Charged black holes have no hair

PTC, P.Tod,

NI preprint NI05067-GMR

Theorem I [PTC, P.Tod, NI preprint NI05067-GMR]

Globally hyperbolic,
electro-vacuum,
static,
regular black hole
=
Reissner-Nordström or Majumdar-Papapetrou
in the exterior region

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regular: contains a spacelike surface which is the union of a **compact** set and a **finite** number of asymptotically flat ends, with boundary lying on the event horizons

The standard **Majumdar–Papapetrou** metrics:

$$ds^2 = \phi^{-2}dt^2 + \phi^2(dx^2 + dy^2 + dz^2)$$

$$\phi = 1 + \sum_{i=1}^{N} \frac{m_i}{\|\vec{x} - \vec{x}_i\|}, \qquad m_i > 0$$

(N=1 — degenerate Reissner–Nordström in unusual coordinates)
For every finite N this is a space–time with N black holes, each having a *degenerate* event horizon.

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$\mathscr{H}$ degenerate or extreme if $\kappa = 0$

Previous versions of Theorem I

Israel, Robinson, Masood-ul-Alam, Ruback, Simon:

Assume moreover that **all horizons non-degenerate**

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In this work we remove the **spurious red hypotheses**

“Near horizon geometry has no hair”

(compare Reall hep-th/0211290)

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Gauss coordinates near a degenerate horizon $\mathscr{H} = \{r = 0\}$

(see Isenberg, Moncrief, CMP 1983) Killing vector $X = \partial_u$

$$g = A \underbrace{r^2}_{\text{degenerate}} du^2 - 2du\,dr - 2rh_a dx^a du - h_{ab} dx^a dx^b$$

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Theorem II [PTC, P.Tod]
degenerate, static, elvac, S^2 crosssection implies:

$$A = \underbrace{\mathring{A}}_{Constant>0} + O(r)\ , \qquad h_a = O(r)\ ,$$

$$\mathring{A} h_{ab} = \underbrace{\mathring{h}_{ab}}_{\text{unit round metric on } S^2} + O(r)\ ,$$

$$\partial_r \underbrace{\varphi}_{\text{electric potential}} = \pm\sqrt{\mathring{A}} + O(r)\ .$$

Idea of the proof:

$$g \underbrace{\approx}_{\text{leading order}} \mathring{A} r^2 du^2 - 2du\,dr - 2r\mathring{h}_a dx^a du - \mathring{h}_{ab} dx^a dx^b$$

staticity: $X \wedge dX = 0 \Longrightarrow \mathring{h}_a = \partial_a \lambda$

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Einstein-Maxwell equations:

$$(*)\ \mathring{R}_{ab} = \tfrac{1}{2}\mathring{\nabla}_a \lambda \mathring{\nabla}_b \lambda - \mathring{\nabla}_a \mathring{\nabla}_b \lambda + \underbrace{C}_{\text{const.}} e^{2\lambda} \mathring{h}_{ab}$$

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Remark: As emphasised by Reall in his September lecture at the NI, it would be of interest to understand the non-static equivalent of $(*)$

compare Lewandowski Pawłowski gr-qc/0208032