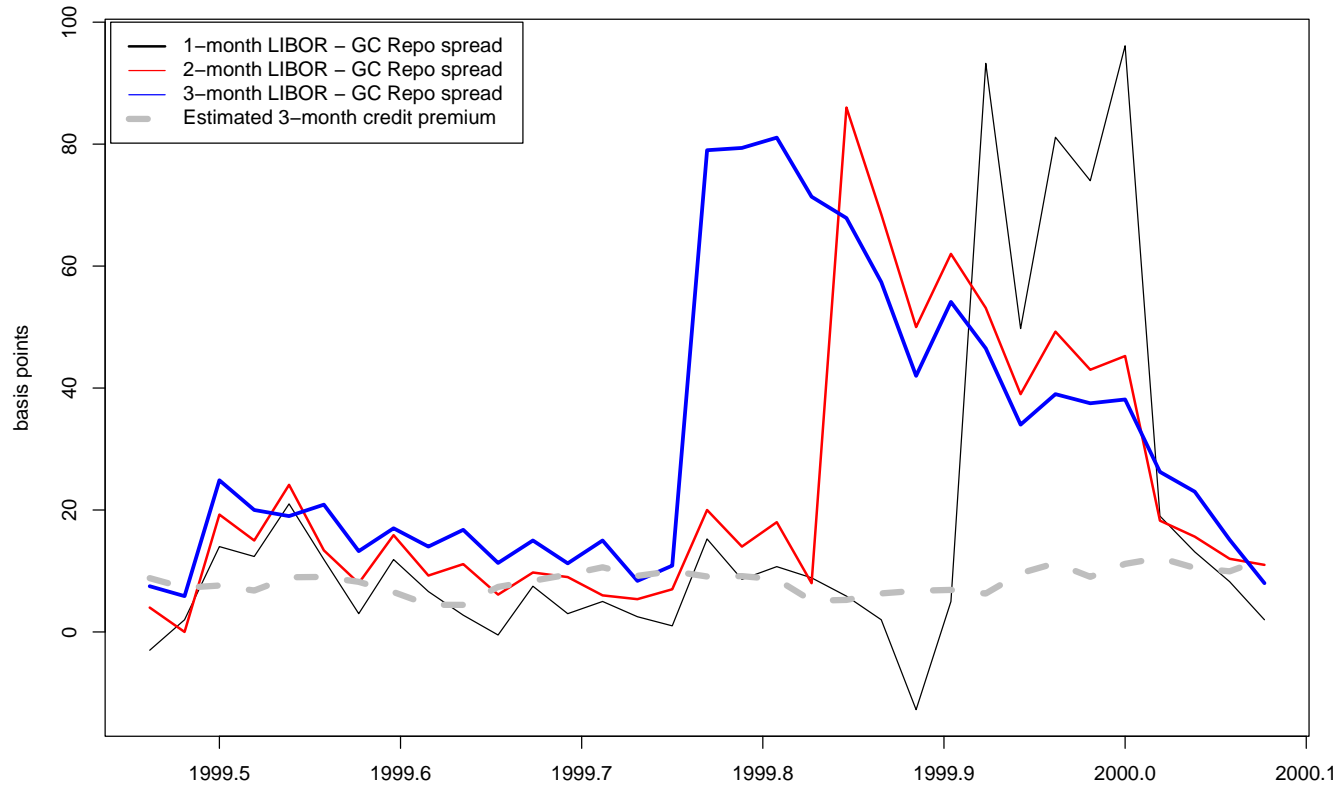


Comparing LIBOR-GC repo rates and our estimated AA short credit spread

Conclusion

- We obtain a decomposition of swap spreads into convenience yield, credit and a swap factor
- We identify a strong MBS duration-related component in swap spreads whose correlation with a key duration MBS index is 0.86 after 2000.
- LIBOR-GC repo is too volatile as measure of short term AA credit risk
At 2-yr maturity, swap is closer to riskless rate. At 5-yr maturity, the Treasury yield is closer

Estimated 3-month credit premium and 1-, 2-, and 3-month LIBOR – GC repo



Parameters of the state variables

	k	θ	α	β	λ	k^*
X_1	-0.2881	-0.0269	0.0007 (0.000037) (0.00008)	0.0051 (0.000016) (0.00066)	-10.3813 (5.660690) (20.34299)	-0.2351 (0.000442) (0.00476)
X_2	-0.6455	-0.0088	0.0007 (0.000079) (0.00012)	0.0005 (0.004648) (0.00202)	-8.1534 (7.888439) (22.06073)	-0.6418 (0.022097) (0.01372)
X_3	-0.2246	-1.4849	0.4238 (0.075498) (0.06711)	0.0000 (0.003863) (0.00800)	-0.7868 (0.181224) (0.47994)	-0.2246 (0.004944) (0.00852)
X_4	-0.0025	0.0013	0.0001 (0.000001) (0.03434)	0.8729 (0.112667) (0.04095)	0.0414 (0.006340) (0.03584)	-0.0387 (0.000108) (0.00295)
X_5	-0.0066	0.0001	0.0000	0.0011 (0.000031) (0.00016)	397.9246 (9.771331) (0.01376)	-0.4468 (0.004064) (0.02232)
X_6	-0.0634	-0.0310	0.0000 (0.000000) (0.00000)	0.0001 (0.000001) (0.00003)	-355.7386 (28.276065) (226.44959)	-0.0234 (0.008918) (0.01891)

Other parameters

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	σ^2
	0.065818 (0.000367) (0.00050)	0.359701 (0.030184) (0.04760)	-23.855810 (0.008164) (1.48397)	0.029284 (0.000018) (0.00034)	0.000002 (0.007488) (0.02321)	0.000001 (0.000000) (0.00000)
	ν_1	ν_2	ν_3	ν_4	ν_5	
	0.002178000 (0.000173) (0.00021)	0.003490162 (0.000216) (0.00023)	0.008656542 (0.000173) (0.00044)	0.016696560 (0.000620) (0.00077)	0.024827600 (0.001152) (0.00121)	