Conformal scattering and the Goursat problem : general ideas and what perspectives for black holes?

Seminar, Newton Institute, Cambridge, 6^{th} september 2005, 15:30.

1 Introduction

This talk is about time-dependent scattering in general relativity. Dietrich Häfner has amply described the topic in his talk on friday. So let me just summarize the essential ideas.

- Fixed background space-time, Lorentzian metric, globally hyperbolic $\mathcal{M} = \mathbb{R}_t \times \Sigma$, Σ 3-manifold with asymptotic ends.
- Study the behaviour in the asymptotic ends of solutions of covariant field equations on $\mathcal{M}$. Does so usually by comparison with simplified dynamics (ideally asymptotic profiles, i.e. the flow of a null congruence) in each asymptotic region : construction of wave operators and completeness. The terminology "time-dependent" means that the construction is based on the dynamics (the evolution) and not on stationary solutions of the form $e^{i\omega t}\phi(x)$.

To this day, most of the works in this domain are constructions outside black holes : for Schwarzschild or other spherical black holes (approximately chronologically) J. Dimock [15], J. Dimock and B. Kay [16, 17, 18], A. Bachelot [1, 2, 3, 4, 5], A. Bachelot and A. Motet-Bachelot [6], J.-P. Nicolas [31], W.M. Jin [26], F. Melnyk [29, 30], T. Daudé (PhD thesis, 2004) ; for Kerr or Kerr-Newmann (approximately chronologically) D. Häfner [23], D. Häfner and J.-P. Nicolas [24], T. Daudé (PhD thesis, 2004).

As I just mentionned, time-dependent scattering in relativity deals with covariant field equations, i.e. equations that derive their structure entirely from that of the metric. **The fundamental information for studying the scattering properties of such equations is therefore the asymptotics of the metric in the asymptotic ends of Σ.** All time-dependent scattering theories cited above are constructed following the same essential procedure :

1. Encoding of the fundamental information in an analytic expression of the metric via a choice of coordinate system and (for non scalar equations) of local frame. This step can already be quite critical (Schwarzschild, good choice of radial variable ; Kerr, example of our paper with Dietrich, we chose a tetrad that allows a spherically

symmetric comparison without long range terms, which accounts for the fact that the effects of the rotation are short range).

2. The information is then extracted via spectral techniques to obtain the scattering theory itself. These techniques are unstable with respect to time dependence of the coefficients of the equation : even for so-called time-dependent method, the time dependence allowed for the coefficients is very simple and means essentially that the methods can only be applied in stationary situations (or at least locally stationary, such as for the Kerr metric). This is a purely technical limitation, due to the methods used, not to the nature of the problem studied.

2 A geometrical alternative : "conformal scattering"

A different approach to scattering theory is to use the geometrical information of the asymptotics of the metric more directly to infer the asymptotic properties of the field. A natural tool for doing this is Penrose's conformal compactification. It allows us to interpret the complete scattering theory as the well-posedness of a Goursat problem on null infinity.

Not a new idea : Goursat problem on null infinity studied by R. Penrose in 1963 [33]. First used in the framework of time dependent scattering by F.G. Friedlander [20, 21], for the wave equation on static spacetimes with regular conformal structure at spacelike infinity. Idea used also by J.C. Baez, I.E. Segal and Z.F. Zhou [7] for a non linear wave equation on flat spacetime. So this is all for static spacetimes.

- Technique based on conformal compactifications, hence carries limitations of its own (these depend also of our understanding of the geometries studied and of the technique itself, there is therefore some hope that they will become less important) :
    1. spacetimes whose asymptotic structure is well described by a conformal compactification and whose conformal metric is regular enough at infinity (essentially, in the present state of things, asymptotically simple space-times with enough regularity at null and timelike infinity) ;
    2. conformally invariant equations or at least conformally "controllable".
- But the technique is totally indifferent to time dependence. Generic time dependence can be allowed.

A first result has been published, in collaboration with L. Mason [28]. Work on asymptotically simple space-times with regular null and timelike infinities. Construction of a scattering operator via conformal techniques : do a picture with a "vertical slice" of the compactified spacetime, a Cauchy hypersurface Σ_0, data in $\mathcal{C}_0^\infty(\Sigma_0)$, trace operators $T^\pm$ on $\mathscr{I}^\pm$, natural candidate for scattering operator is $S := T^+ \left(T^-\right)^{-1}$. The whole work is therefore to show that $T^\pm$ are isomorphisms, i.e. to solve the Goursat problem. Note that the trace operators associate to the data only a part of the trace of the solution, called scattering data, that comes out naturally in the energy estimates needed to solve the Goursat problem. The construction is done for :

- **massless Dirac fields :** no restriction, conformally invariant closed 3-form ;
- **Maxwell fields :** closed 3-form but not conformally invariant, at least not fully, the normal vector field is not rescaled, not applicable for energy estimates involving $\mathscr{I}$; estimates are easy away from i_0 because we have a symmetric hyperbolic system on a regular globally hyperbolic space-time ; near i_0 we need an exact Killing vector field to have exactly conserved quantities, otherwise, the singularity is a problem ; our construction is valid only for the space-times of Chrusciel-Delay and Corvino-Schoen, the exact Killing vector field near i_0 is useful for obtaining the energy estimates ;
- **massless scalar fields :** the construction is not quite complete, the Goursat problem is solved, it is easier than for Dirac and Maxwell, but for the energy estimates, we have a problem with the scalar curvature, it is said how to obtain them, but this is not done explicitely and again we need the exact symmetry near i_0.

The construction for the wave equation is now complete. Extensions to nonlinear cases is in progress.

The resolution of the Goursat problem is of course the fundamental construction in this work. It follows the ideas of a work by L. Hörmander [25] in which he describes a technique based on energy estimates. His method is well adapted to scattering in that it gives convergences in minimum regularity spaces ; in addition, we slightly modify his approximation of solutions, this allows us to show the equivalence with an analytic scattering theory, defined in terms of classical wave operators whose comparison dynamics are asymptotic profiles.

In order to work with completely generic asymptotically simple space-times, without explicit symmetry near i_0, one possibility is to handle the Goursat problem with very little regularity (at least at one point) since the metric is not much better than Lipschitz near i_0. Also for the space-times of Christodoulou and Klainerman, that are the most general solutions (in terms of regularity) of the Einstein vacuum equations that are asymptotically simple (almost, not quite regular enough at $\mathscr{I}$), we need weaker regularity for the Goursat problem.

3 Hörmander's work (J.F.A. 1990)

What we describe is a slightly simplified version of Hörmander's work for the fully characteristic Cauchy problem. His work in fact treats the Cauchy problem, the Goursat problem and anything in between where the initial data surface is allowed to be locally spacelike or null. However, we are interested here in the Goursat problem.

3.1 Geometrical framework

- X (space) a $\mathcal{C}^\infty$ compact manifold without boundary, $\dim_{\mathbb{R}} X = n \geq 1$.
- $\tilde{X} = \mathbb{R}_t \times X$ (spacetime).

- $g(t) \in \mathcal{C}^\infty\left(\tilde{X}\,;\ T^*X \odot T^*X\right)$ a time-dependent Riemannian metric on X.

- Hypersurface Σ on which the initial data are fixed, defined as a graph

$$\Sigma = \{(\phi(x), x)\ ;\ x \in X\} \tag{1}$$

  where $\phi\ :\ X \to \mathbb{R}$ is Lipschitz-continuous (which authorizes singularities like the vertex of a cone). Σ is assumed to be characteristic (or null) : this means that at each point where ϕ is differentiable (hence for almost every $x \in X$, since ϕ is Lipschitz-continuous and therefore differentiable almost everywhere), the normal (or even simpler the co-normal) to Σ is null for the Lorentzian metric

$$\tilde{g} = \mathrm{d}t^2 - g\,,$$

  i.e.

$$g^{\alpha\beta}(\phi(x), x)\partial_\alpha\phi(x)\partial_\beta\phi(x) = 1\,,\ \text{for a.e. } x \in X\,.$$

3.2 Analytical framework

- We choose a smooth measure $\mathrm{d}\nu$ on X, for example that associated with the metric $g(0)$, that, in a local system of coordinates, is written $\mathrm{d}\nu = \gamma\mathrm{d}x$, where γ is a smooth density.

- Wave equation :

$$\Box u + L_1 u = 0\,, \tag{2}$$

  where

$$\Box := \partial_t^2 - \gamma^{-1}\partial_\alpha\left(\gamma g^{\alpha\beta}\partial_\beta\right)$$

  is the simplified d'Alembertian and

$$L_1 := b^0\partial_t + b^\alpha\partial_\alpha + c\,,\ b^0\,,\ b^\alpha\,,\ c\ \in \mathcal{C}^\infty\left(\tilde{X}\right)\,.$$

- On X we have natural Sobolev spaces $H^s(X)$, $s \in \mathbb{R}$, defined via local charts. On $\tilde{X}$ we shall only use local Sobolev spaces $H^s_{\mathrm{loc}}(\tilde{X})$.

- Σ is a Lipschitz hypersurface, we can therefore define on Σ the spaces $H^s(\Sigma)$ for $-1 \le s \le 1$ that are canonically isomorphic to the corresponding $H^s(X)$ via parametrization (1). We have a natural norm on $H^1(\Sigma)$: two possible expressions

  (i) $u \in H^1(\Sigma)$, lift on Σ of $v \in H^1(X)$,

$$\|u\|^2_{H^1(\Sigma)} = \int_X \left(|v(x)|^2 + g^{\alpha\beta}(\varphi(x), x)\partial_\alpha v(x)\partial_\beta v(x)\right)\mathrm{d}\nu(x)\,;$$

  (ii) $u \in H^1(\Sigma)$, trace on Σ of $\psi \in H^{3/2}_{\mathrm{loc}}(\tilde{X})$,

$$\|u\|^2_{H^1(\Sigma)} = \int_\Sigma \left(|\psi|^2 + g^{\alpha\beta}\left(\partial_\alpha\psi + \partial_\alpha\varphi\partial_t\psi\right)\left(\partial_\beta\psi + \partial_\beta\varphi\partial_t\psi\right)\right)\mathrm{d}\nu_\Sigma\,,$$

  where $\mathrm{d}\nu_\Sigma$ is the lift on Σ via (1) of the measure $\mathrm{d}\nu$.

- Energy norm on $X_t := \{t\} \times X$:

$$\begin{aligned} E(t,u) &= \|u\|^2_{H^1(X_t)} + \|\partial_t u\|^2_{L^2(X_t)} \\ &= \int_{X_t} \left(|u|^2 + g^{\alpha\beta}\partial_\alpha u \partial_\beta u + |\partial_t u|^2 \right) \mathrm{d}\nu \, . \end{aligned}$$

- Cauchy problem for (2) : well known to be well-posed in the energy space, i.e. for data $u_{|t=0} \in H^1(\Sigma)$ and $\partial_t u_{|t=0} \in L^2(\Sigma)$. With equivalence locally uniform in time between the energy norms at any two given times

$$E(t,u) \leq e^{C_1(T,g,L_1)|t-s|} E(s,u) \, , \; \forall t,s \in [-T,T] \, . \tag{3}$$

  We also have regularity properties of the solutions : if u_0, $u_1 \in \mathcal{C}^\infty(X)$, then the associated solution u belongs to $\mathcal{C}^\infty(\tilde{X})$.

- We denote by $\mathcal{E}$ the space of solutions of (2) in

$$\mathcal{F} := \mathcal{C}^0 \left(\mathbb{R}_t \, ; \; H^1(X) \right) \cap \mathcal{C}^1 \left(\mathbb{R}_t \, ; \; L^2(X) \right) \, .$$

  Thanks to energy estimate (3), we see that we can choose on $\mathcal{E}$ the $H^1 \times L^2$ norm of the initial data at $t = 0$ and that makes it a Banach space isomorphic to $H^1(X) \times L^2(X)$.

- Energy norm on Σ :

$$E_\Sigma(u) = \|u_{|\Sigma}\|^2_{H^1(\Sigma)} \, ,$$

  all information conerning the time derivative of u restricted to Σ is apparently lost.

3.3 Hörmander's theorem

We consider the operator T_Σ that to solutions $u \in \mathcal{E} \cap \mathcal{C}^\infty(\tilde{X})$ associates $u_{|\Sigma}$.

Theorem 1 (Hörmander 1990). *The operator T_Σ extends in a unique manner as a linear continuous map, still denoted T_Σ, from $\mathcal{E}$ to $H^1(\Sigma)$. Moreover, T_Σ is an isomorphism.*

3.4 structure of the proof

- Two fundamental energy estimates : let $T > \max\{|\varphi(x)| \, , \; x \in X\}$, there exists $C_2(T,g,L_1) > 0$, continuous in T, the Lipschitz norms of g and g^{-1} and the L^∞ norms of the corfficients of L_1 on $]-T,T[\times X$, such that, for all $u \in \mathcal{E} \cap \mathcal{C}^\infty(\tilde{X})$,

$$\|T_\Sigma u\|^2_{H^1(\Sigma)} = E_\Sigma(u) \leq C_2(T,g,L_1)\|u\|^2_{\mathcal{E}} = E(0,u) \, , \tag{4}$$

$$E(0,u) \leq C_2(T,g,L_1) E_\Sigma(u) \, . \tag{5}$$

  These are standard energy estimates proved for smooth solutions by integration by parts (Stokes's formula really).

  Estimate (4) allows to extend T_Σ in a unique manner as a linear continuous map from $\mathcal{E}$ to $H^1(\Sigma)$. Estimate (5) not only tells us that T_Σ is one-to-one, it also gives us a reciprocal estimate, hence, for proving the surjectivity of T_Σ, we only need to define an inverse to T_Σ on a dense subspace of $H^1(\Sigma)$.

- Surjectivity of T_Σ : we construct a solution to the Goursat problem for some data v that is the lift on Σ of a smooth function (still denoted v) on X (this is our dense subspace of data). This is done in three steps (note that we will need some data w for the time derivative of u on Σ in each approximation, provided we always keep the same w, the choice is totally irrelevant, we simply take $w = 0$) :

  1. Σ smooth and spacelike, the result is clear (usual Cauchy problem) ;
  2. Σ Lipschitz and spacelike, we approach Σ by smooth spacelike surfaces and use compactness arguments.
  3. Σ Lipschitz and null : we approach this situation by the previous one by slowing down the propagation of the equation ; we consider, for $0 < \lambda < 1$,

     $$\Box_\lambda u + L_1 u = 0\,,\ \Box_\lambda = \partial_t^2 - \lambda\gamma\partial_\alpha\left(\gamma g^{\alpha\beta}\partial_\beta\right)\,. \tag{6}$$

     Σ is uniformly spacelike for (6). Then use weak convergence and compactness.

3.5 Our slightly different version more adapted to scattering

In our scattering constructions, we adopt Hörmander's technique, but we prove existence in a slightly different way. We describe the construction for future infinity.

We consider on our physical spacetime a global time function t and the foliation by the level hypersurfaces $\{\Sigma_t\}_{t\in\mathbb{R}}$ of t. Say that Σ_0 is the surface on which we fix the initial data for the Cauchy problem. The gradient of t will serve to identify the points on the different slices. We consider a compact domain $K \subset \Sigma$, large enough so that outside $\mathbb{R}_t^+ \times K$ we can define a congruence of outgoing null geodesics, let us denote $\mathcal{C}$ such a congruence.

Now consider some scattering data for our equation $\hat{\phi}_{\mathscr{I}^+} \in \mathcal{C}_0^\infty(\mathscr{I}^+)$ (for the wave equation, it will simply be the trace of u, for Dirac or Maxwell the trace of one component of the spinor field. We project this data onto Σ_t along $\mathcal{C}$ and cut off inside K (with a smooth cut-off if necessary). We complete the data for the Cauchy problem with identically zero functions. We propagate backwards down to Σ_0 using the full dynamics. We show that as $t \to +\infty$, the data we obtain on Σ_0 tends to the initial data for a solution whose image by T^+ is precisely $\hat{\phi}_{\mathscr{I}^+}$ (strong limit in the energy space) , i.e. we have constructed an inverse to T^+ on a dense domain. Moreover we have constructed it as a classical wave operator : free dynamics (null geodesic flow), followed by cut-off, followed by backwards physical dynamics.

Hence, we recover a complete scattering theory defined in terms of classical wave operators, in a generically non stationary framework where usual analytic scattering techniques are not applicable.

3.6 Hörmander's remark

At the end of his paper, Hörmander remarks that although all the results are expressed and proved for a smooth metric and smooth coefficients of L_1, the bounds in the estimates only depend on the Lipschitz norm of the metric and the L^∞ norm of the coefficients of L_1 and so this is the proper generality of the theorem.

The problem is that all parts of the proof rely heavily of regular solutions that allow to perform integrations by parts to prove energy estimates. It in fact suffices to have $H^2_{\rm loc}$ solutions to be able to make the whole proof run in the same manner. However, for the regularity setting proposed by Hörmander, we do not have access to such solutions.

This remark is not proved in Hörmander paper and to my knowledge it has not been checked ever since.

4 Extension to lower regularity

This is a recently submitted work [32]

4.1 Cauchy problem for a Lipschitz metric

Theorem 2. *We assume that g is Lipschitz-continuous on $\tilde{X}$ and the coefficients of L_1 are locally L^∞ on $\tilde{X}$. Then, for any $u_0 \in H^1(X)$, $u_1 \in L^2(X)$, there exists a unique*

$$u \in \mathcal{F} := L^\infty_{\rm loc}(\mathbb{R}_t\,;\ H^1(X)) \cap \mathcal{C}^1(\mathbb{R}_t\,;\ L^2(X))$$

solution of (2) associated with the data u_0, u_1 at, say, $t = 0$. Moreover, u satisfies estimate (3) for almost all s, t and it turns out that $u \in \mathcal{F}$ (i.e. the solutions are in fact continuous in time with values in $H^1(X)$). Hence, we still denote by $\mathcal{E}$ the space of solutions in $\tilde{\mathcal{F}}$.

Corollary 4.1. *Under the same regularity hypotheses as theorem 2, we consider the equation*

$$\partial_t^2 u - g^{\alpha\beta}\partial_\alpha\partial_\beta u = 0\,, \tag{7}$$

this is equation (2) with

$$L_1 = \gamma^{-1}\partial_\alpha\left(\gamma g^{\alpha\beta}\right)\partial_\beta\,,$$

hence the Cauchy problem for (7) is well-posed in the same function space as for (2). Moreover, if the initial data

$$(u_0, u_1) \in H^2(X) \times H^1(X)\,,$$

the associated solution u of (7) belongs to

$$\bigcap_{l=0}^{2} \mathcal{C}^l\left(\mathbb{R}_t\,;\ H^{2-l}(X)\right)\,.$$

The proof of the corollary is trivial, we just commute derivatives into the equation.

Ideas of the proof of theorem 2.

1. **Uniqueness.** Obtained by regularizing purely in space. We obtain energy estimate (3) (for all t, s such that $u(t), u(s) \in H^1(X)$).

2. **Existence.** We regularize the metric and coefficients of L_1. The rest makes use of weak convergence techniques and compactness arguments. We see that $u(t) \in H^1(X)$ for all t.

3. **Continuity in time.** Weak continuity in time plus energy estimate (3).

4.2 Goursat problem for a weekly regular metric

The trace operator T_Σ associates to a solution $u \in \mathcal{E}$ its trace on Σ, this is well defined in $\mathcal{L}(\mathcal{E}\,;\ L^2(\Sigma))$ by standard trace theorems, we need energy estimates to show it is valued in H^1 and one-to-one and then we also need to prove surjectivity.

1. **Estimates (4) and (5).** Idea is to regularize the solution so that the approximating functions satisfy analogues of (4) and (5), and converge strongly in $H^1(\Sigma)$ and $H^1(X)$; will guarantee that the energy estimates remain valid in the limit.

   We write equation (2) as (7) plus first order perturbation $\tilde{L}_1$. Then regularize the first order perturbation and the initial data. We get solutions u_k in $H^2_{\mathrm{loc}}(\tilde{X})$ thanks to the corollary above. They all satisfy estimates of the type (4), (5), that can simply be proved by integration by parts, with constants uniform in k. All the equations have the same principal part, this allows to get for $u_k - u_l$ estimates of the type (4), (5) with a source term $(L_1^k - L_1^l)u_l$. We simply need to read the regularity of the coefficients of $\tilde{L}_1$ that guarantees that this term tends to zero in say $L^1_{\mathrm{loc}}(\mathbb{R}_t\,;\ L^2(X))$. We see that we need

$$g \in L^\infty_{\mathrm{loc}}(\mathbb{R}_t\,;\ \mathcal{C}^1(X)) \cap W^{1,\infty}_{\mathrm{loc}}(\mathbb{R}_t\,;\ \mathcal{C}^0(X))\,, \tag{8}$$

$$b^0\,,\ b^\alpha \in L^\infty_{\mathrm{loc}}(\mathbb{R}_t\,;\ \mathcal{C}^0(X))\,, \tag{9}$$

$$c \in L^\infty_{\mathrm{loc}}(\tilde{X})\,. \tag{10}$$

2. **Surjectivity.** Regularize metric, coefficients of L_1 and slow down propagation, plus compactness.

Theorem 3. *Under the regularity assumptions (8)-(10), the operator T_Σ is an isomorphism from $\mathcal{E}$ onto $H^1(\Sigma)$.*

5 Perspectives for black hole space-times

The conformal geometry of black hole space-times is quite different from that of asymptotically simple space-times :

- horizon : this means that there will be four trace operators, one for each asymptotic region for the future and the past ;
- singularity at timelike infinity : this is where the whole structure collapses.

If the conformal scattering constructions are to be extended to black hole space-times, it means that we must understand the singularity of the conformal metric at timelike infinity. This is in itself a whole research project (judging from the time people have spent on spacelike infinity). But it seems just as important to understand. The recent work of M. Dafermos and I. Rodnianski gives strong hopes to construct the scattering theory for the wave equation on spherically symmetric blach-hole space-times in a conformal manner using their results. This seems an interesting path that could be pursued for other equations. Entensions to other geometries seems however very difficult for the moment.

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