Time asymmetric spacetimes near spatial and null infinities

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Definition (Asymptotic simplicity). *A smooth spacetime* $(\widetilde{M}, \widetilde{g})$ *is called* asymptotically simple *if there exists a smooth, oriented, time-oriented, causal spacetime* (M, g) *and on* M *a smooth function* $\boldsymbol{\Omega}$ *such that:*

i) M *is a manifold with boundary* $\mathscr{I}$*,*

ii) $\boldsymbol{\Omega} > 0$ *on* $M \backslash \mathscr{I}$ *and* $\boldsymbol{\Omega} = 0$*,* $d\boldsymbol{\Omega} \neq 0$ *on* $\mathscr{I}$*,*

iii) there exists an embedding $\boldsymbol{\Phi}$ *of* $\widetilde{M}$ *onto* $\boldsymbol{\Phi}(\widetilde{M}) = M \backslash \mathscr{I}$ *such that* $\boldsymbol{\Omega}^2 \boldsymbol{\Phi}^{-1*} \widetilde{g} = g$*,*

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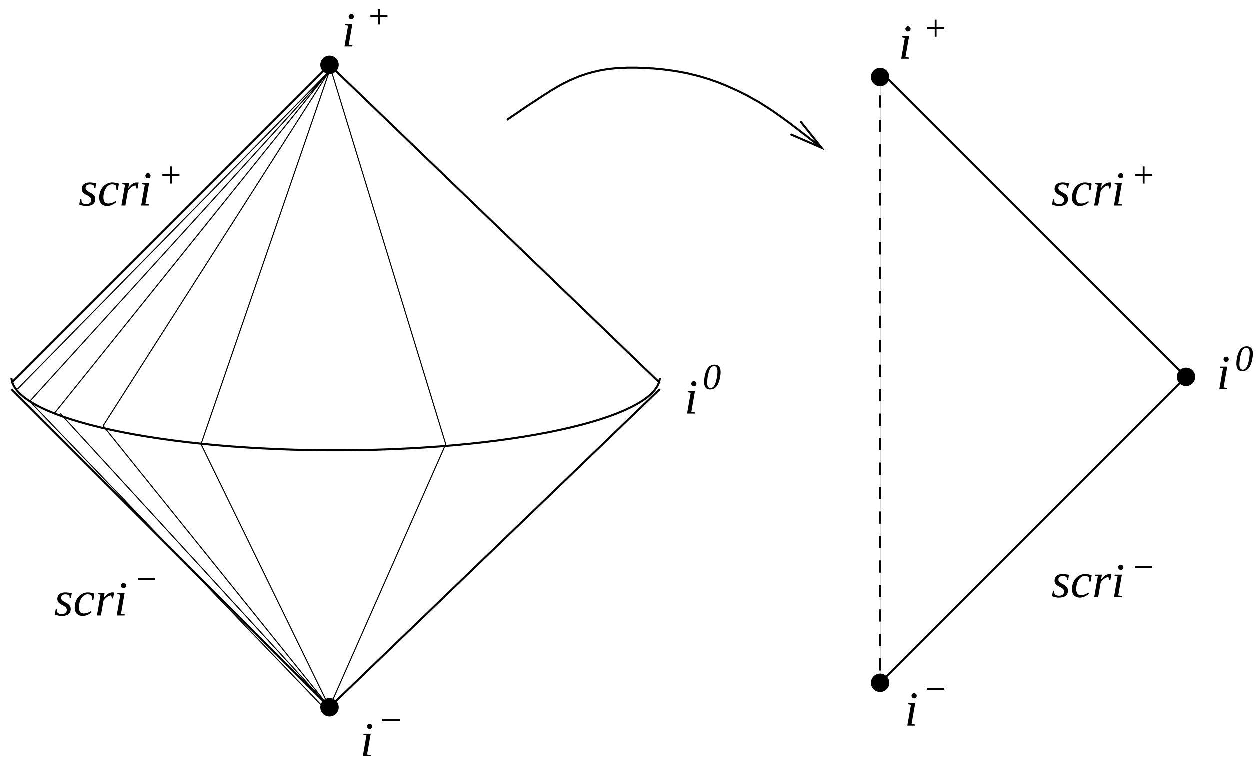
iv) each null geodesic of $(\widetilde{M}, \widetilde{g})$ *acquires two distinct endpoints on* $\mathscr{I}$.

- That is, we do not look to the spacetime itself $(\widetilde{M}, \widetilde{g}_{ab})$ but to a compactified one with boundary —*unphysical*, (M, g_{ab})— which is obtained by means of a conformal factor.



- The (conformal) boundary —where $\Omega = 0$— of the unphysical manifold can be shown to correspond to the points at "infinity" in the physical spacetime.

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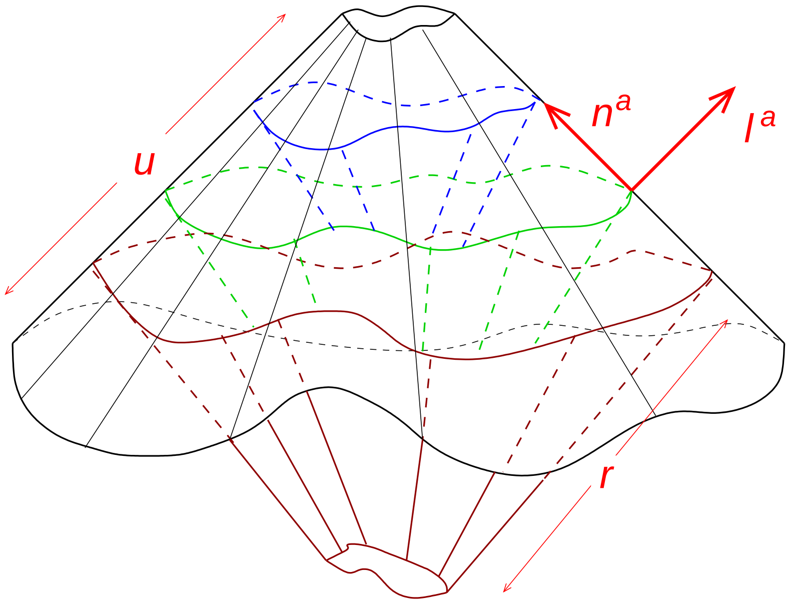
- The proposal is an attempt to provide a characterisation of isolated systems in general relativity...

Peeling behaviour: a prescription for the decay of the components of the Weyl tensor along generators of light cones opening towards null infinity.



Describe the "free" gravitational field by means of the Weyl spinor $\widetilde{\Psi}_{ABCD}$ with respect to an adapted tetrad (NP gauge):

$$\widetilde{C}_{abcd} \text{``=''} \widetilde{\Psi}_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} + \overline{\widetilde{\Psi}}_{A'B'C'D'}\epsilon_{AB}\epsilon_{CD},$$

such that

$$\widetilde{\nabla}^{AA'}\widetilde{\Psi}_{ABCD} = 0.$$

- The gravitational field *peels* if

$$\widetilde{\Psi}_{ABCD} = \frac{[N]_{ABCD}}{r} + \frac{[III]_{ABCD}}{r^2} + \frac{[II]_{ABCD}}{r^3} + \frac{[I]_{ABCD}}{r^4} + O\left(\frac{1}{r^5}\right).$$

- The Weyl spinor has 5 complex components (with respect to the NP frame): $\widetilde{\Psi}_0, \ldots, \widetilde{\Psi}_4$.

  If the spacetime *peels* then:

$$\widetilde{\Psi}_4 = O(1/r) \qquad \text{“radiation field”}$$
$$\widetilde{\Psi}_3 = O(1/r^2)$$
$$\widetilde{\Psi}_2 = O(1/r^3)$$
$$\widetilde{\Psi}_1 = O(1/r^4)$$
$$\widetilde{\Psi}_0 = O(1/r^5)$$

- NB. The above decay can be obtained via an *(asymptotic) characteristic initial value problem* if $\widetilde{\Psi}_0 = O(1/r^5)$ is prescribed.

Questions...

The usefulness of Penrose's proposal depends on being able to answer the following questions:

(a) are there non-trivial solutions to the Einstein field equations which are asymptotically simple?

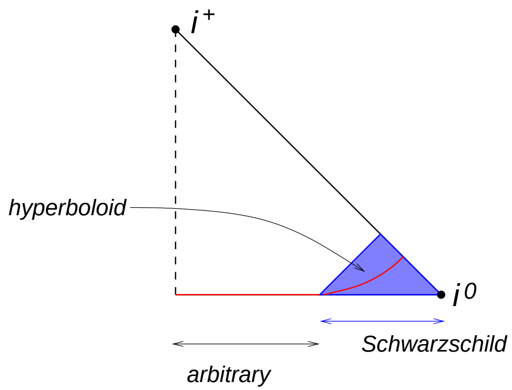
(b) how big is this class of solutions?

(c) how can we characterise them in terms of initial data?

(d) what is the physical content of the proposal? What are we leaving aside? Are the solutions one can construct useful?

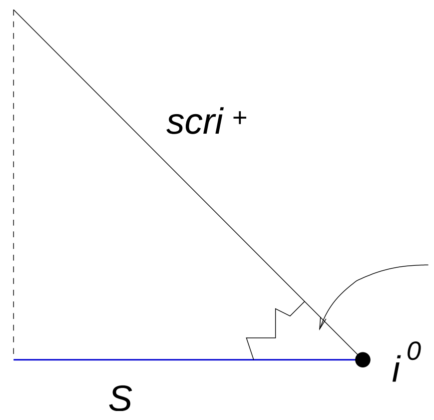
(Non-trivial) Solutions to the Einstein vacuum equations with Peeling do exist[a]....

- Their construction mainly consists of two ingredients:
  (a) Corvino-type data. These are solutions to the constraint equations which are fairly arbitrary inside a compact set and outside it they are Schwarzschildean.
  (b) Friedrich's hyperboloidal existence theorem. Hyperboloidal data —i.e. data prescribed on an asymptotically null hypersurface— close to Minkowskian hyperboloidal data have a development with a complete null infinity.



[a]See e.g. P.T. Chruściel & Delay, *Existence of non-trivial, vacuum, asymptotically simple spacetimes*, Class. Quantum Grav. **19**, L71 (2002).

- The crucial lesson to be extracted from this example is that "the decision" whether a spacetime has a Peeling null infinity or not is made in arbitrarily small neighbourhood of spacelike infinity...



- The location of the intersection hyperboloids with $\mathscr{I}^+$ is irrelevant for Friedrich's theorem [a].

[a] H. Friedrich. *On the existence of n-geodesically complete or future complete solutions of Einstein's equations with smooth asymptotic structure*. Comm. Math. Phys. **107**, 587 (1986)

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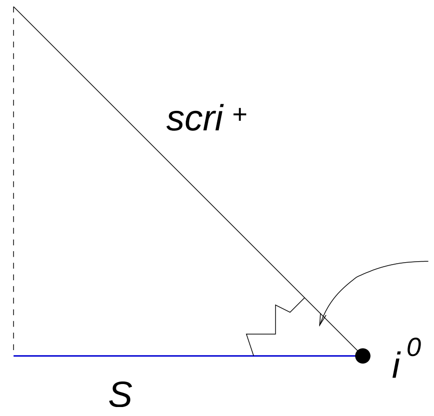

- The location of the intersection hyperboloids with $\mathscr{I}^+$ is irrelevant for Friedrich's theorem [a].
- Thus, if one wants to construct solutions with the Peeling behaviour then one has to examine with care what happens at spatial infinity...

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The regular Cauchy problem at spatial infinity:

(i) the data and equations are regular,

(ii) spacelike and null infinity have a finite representation with their structure and location known *a priori*,

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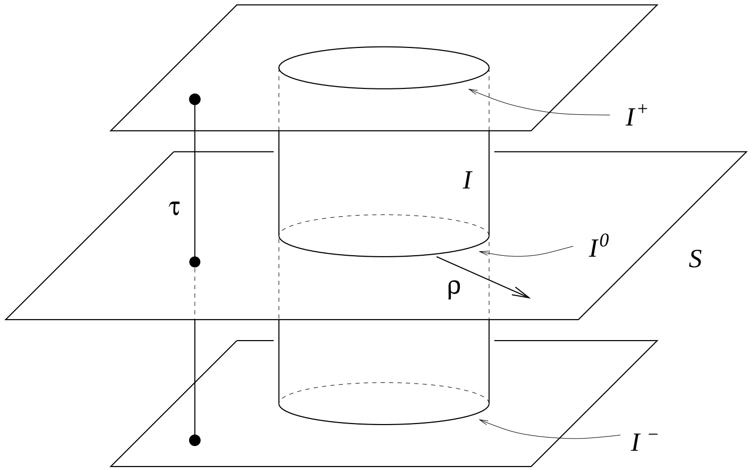
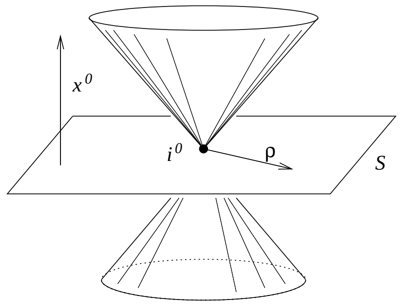
(iii) the setting relies on general properties of conformal structures.

On the implementation:

- Use Weyl connections and the extended Conformal Einstein Field Equations in spinorial form.
- Use conformal geodesics to construct a system of conformal Gaußian coordinates.

Blow-up of i^0:





The conformal factor is given by:

$$\Theta = \frac{\Omega}{\omega}(1 - \tau^2),$$

where

$$\Omega/\omega = \rho + O(\rho^2),$$

is given in terms of initial data on $\mathcal{S}$.

The conformal propagation equations near spatial infinity:

- The unknowns are given by the components of the frame, connection, and Ricci tensor:

$$v = (c^{\mu}_{AB}, \Gamma_{ABCD}, \Phi_{ABCD}),$$

 and the components of the Weyl spinor

$$\phi = (\phi_0, \phi_1, \phi_2, \phi_3, \phi_4).$$

- The evolution equations are of the form:

$$\partial_\tau v = Kv + Q(v, v) + L\phi,$$
$$A^0\phi + A^\alpha \partial_\alpha \phi = B(\Gamma_{ABCD})\phi.$$

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  - Standard methods of symmetric hyperbolic systems cannot be used to analyse the equations near $I^\pm$.

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- The cylinder I is a total characteristic of the propagation equations:
  - The equations reduce to an interior system on I.
- Differentiation of the propagation equations with respect to ρ and evaluation on I renders a hierarchy of interior equations:

$$\partial_\tau v^{(p)} = K v^{(p)} + Q(v^{(0)}, v^{(p)}) + Q(v^{(p)}, v^{(0)}) + \sum_{j=1}^{p-1} \Big(Q(v^{(j)}, v^{(p-j)}) + L^{(j)} \phi^{(p-j)} \Big) + L^{(p)} \phi^{(0)},$$

$$A^{0,(0)} \partial_\tau \phi^{(p)} + A^{C,(0)} \partial_C \phi^{(p)} = B(\Gamma^{(0)}_{ABCD}) \phi^{(p)} + \sum_{j=1}^{p} \begin{pmatrix} p \\ j \end{pmatrix} \Big(B(\Gamma^{(j)}_{ABCD}) \phi^{(p-j)} - A^{\mu,(j)} \partial_\mu \phi^{(p-j)} \Big),$$

which can be solved recursively —the equations are linear and decoupled.

- The solution jets:

$$J^{(p)}(v) = \{v^{(0)}, \ldots, v^{(p)}\} \qquad J^{(p)}(\phi) = \{\phi^{(0)}, \ldots, \phi^{(p)}\}$$

can be thought as the coefficients in the formal expansions:

$$v \sim \sum_{p\geq 0} \frac{1}{p!} v^{(p)}(\tau, \theta, \varphi)\rho^p, \qquad \phi_j \sim \sum_{p\geq 0} \frac{1}{p!} \phi_j^{(p)}(\tau, \theta, \varphi)\rho^p.$$

- They are completely determined by the expansions of the initial data on $\mathcal{S}$ near spatial infinity.

- Thus, one can relate properties of the initial data with the asymptotic behaviour of the spacetime near null and spatial infinities.

Obstructions to the smoothness of null infinity:

Due to the degeneracy of the Bianchi propagation equations at the critical sets $I^{\pm}$, any hint of non-smoothness is bound to arise first in the coefficients $\phi^{(p)}$.

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- Decompose $\phi^{(p)}$ in spherical harmonics:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \;\; {}_{j-2}Y_{lm}$$

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- A first analysis of the equations at the level of the linearised Bianchi equations —spin-2 zero-rest-mass fields— reveals that the coefficients

$$a_{j;p,p,m}(\tau) \longleftrightarrow {}_{j-2}Y_{pm}, \qquad m = -p, \ldots, p$$

develop a certain type of logarithmic singularities at $\tau = \pm 1$.

- More precisely,

$$a_{j;p,p,m}(\tau) = A(1-\tau)^{p-2+j}(1+\tau)^{p+2-j}\ln(1-\tau) \\ + B(1-\tau)^{p-2+j}(1+\tau)^{p+2-j}\ln(1+\tau). + (\text{polynom in } \tau)$$

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- These singularities can be precluded by imposing a certain regularity condition at the initial hypersurface:

$$a_{0;p,p,m}(0) = a_{4;p,p,m}(0), \qquad m = -p, \ldots, p.$$

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- Any further analysis of other possible obstructions will involve the particular structure of the data upon consideration.

Selecting the initial data:

- Construct maximal initial data $(\tilde{h}_{\alpha\beta}, \tilde{\chi}_{\alpha\beta})$ by means of the conformal Ansatz:

$$\tilde{h}_{\alpha\beta} = \vartheta^4 h_{\alpha\beta}, \qquad \tilde{\chi}_{\alpha\beta} = \vartheta^{-2}\psi_{\alpha\beta},$$

so that the constraint equations reduce to:

$$D^\alpha \psi_{\alpha\beta} = 0,$$
$$\left(D^\alpha D_\alpha - \frac{1}{8}r\right)\vartheta = \frac{1}{8}\psi_{\alpha\beta}\psi^{\alpha\beta}\vartheta^{-7}.$$

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- Static and stationary initial data are expected to play a privileged role:
  - they should not develop singularities at the critical sets!

- To be able to consider stationary data one has to move away from conformal metrics which are smooth at infinity:

$$h_{\alpha\beta} = h^{(1)}_{\alpha\beta} + |x|^3 h^{(2)}_{\alpha\beta},$$

where $h^{(1)}_{\alpha\beta} = O(1)$ and $h^{(2)}_{\alpha\beta} = O(1)$ are smooth.

- To solve the momentum constraint write:

$$\psi_{\alpha\beta} = \psi^{A}_{\alpha\beta} + \psi^{J}_{\alpha\beta} + \psi^{Q}_{\alpha\beta} + \psi^{\lambda}_{\alpha\beta} + \mathcal{L}(u)_{\alpha\beta},$$

where

$$\psi^{A}_{\alpha\beta} = \frac{A}{\rho^3}\left(3n_\alpha n_\beta - \delta_{\alpha\beta}\right),$$

$$\psi^{J}_{\alpha\beta} = \frac{3}{\rho^3}\left(n_\beta \epsilon_{\gamma\alpha\rho} J^\rho n^\gamma + n_\alpha \epsilon_{\rho\beta\gamma} J^\gamma n^\rho\right),$$

$$\psi^{Q}_{\alpha\beta} = \frac{3}{2\rho^2}\left(Q_\alpha n_\beta + Q_\beta n_\alpha - (\delta_{\alpha\beta} - n_\alpha n_\beta) Q^\gamma n_\gamma\right),$$

$$\psi^{\lambda}_{\alpha\beta} = O(1/\rho) \qquad \text{(higher multipoles).}$$

- Both ϑ and u_α satisfy elliptic equations:
  - Thus, one can only construct expansions which are consistent with the constraint equations.
- In particular:

$$\vartheta = \frac{1}{\rho} + W,$$

where

$$W = \frac{m}{2} + \sum_{p=1}^{\infty} \frac{1}{p!} w_{p;l,m} Y_{lm} \rho^p,$$

and the $w_{p;l,m}$'s are functions of the parameters of the freely specifiable data!

Further obstructions to the smoothness of null infinity:

$$\phi_j^{(p)} = \sum_{l=|j-2|}^{p} \sum_{m=-l}^{l} a_{j;p,l,m}(\tau) \;\; {}_{j-2}Y_{lm}$$

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  - If $\Upsilon^{\pm}_{p;l,m} = 0$ for given p, l, m then a certain subset of the logarithmic singularities is not present.
  - The obstructions are expressible in terms of the initial data.

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  - In particular, for $p = 5$, one has quadrupolar obstructions (harmonics ${}_{j-2}Y_{2m}$) of the form:

$$\Upsilon^{+}_{5;2,m} = \Upsilon^{-}_{5;2,m} = m \times (\text{quadrupole}) + (\text{dipole})^2 + J^2.$$

    The obstructions are of a time symmetric nature.

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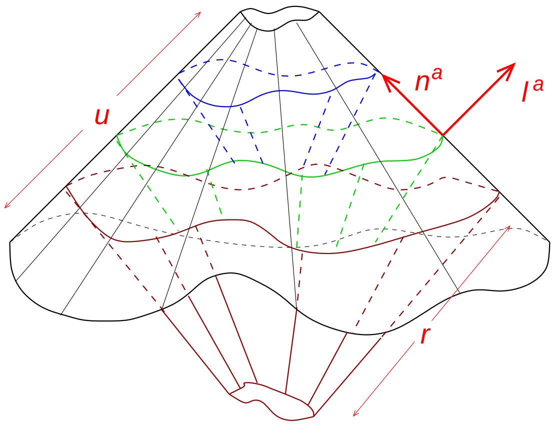
- And so on...

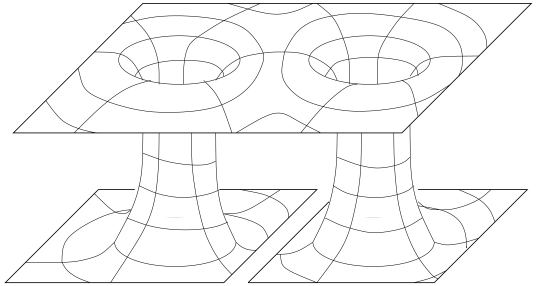
How does this translate into the NP gauge?



$$\widetilde{\Psi}_0 \sim k_0 \sum_m \Upsilon^+_{5;2,m} \ln r / r^4 + \psi_0^4 / r^4 + \cdots,$$
$$\widetilde{\Psi}_1 \sim k_1 \sum_m \Upsilon^+_{5;2,m} \ln r / r^4 + \psi_1^4 / r^4 + \cdots,$$
$$\widetilde{\Psi}_2 \sim \psi_2^3 / r^3 + \cdots,$$
$$\widetilde{\Psi}_3 \sim \psi_3^2 / r^2 + \cdots,$$
$$\widetilde{\Psi}_4 \sim \psi_4^1 / r + \cdots.$$

- The spacetime cannot be *stationary* if $\Upsilon^+_{5;2,m}$ —stationary spacetimes peel.
- The existence of time asymmetric obstructions opens room for the existence of spacetimes where $\mathscr{I}^+$ and $\mathscr{I}^-$ have different degrees of differentiability.
  - E.g. $\mathscr{I}^+$ could peel, but $\mathscr{I}^-$ could be polyhomogeneous with
$$\widetilde{\Psi}_4 = O(r^{-5} \ln r).$$

An example: *Brill-Lindquist data*



$$\widetilde{h}_{\alpha\beta} = \left(1 + \frac{m_1}{2|\vec{x} - \vec{x}_1|} + \frac{m_2}{2|\vec{x} - \vec{x}_2|}\right)\delta_{\alpha\beta},$$

$$\widetilde{\chi}_{\alpha\beta} = 0.$$

- In this case one finds,

$$\widetilde{\Psi}_0 = \Upsilon(k_0 + k_1 \ln r)\, r^{-4} + \cdots + k_0' \Upsilon \ln^3 r / r^6 + \cdots$$

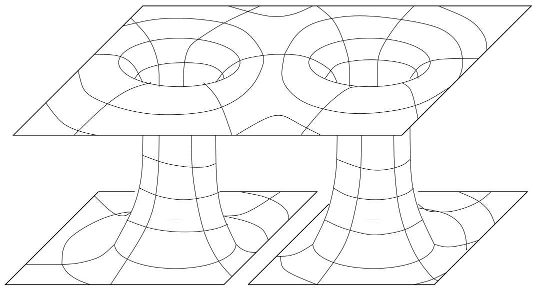
$$\widetilde{\Psi}_1 = \Upsilon(k_2 + k_3 \ln r)\, r^{-4} + \cdots$$

$$\widetilde{\Psi}_2 = O(r^{-3})$$

$$\vdots$$

where $\Upsilon = m_1 m_2 |\vec{x}_1 - \vec{x}_2|^2$.

An example: *Brill-Lindquist data*



$$\widetilde{h}_{\alpha\beta} = \left(1 + \frac{m_1}{2|\vec{x} - \vec{x}_1|} + \frac{m_2}{2|\vec{x} - \vec{x}_2|}\right)\delta_{\alpha\beta},$$

$$\widetilde{\chi}_{\alpha\beta} = 0.$$

- In this case one finds,

$$\widetilde{\Psi}_0 = \Upsilon(k_0 + k_1 \ln r) r^{-4} + \cdots + k_0' \Upsilon \ln^3 r / r^6 + \cdots$$

$$\widetilde{\Psi}_1 = \Upsilon(k_2 + k_3 \ln r) r^{-4} + \cdots$$

$$\widetilde{\Psi}_2 = O(r^{-3})$$

$$\vdots$$

where $\Upsilon = m_1 m_2 |\vec{x}_1 - \vec{x}_2|^2$.

- Similar behaviour occurs for Bowen-York data!

Does the vanishing of the obstructions imply stationary data?

- If the data is time symmetric then the vanishing of the hierarchy of obstructions up to order p=7 implies that there is a static data such that:

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- So one is led to conjecture:

  Conjecture. *If the development of a time symmetric data set admits a smooth conformal completion at either future or past null infinity then the initial data set is static in a neighbourhood of spatial infinity.*

- If the initial data is conformally flat (but not necessarily time symmetric), then the vanishing of the obstructions up to $p = 7$ implies:

$$\vartheta = \frac{1}{\rho} + \frac{m}{2} + O(\rho^4), \qquad \psi_{\alpha\beta} = \psi^A_{\alpha\beta} + O(1).$$

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- The conjecture has to be slightly modified:

  Conjecture. *If the time development of (time asymmetric) conformally flat initial data admits a smooth conformal extension at both future and past null infinity, then the initial data is Schwarzschildean in a neighbourhood of infinity.*

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  - May be necessary to restrict the class of data —but keeping the time asymmetry and the non-conformal flatness.

Some references:

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