The Extreme Distortion of Black Holes due to Matter

David Petroff

Friedrich-Schiller-Universität Jena

in collaboration with: Marcus Ansorg (AEI Golm)

Part of project B1 of the SFB “Gravitationswellenastronomie”

project leaders: Reinhard Meinel, Gernot Neugebauer

Outline

1 Introduction

Motivation

- ring of matter surrounding Black Hole is a likely intermediate stage of binary star coalescence (e.g. Shibata, Taniguchi & Uryu) and single star collapse (e.g. Shapiro)
- many researchers speculate that accretion of matter from ring onto Black Hole could explain gamma ray bursts (e.g. Ruffert & Janka)
- of interest for modelling central regions of galaxies
- **distortion of Black Hole due to presence of surrounding matter can be studied**
- provides initial data for time evolution programs

Previous Work

- Will (1974,1975) handled infinitesimal ring around slowly rotating Black Hole perturbatively
- Abramowicz and others (1983,1984,1992) discussed self-gravitating disc around (Newtonian) pseudo Black Hole
- Lanza (1992) studied an infinitesimal disc around a Black Hole numerically
- Nishida & Eriguchi (1992) studied (in integral formulation) polytropic, differentially rotating ring around Black Hole numerically
- Rezzolla, Yoshida, Zanotti, Montero & Font (2003, 2004, 2005) looked at ring dynamics in background Kerr metric

This Talk

- stationary, axisymmetric, uniformly rotating, self-gravitating ring of constant density around Black Hole solved numerically to extremely high accuracy

2 Basic Equations and Numerical Methods

- stationary, axially symmetric, purely rotational fluid:

$$ds^2 = -e^{2\nu}\,dt^2 + \varrho^2 B^2 e^{-2\nu}\,(d\varphi - \omega\,dt)^2 + e^{2\mu}\left(d\varrho^2 + d\zeta^2\right)$$

  metric potentials ν, B, ω, μ are functions of ϱ and ζ
  μ found using line integral

- perfect fluid: $T^{ab} = (\varepsilon + p)u^a u^b + p g^{ab}$

- vanishing divergence of energy-momentum tensor $\longrightarrow$

$$(1 + p/\varepsilon)e^{\nu}\sqrt{1 - v^2} = \text{constant} =: e^{V_0}$$

  with $v := \varrho B e^{-2\nu}(\Omega_\mathrm{r} - \omega)$

Boundary Conditions (Bardeen; Carter; Hawking 1973)

- the coordinates can always be chosen in such a way that the event horizon is located at

$$r = \text{constant} =: r_\text{c} \qquad (r^2 := \varrho^2 + \zeta^2)$$

- r_c is physical parameter of Black Hole (for Schwarzschild: $r_\text{c} = M/2$)
- on the event horizon

$$e^{2\nu} = 0, \qquad \omega = \Omega_\text{c}, \qquad B = 0$$

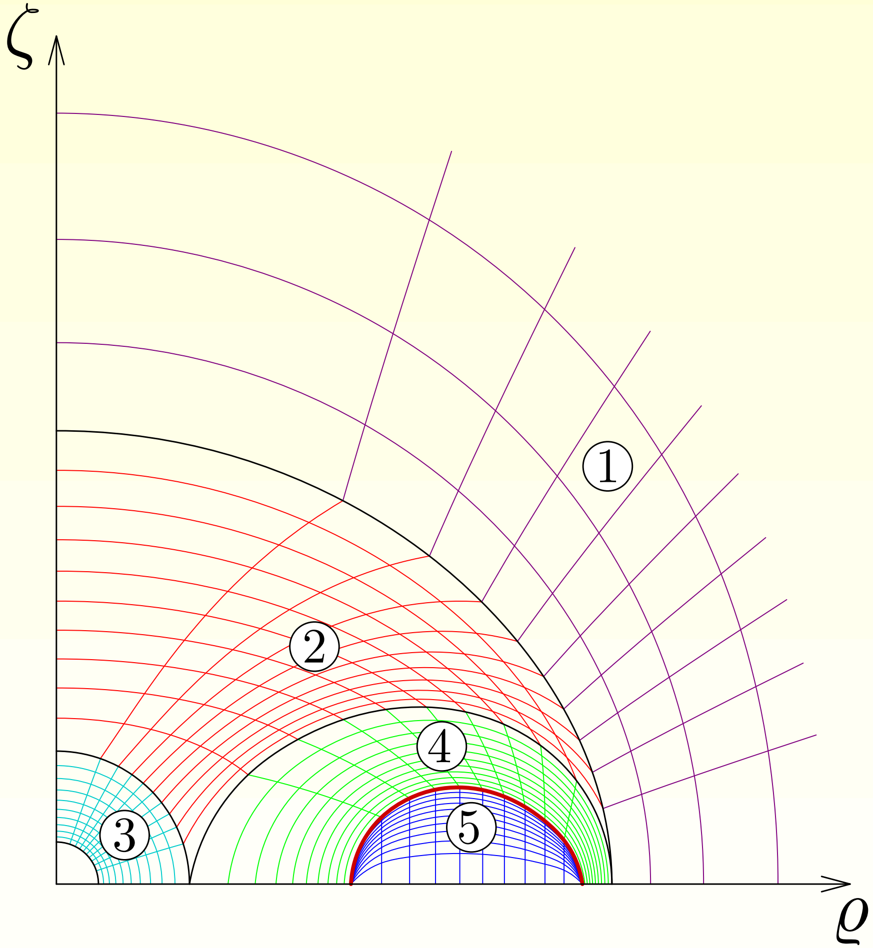
- $u := \nu - \ln B$ is regular (even on the horizon)

Einstein's equations, boundary conditions, regularity, asymptotic behaviour and transition conditions form complete set of equations (free boundary problem)

The Numerical Methods

Multi-domain, pseudo-spectral method:

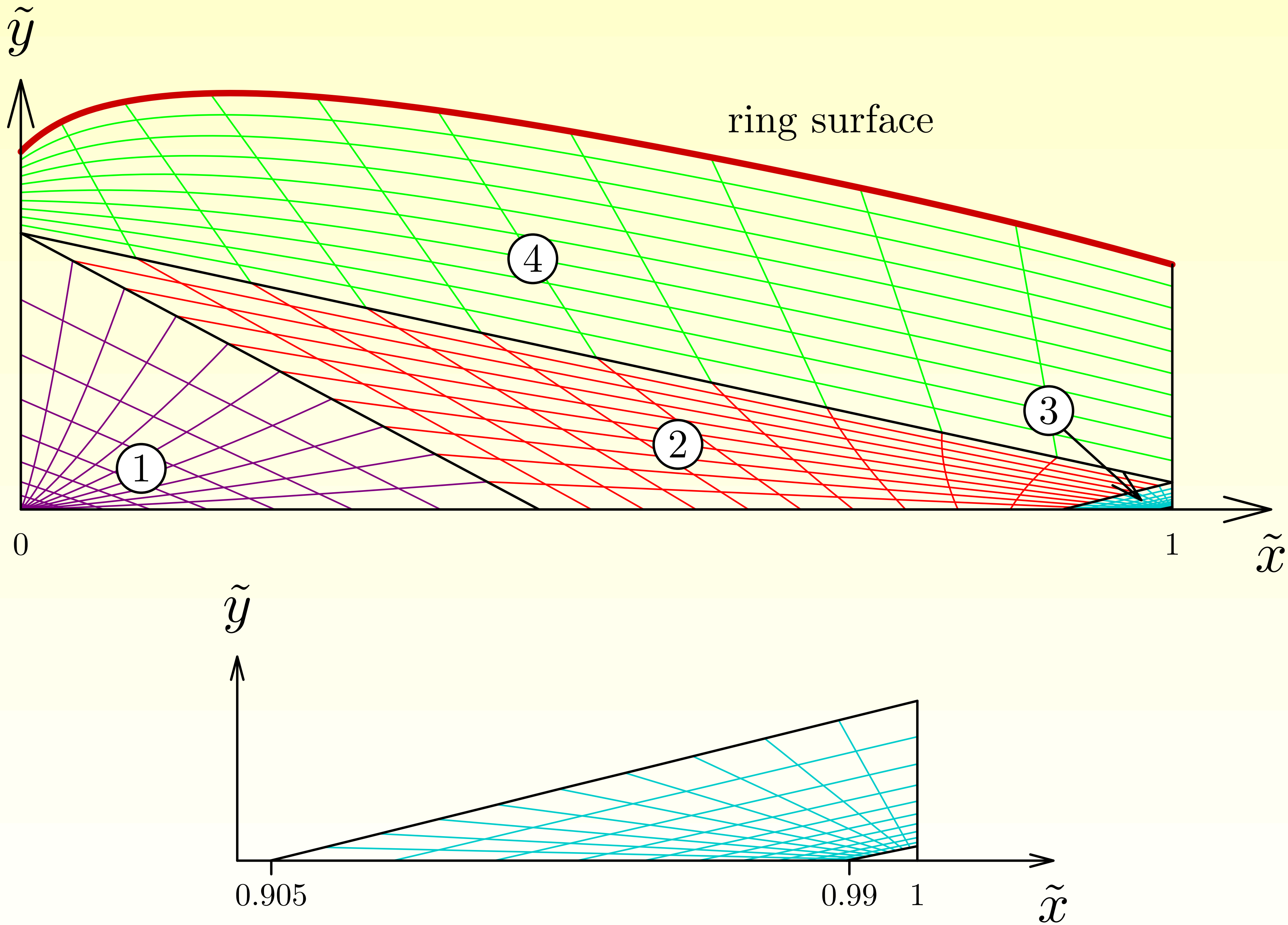
- divide ϱ-ζ space into various domains



- compactify (domain 1) and map each domain onto a coordinate square $(\varrho, \zeta) \longrightarrow (s, t)$ with $s, t \in [0, 1]$
- expand metric potentials and unknown ring surface in terms of Chebyshev polynomials (surface function enters into coordinate transformation)
- prescribe that Einstein's equations be fulfilled at the "grid points" (zeros of Chebyshev polynomials)
- prescribe that normal derivatives of potentials be continuous at domain boundaries
- asymptotic behaviour, regularity, boundary conditions and continuous behaviour of functions at domain boundaries guaranteed by the representation of the potentials chosen
- $\longrightarrow$ non-linear algebraic system of equations for polynomial coefficients solved using Newton-Raphson method

(Incomplete) List of Problems to Overcome

1. seed solution?
   - uniformly rotating test-ring limit does not exist $\longrightarrow$ Schwarzschild (Kerr) Black Hole cannot be used
   - Newtonian theory: potential for line of mass along axis in coordinates in which this line becomes sphere has functional behaviour like ν
   - $b := 1 - (M/2r)^2$ has behaviour of B, ω chosen to be 0
   - Newtonian problem solved numerically by using existing program (Ansorg, Kleinwächter, Meinel 2003) and adding (linear superposition) central mass
   - for a sufficiently small mass M, this generated a successful relativistic solution with $\Omega_\mathrm{c} = 0$
2. different scales?
   - behaviour of metric functions governed by Black Hole (boundary values), but (often) only in the direct vicinity of the event horizon

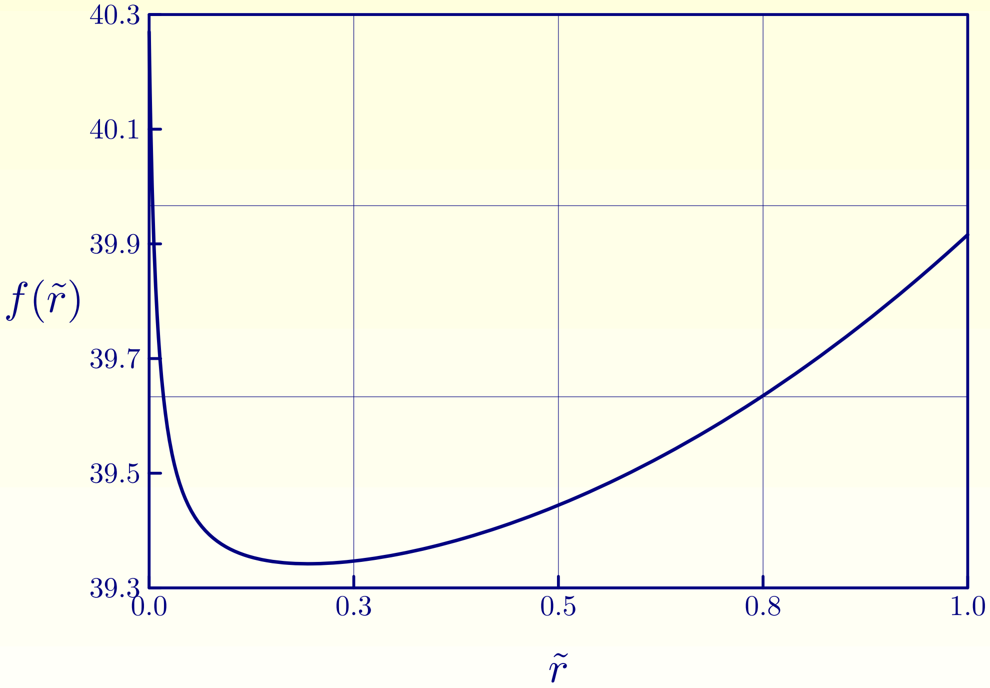


$\tilde{y}$
ring surface
4
3
2
1
0
1
$\tilde{x}$
$\tilde{y}$
0.905
0.99
1
$\tilde{x}$

- qualitative behaviour of typical metric potential can be modelled by linear superposition of point mass surrounded by infinitesimal (but massive) ring

$$f(\tilde{r}) = \frac{\epsilon}{\epsilon + \tilde{r}} + \frac{100}{4 + \tilde{r}} K(\frac{4\sqrt{\tilde{r}}}{4 + \tilde{r}})$$

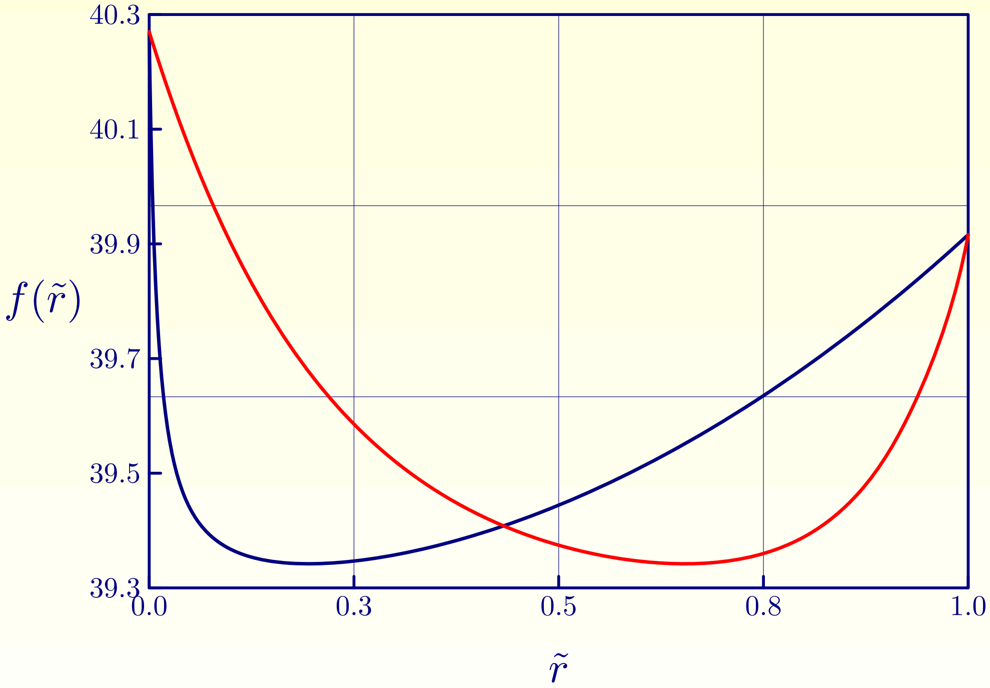
- in linear coordinates:



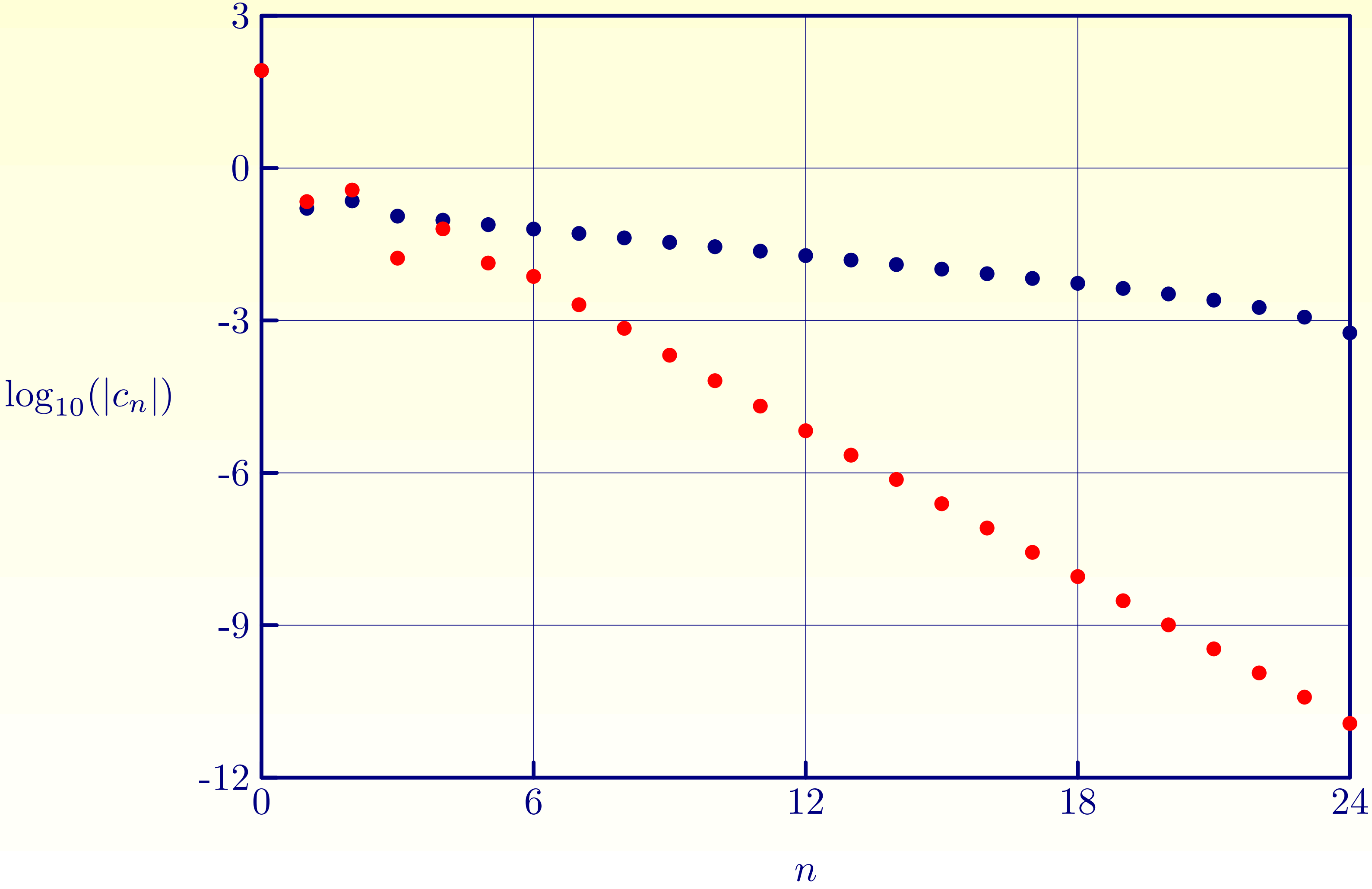
- qualitative behaviour of typical metric potential can be modelled by linear superposition of point mass surrounded by infinitesimal (but massive) ring

$$f(\tilde{r}) = \frac{\epsilon}{\epsilon + \tilde{r}} + \frac{100}{4 + \tilde{r}} K(\frac{4\sqrt{\tilde{r}}}{4 + \tilde{r}})$$

- in linear coordinates and logarithmic coordinates:



- performing a Chebyshev expansion of these functions (as used in the numerical program), one finds for the polynomial coefficients c_n



3 Mass and Angular Momentum

- one of the field equations is

$$\nabla \cdot (\varrho^2 B^3 e^{-4\nu} \nabla \omega) = (\varepsilon + p) f(\nu, \mu, \omega, B)$$

- integrating over $d^3 r$, using the divergence theorem and taking into account the asymptotic behaviour

$$\nu = \frac{-M_{\text{tot}}}{r} + \mathcal{O}(\frac{1}{r^2}) \qquad B = 1 + \mathcal{O}(\frac{1}{r^2})$$

$$\omega = \frac{2J_{\text{tot}}}{r^3} + \mathcal{O}(\frac{1}{r^4}) \qquad \mu = \frac{M_{\text{tot}}}{r} + \mathcal{O}(\frac{1}{r^2})$$

$$\Longrightarrow J_{\text{tot}} = \oint_{\text{BH}} f_1 \, d\Omega + \int_{\text{ring}} f_2 \, d^3 r =: J_{\text{c}} + J_{\text{r}}$$

- a similar procedure yields M_{c} and M_{r} with

$$M_{\text{tot}} = M_{\text{c}} + M_{\text{r}}$$

- it turns out that

$$\kappa := e^{-\mu} \, \frac{\partial}{\partial r} e^{\nu} \bigg|_{r=r_{\mathsf{c}}}$$

is a constant on the horizon

- even in the presence of the ring, the Black Hole parameters obey

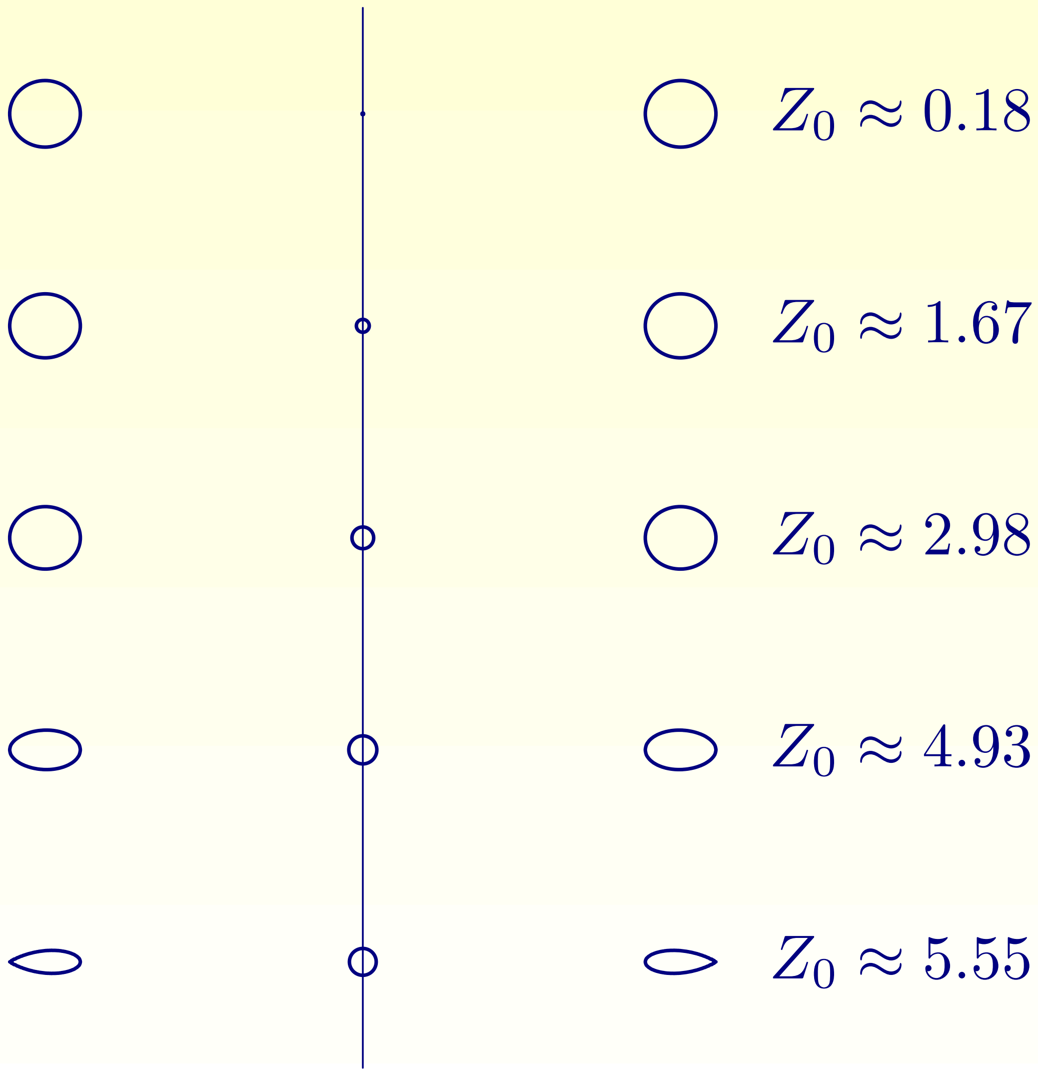
$$M_{\mathsf{c}} = \frac{1}{4\pi}\kappa A_{\mathsf{c}} + 2\Omega_{\mathsf{c}} J_{\mathsf{c}}$$

4 The Solution Space in Bite Sized Pieces

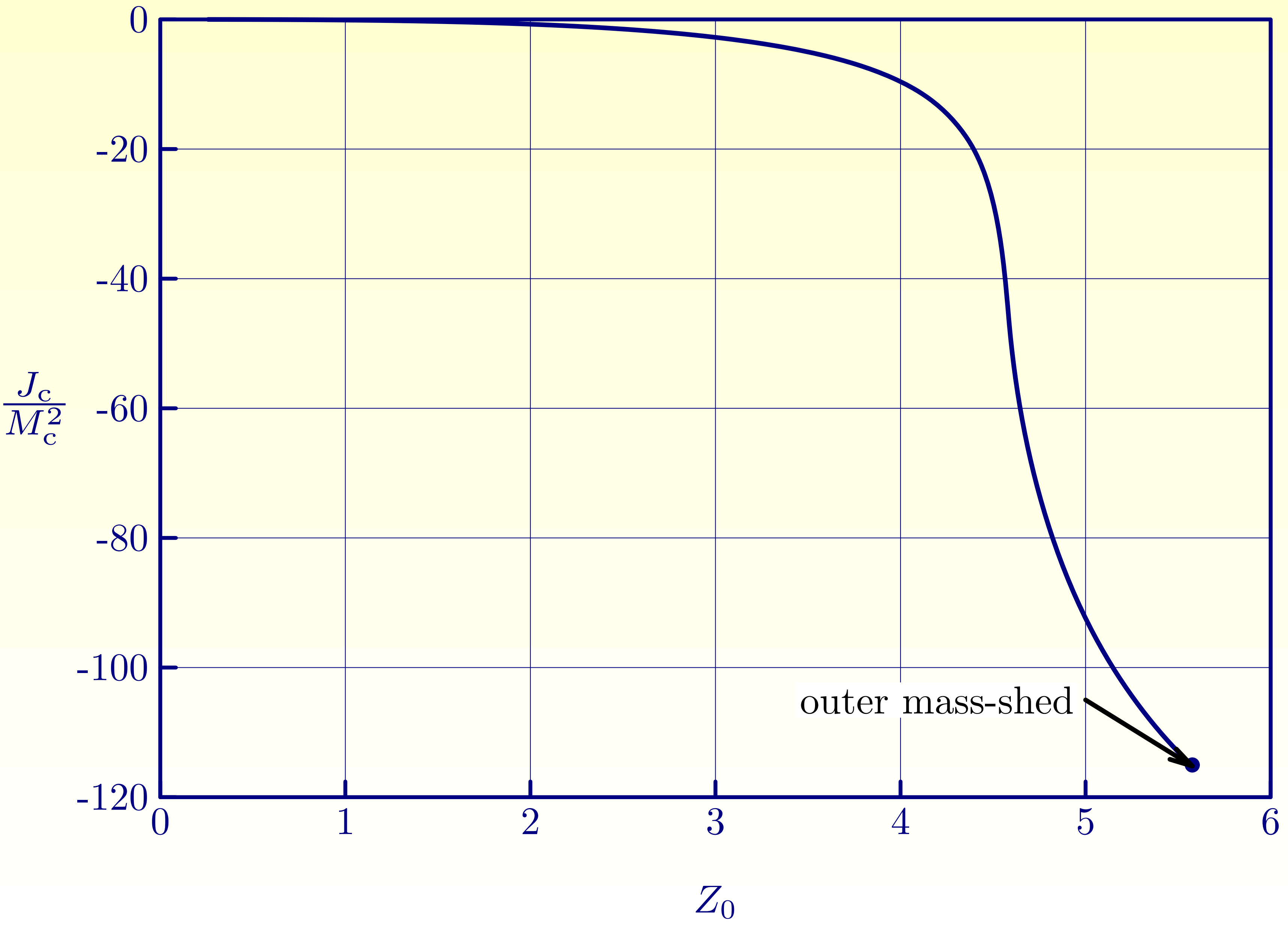
- after fixing equation of state, solution space depends on 4 parameters (e.g. M and J for both ring and Black Hole)
- in this talk we consider rings of constant density
- first approach to understanding solution space: foliate it and study one-parameter sequences (paying special attention to physical boundary sequences)
- in this talk: only configurations with $\Omega_\mathrm{c} = 0$ considered
- special attention will be paid to the mass-shedding limit (inner and outer)
- the inequality $|J_\mathrm{c}/M_\mathrm{c}^2| \leq 1$, which holds for the Kerr Black Hole, will serve as leitmotif and one measure of influence of matter on Black Hole

5 Numerical Results

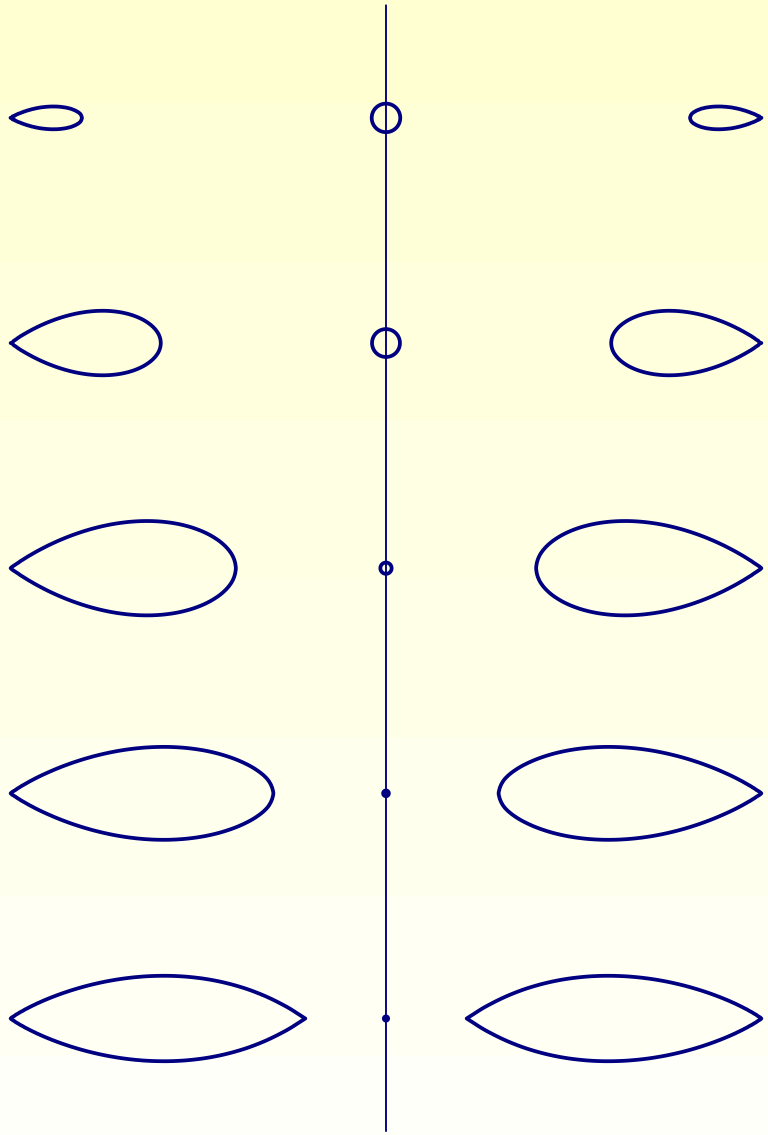
$M_c/M_r = 0.05 \qquad \varrho_i/\varrho_o = 0.8$:



$M_c/M_r = 0.05 \qquad \varrho_i/\varrho_o = 0.8$:



$M_c / M_r = 0.05$ outer mass-shed:



$\varrho_i / \varrho_o = 0.8$

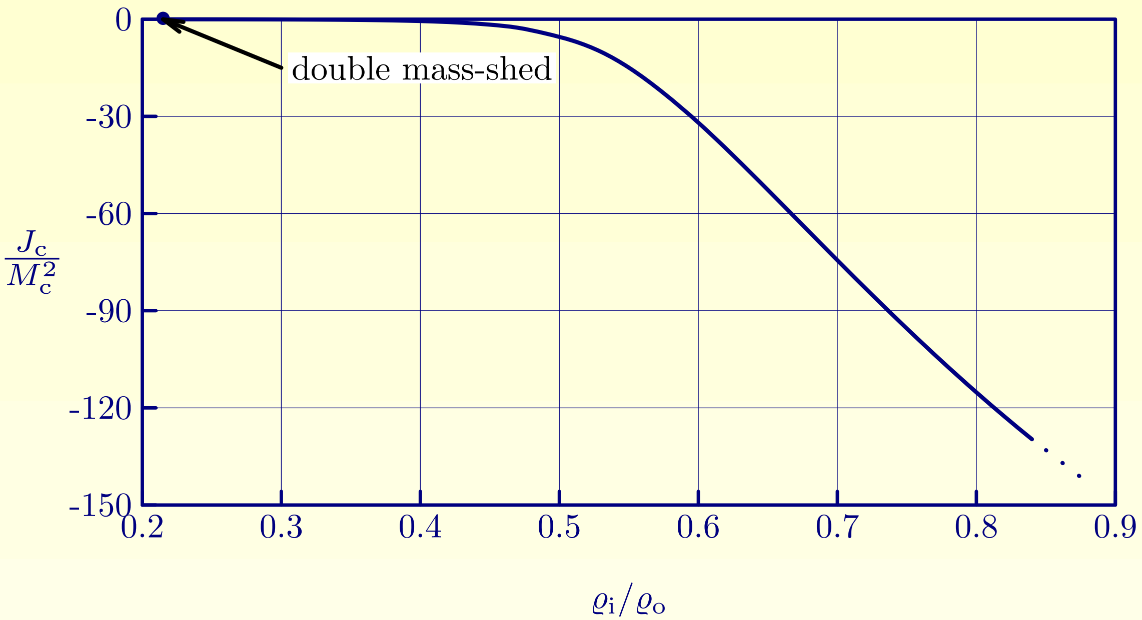
$\varrho_i / \varrho_o = 0.6$

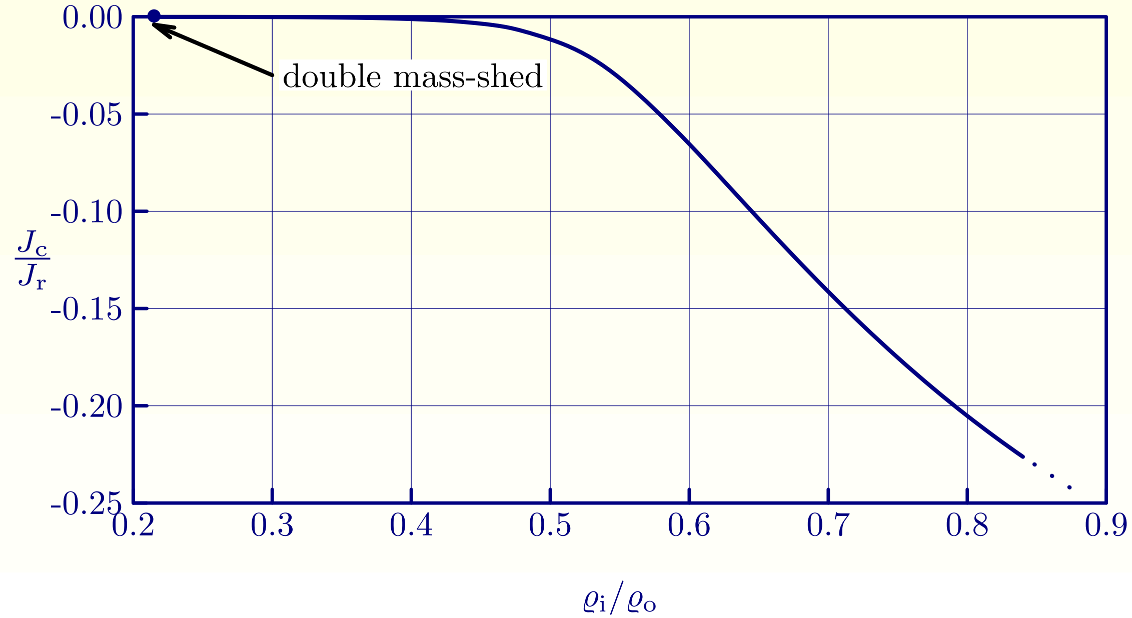
$\varrho_i / \varrho_o = 0.4$

$\varrho_i / \varrho_o = 0.3$

double mass-shed

$M_c/M_r = 0.05$ outer mass-shed:





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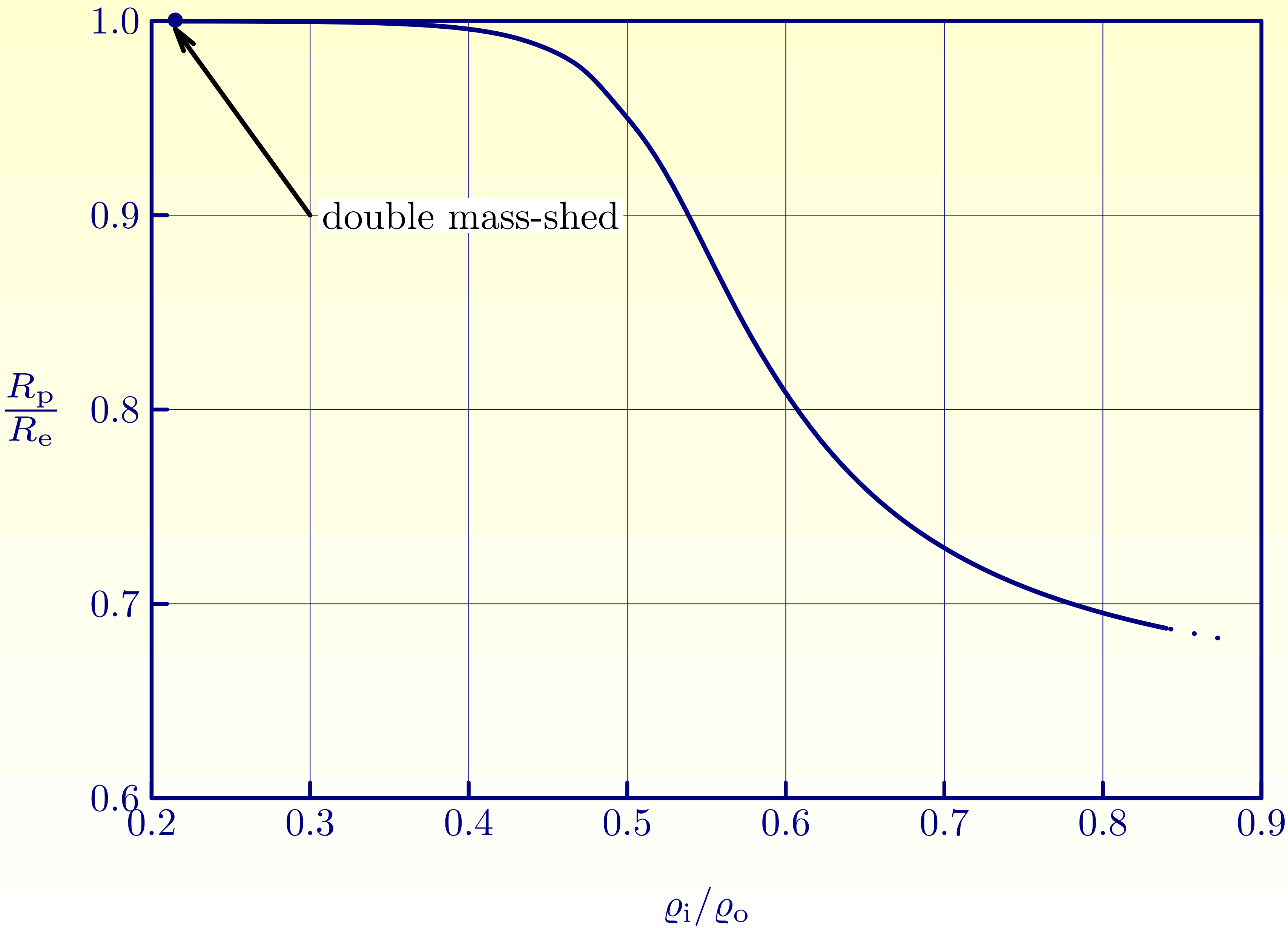
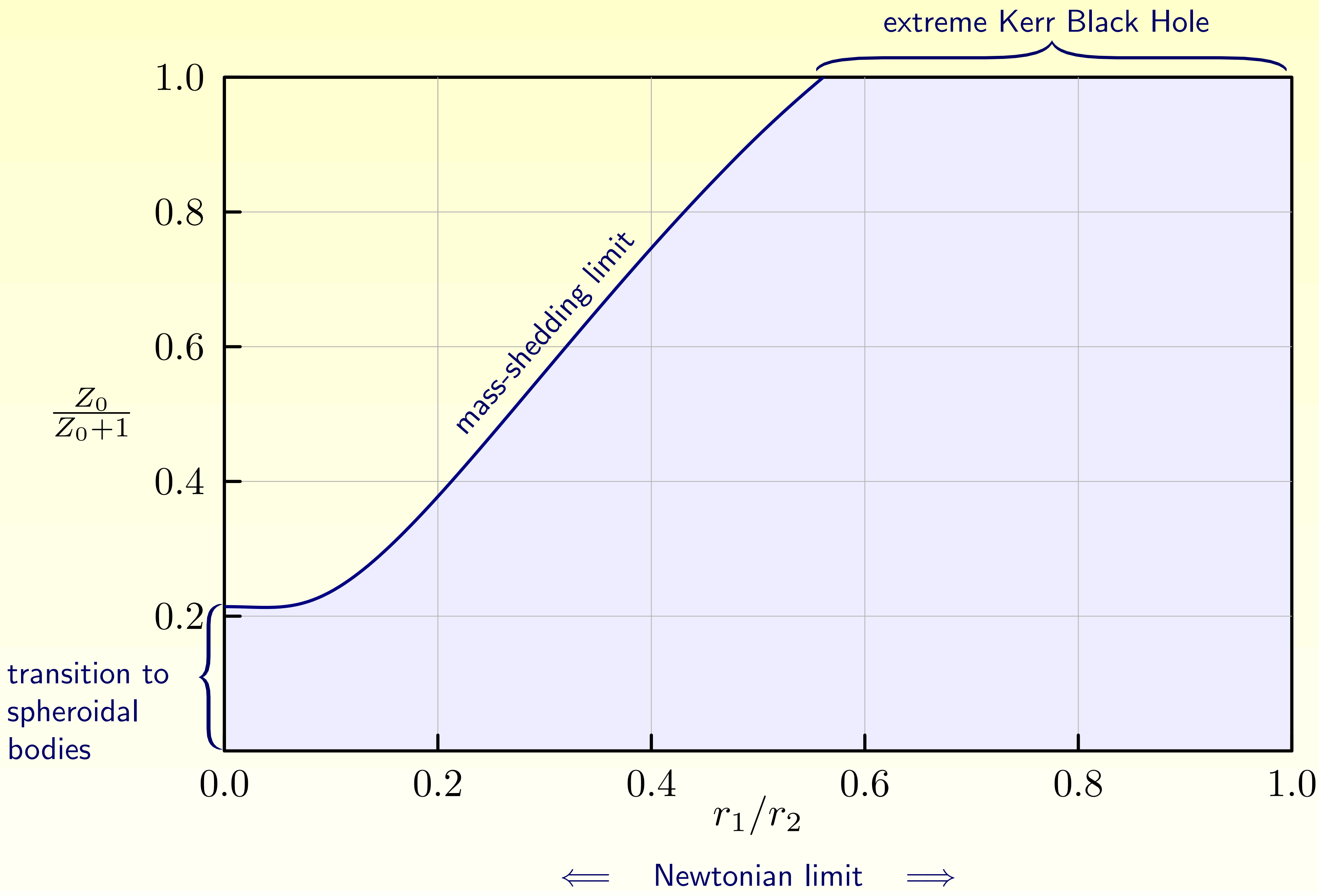
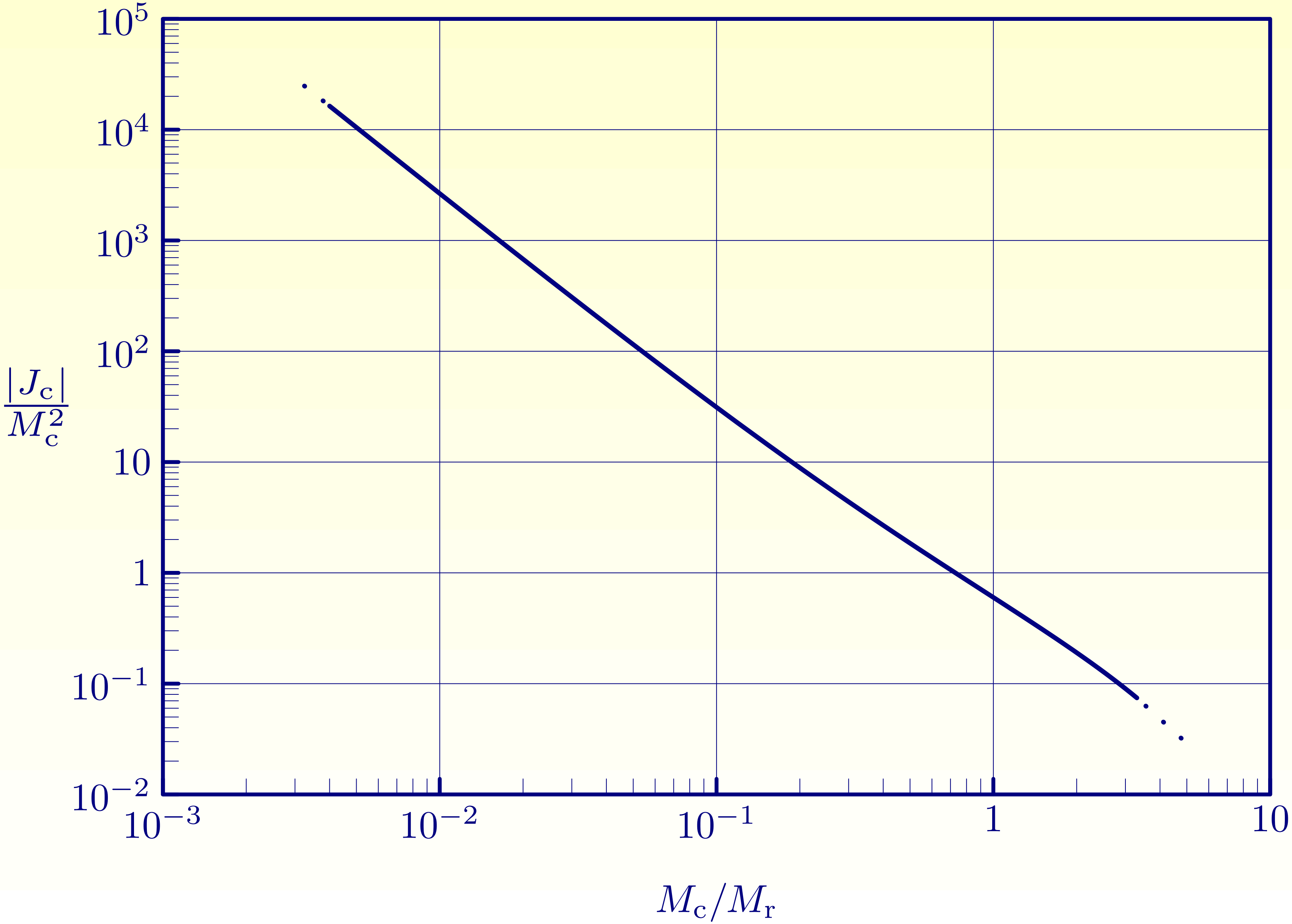




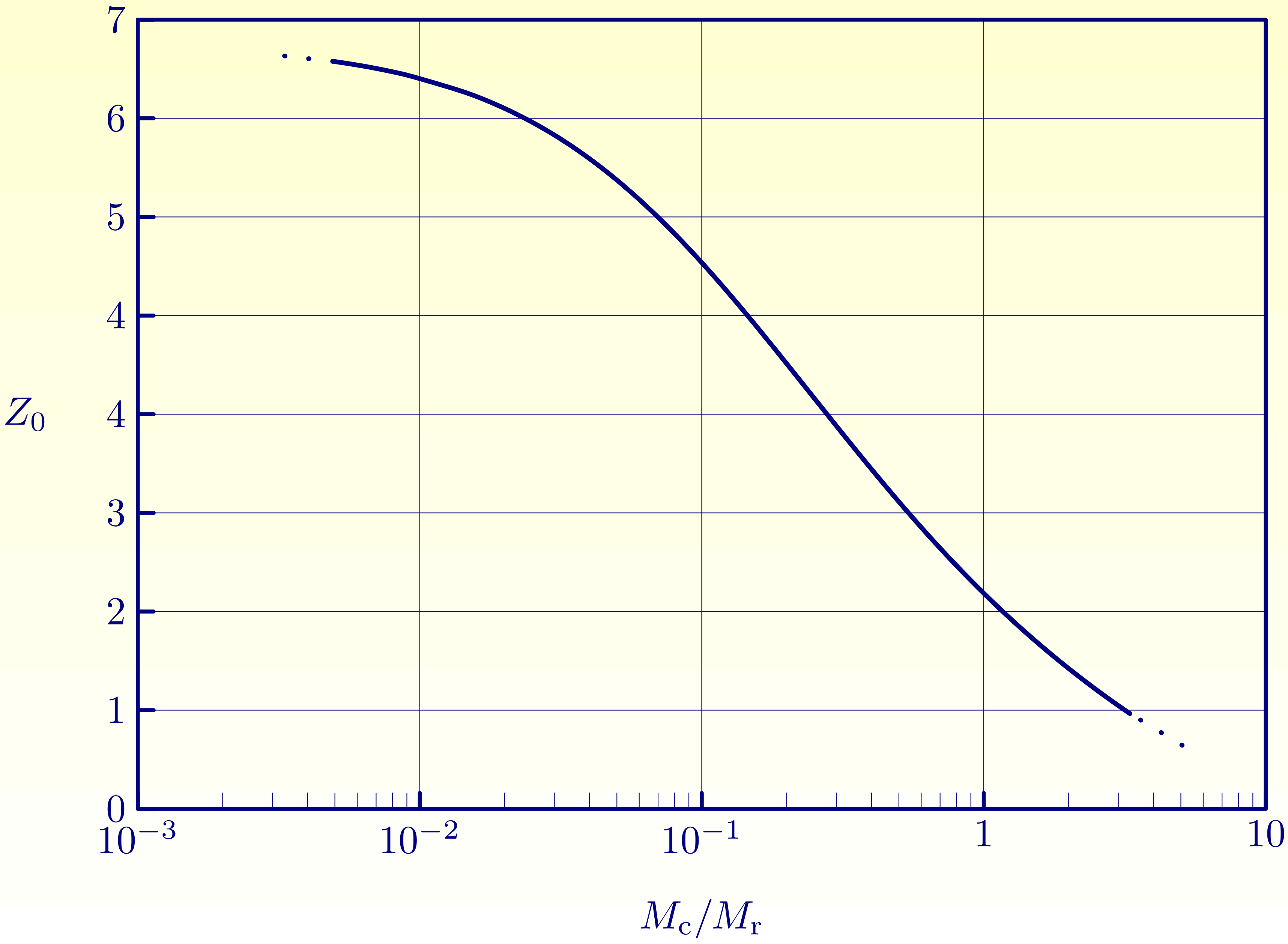
Ansorg, Kleinwächter and Meinel, *Astophys. J.* **582** L87 (2003)

$\varrho_{\mathrm{i}}/\varrho_{\mathrm{o}} = 0.8$ outer mass-shed:

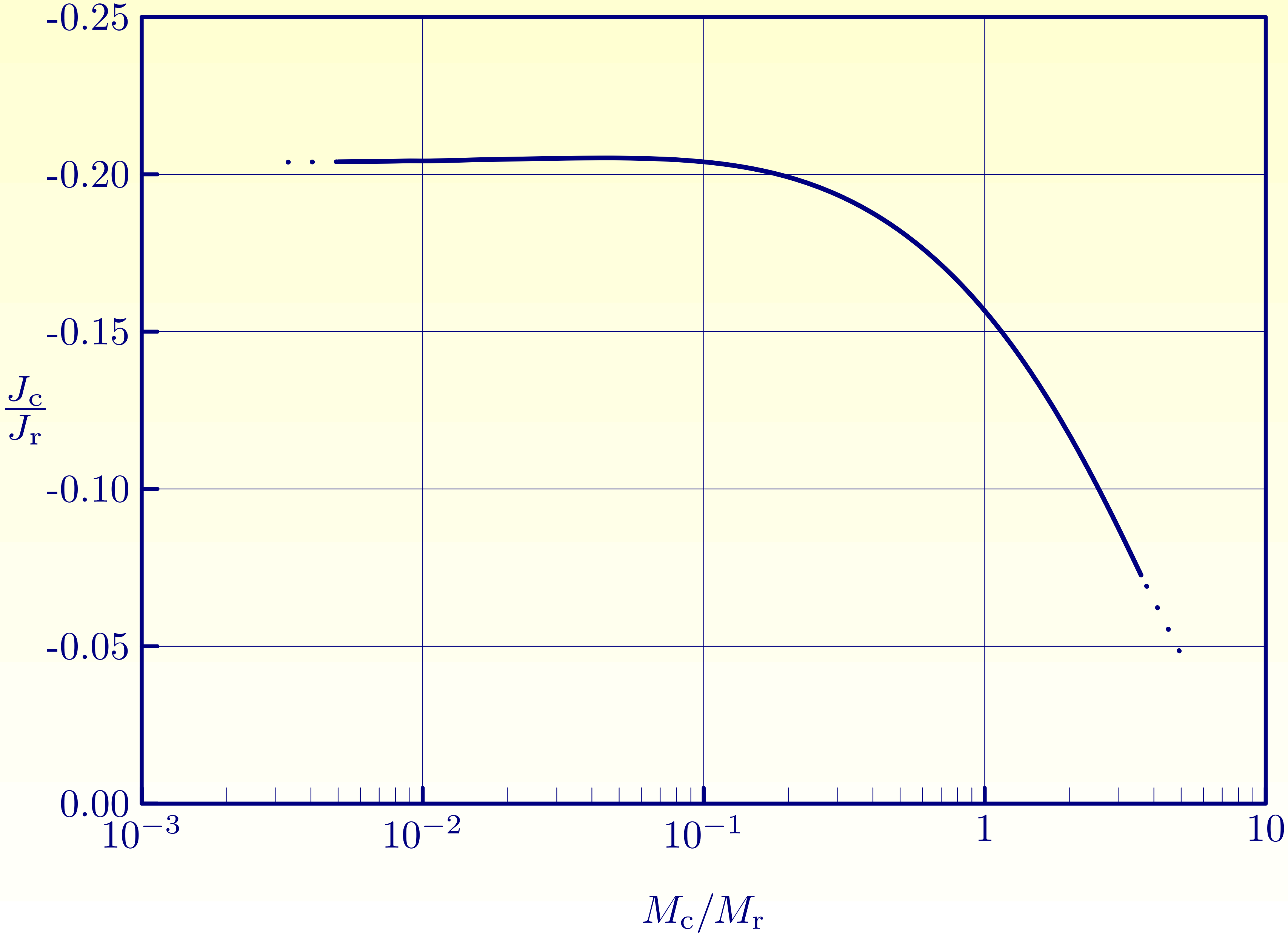


	BH–Ring	Kerr with same $\bar{J}_\mathrm{c}$, κM_c	Kerr with same $\bar{J}_\mathrm{c}$, $\bar{A}_\mathrm{c}$
$\lvert J_\mathrm{c}\rvert/M_\mathrm{c}^2$	3.281340×10^4	$1 - 4.5 \times 10^{-10}$	0.9998
$\bar{J}_\mathrm{c}$	-5.415253×10^{-2}	$\mathbf{-5.42 \times 10^{-2}}$	$\mathbf{-5.42 \times 10^{-2}}$
$\bar{M}_\mathrm{c}$	0.001284647	0.233	0.233
$\bar{\Omega}_\mathrm{c}$	0	-2.15	-2.11
$R_\mathrm{p}/R_\mathrm{e}$	0.6160728	0.608	0.616
$\bar{R}_\mathrm{e}$	0.4654637	0.465	0.465
$\bar{R}_\mathrm{i}$	1.251534	–	–
κM_c	1.494494×10^{-5}	$\mathbf{1.49 \times 10^{-5}}$	0.00951
$\bar{A}_\mathrm{c}$	1.387660	1.36	$\mathbf{1.39}$
$\left\lvert \frac{M_\mathrm{i} - M_\mathrm{a}}{M_\mathrm{a}} \right\rvert$	2.6×10^{-9}	–	–
$\left\lvert \frac{J_\mathrm{i} - J_\mathrm{a}}{J_\mathrm{a}} \right\rvert$	3.0×10^{-9}	–	–

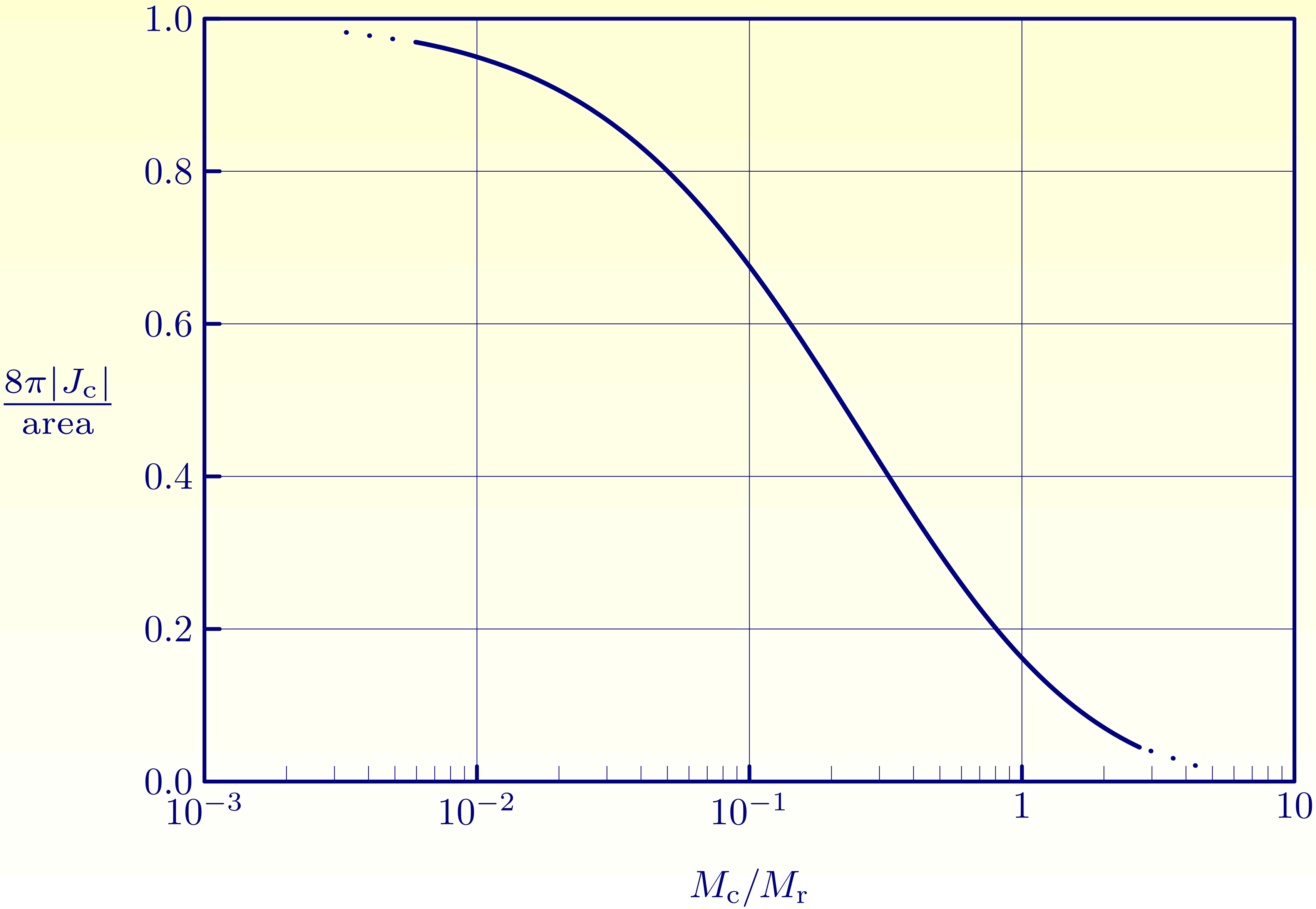
$\varrho_\mathrm{i}/\varrho_\mathrm{o} = 0.8$ outer mass-shed:



$\varrho_i / \varrho_o = 0.8$ outer mass-shed:



$\varrho_{\mathrm{i}}/\varrho_{\mathrm{o}} = 0.8$ outer mass-shed:



6 Future Plans

- study the solution space in detail
  - transition to an inifinitely thin disc (cf. inverse scattering methods, G. Neugebauer & R. Meinel)?
  - transition to a (global) Black Hole??
  - . . .
- consider interaction between matter and Black Hole in analogy to that between Black Holes and coloured solitons (A. Ashtekar, B. Krishnan)
- use configuration as initial data in time-evolution program (L. Baiotti, L. Rezzolla)
- consider various equations of state
- extend program to allow for differential rotation??