Quantum random walks via generating function techniques (Newton Institute 06/0629)

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Joint work with graduate students Andrew Bressler and Wil Brady (one dimension) and with Yuliy Baryshnikov (two dimesions).

Work very much in progress.

Hadamard coin-flip matrix:

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

GF matrix $= (I - yM)^{-1}$ where

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} x & x^{-1} \\ x & -x^{-1} \end{pmatrix}.$$

Entries of the matirix are given by

$$\frac{P(x,y)}{1 - \dfrac{x - x^{-1}}{\sqrt{2}}y - y^2}.$$

where $P(x,y)$ are polynomials (one for each of the four entries).

$$M := \begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & 1 \end{pmatrix}.$$

$\mathrm{GF} = (I - zM')^{-1}$ where

$$M' := M \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & x^{-1} & 0 \\ 0 & 0 & 0 & y^{-1} \end{pmatrix}.$$

Again, matrix entries are rational functions:

$$\frac{P(x,y)}{(1-z^2)(1 - \frac{x+x^{-1}+y+y^{-1}}{4}z + z^2)}.$$

Example of univariate transfer theorem (see Flajolet and Odlyzko 1990): If

$$f(z) = (1-z)^{-\alpha} \log\left(\frac{1}{1-z}\right)^{\beta}$$

then

$$a_n \sim n^{\alpha-1}(\log n)^{\beta}\,.$$

This is proved by a contour integral in the complex plane, with the contour looking like this:

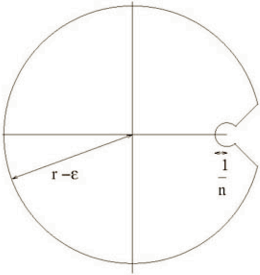

Figure 1: the contour γ

For a survey of what is known about coefficient asymptotics for multivariate generating functions, see:

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math.CO/0512548 Twenty combinatorial examples of asymptotics
derived from multivariate generating functions.
Robin Pemantle, Mark C. Wilson. 89 pages.
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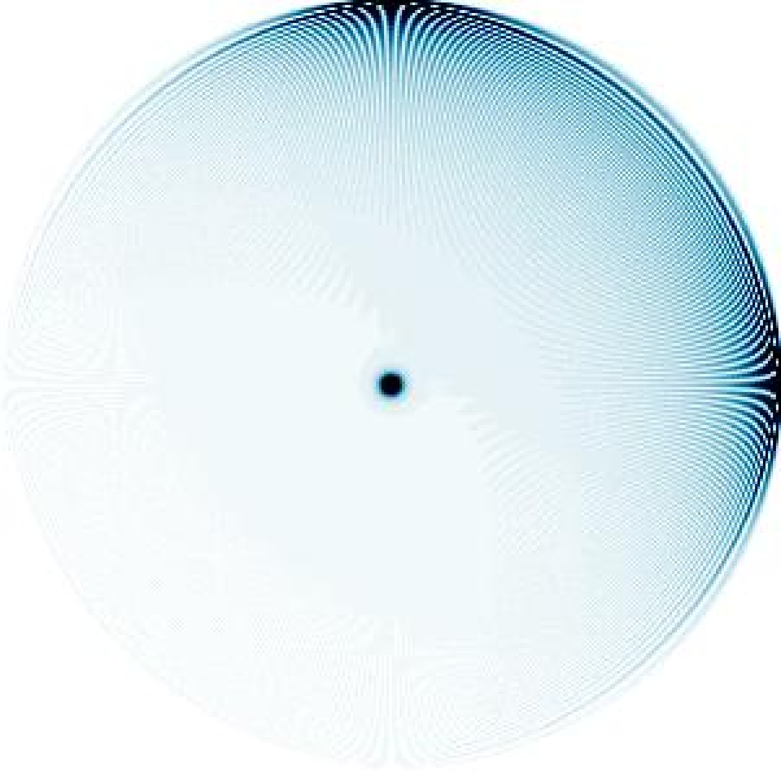
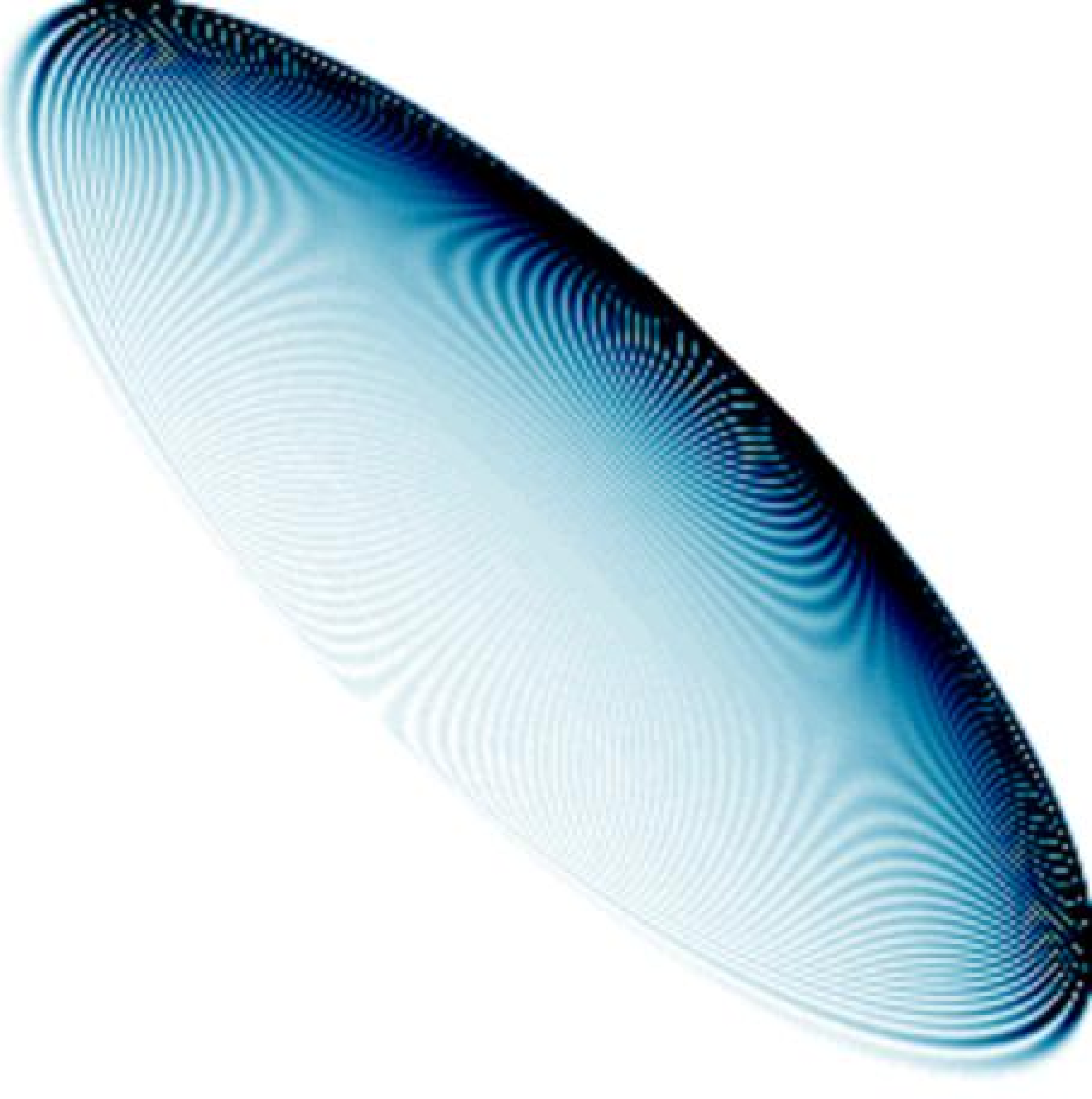
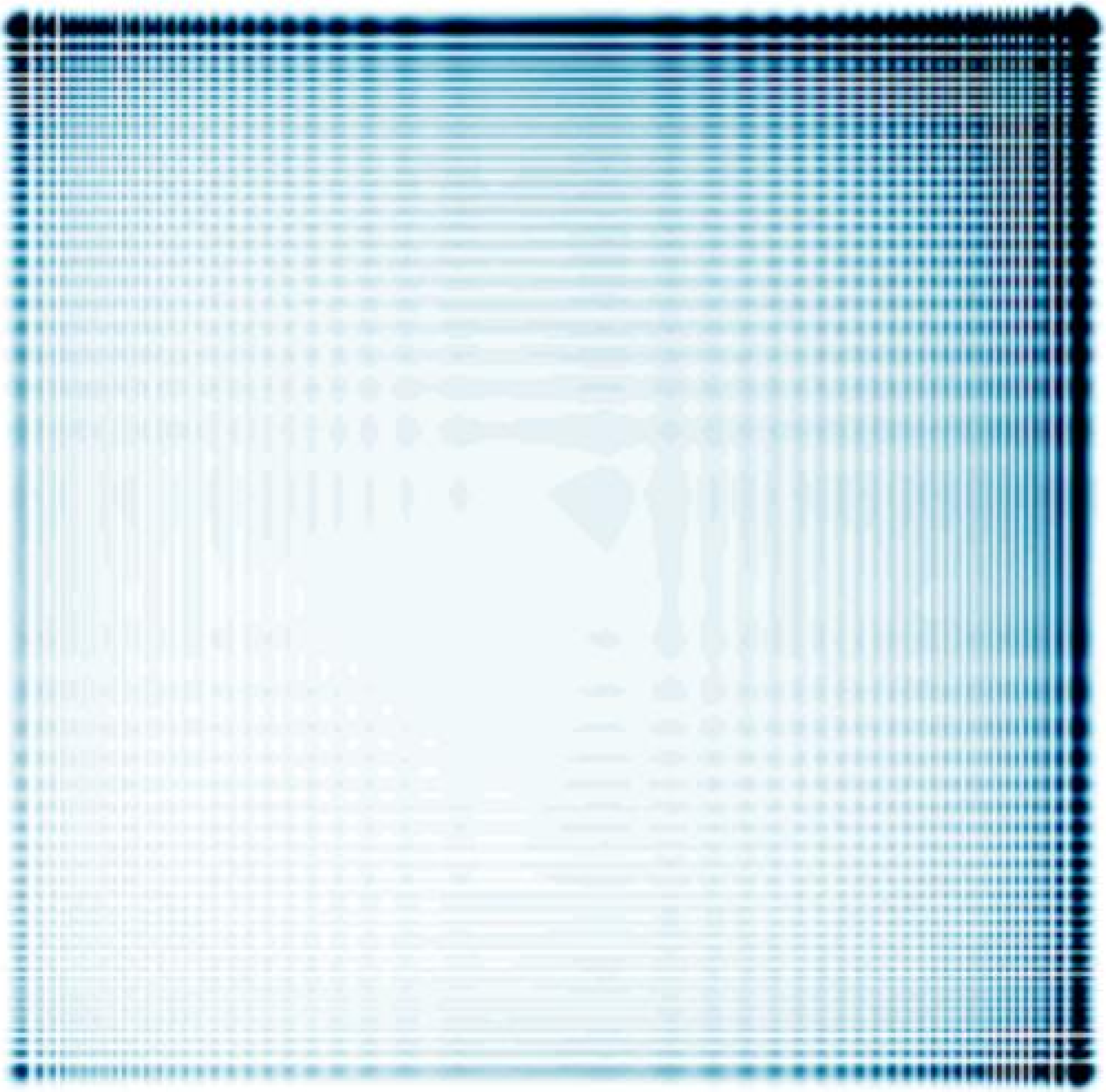
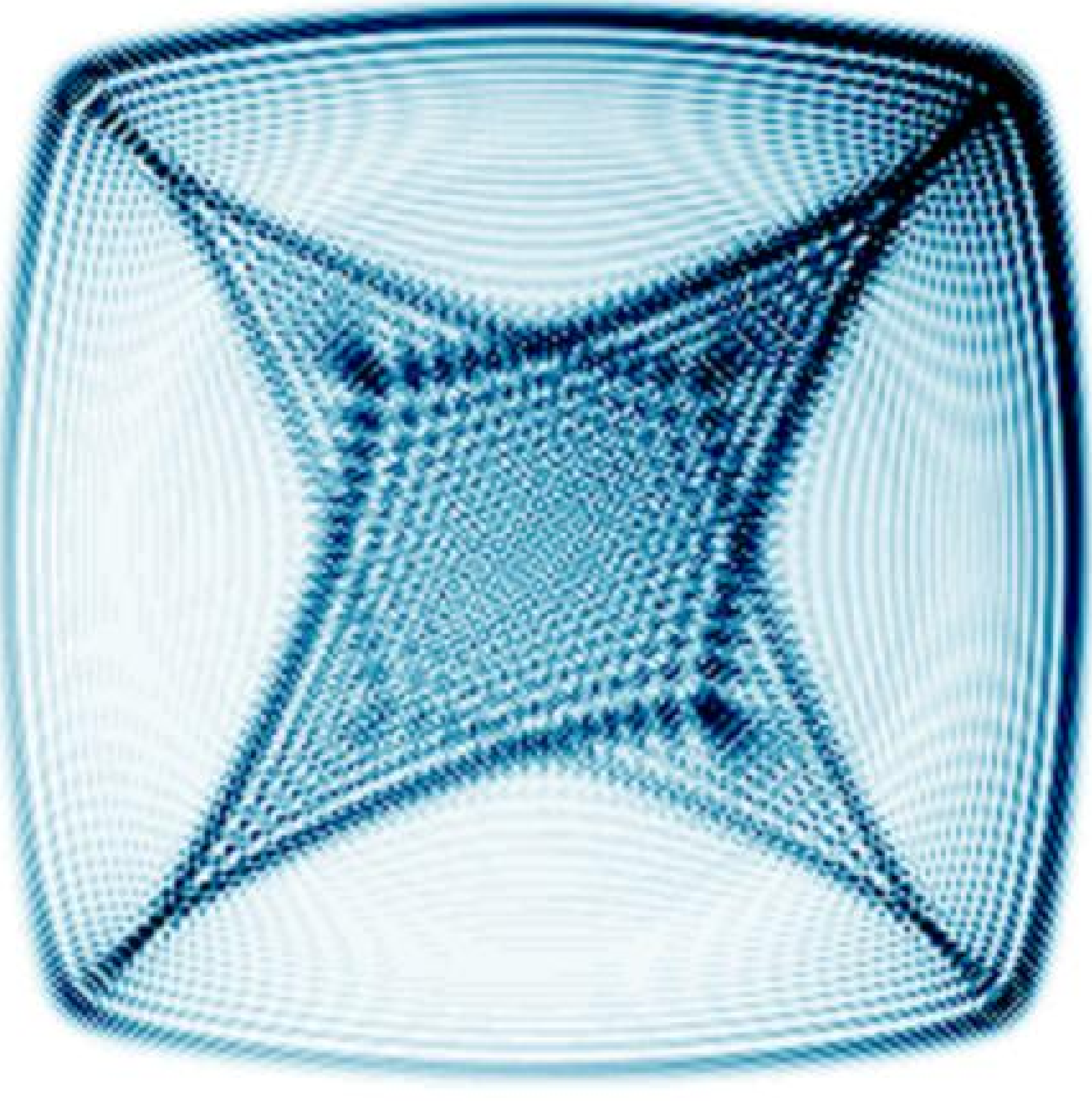
For the QRW generating function, there are two points: $(x(\lambda), y(\lambda))$ and the conjugate of this. Both x and y are unit complex numbers, call them $e^{i\alpha}$ and $e^{i\beta}$. Summing the quantity $C(r+s)^{-1/2}x^{-r}y^{-s}$ over the conjugate pair (the constant C is also conjugated) yields

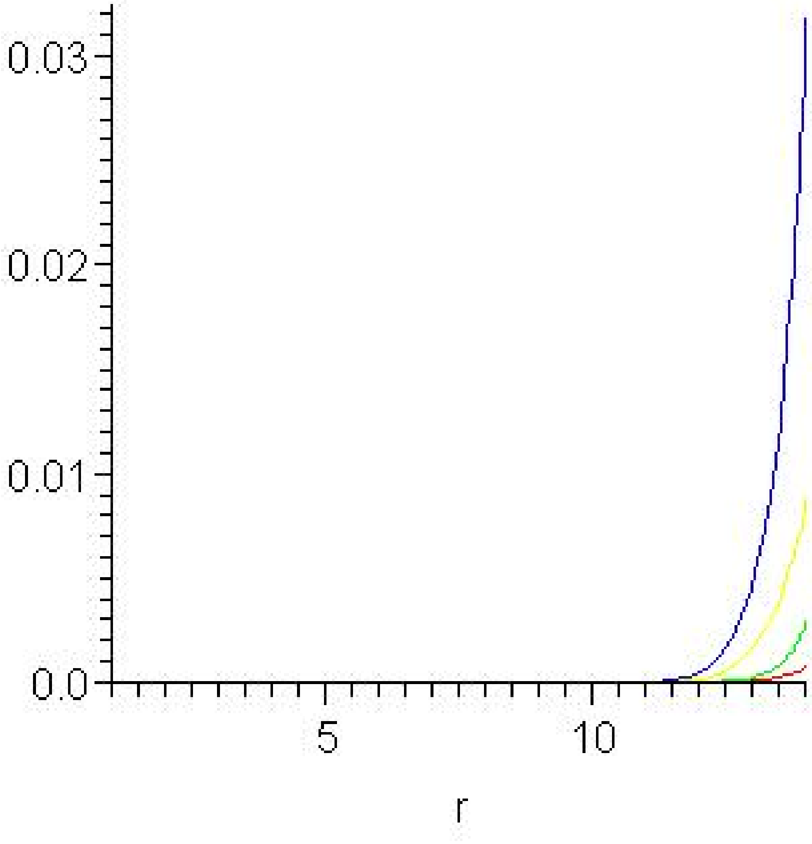
$$|a_{r,s}|^2 \sim \cos^2(r\alpha + s\beta)C(\lambda)\,.$$











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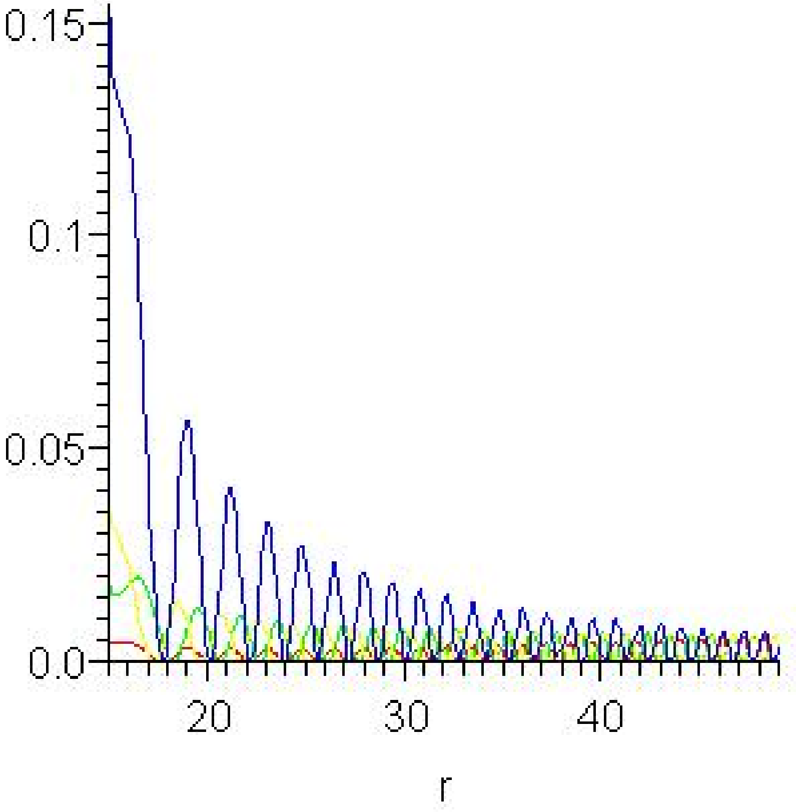
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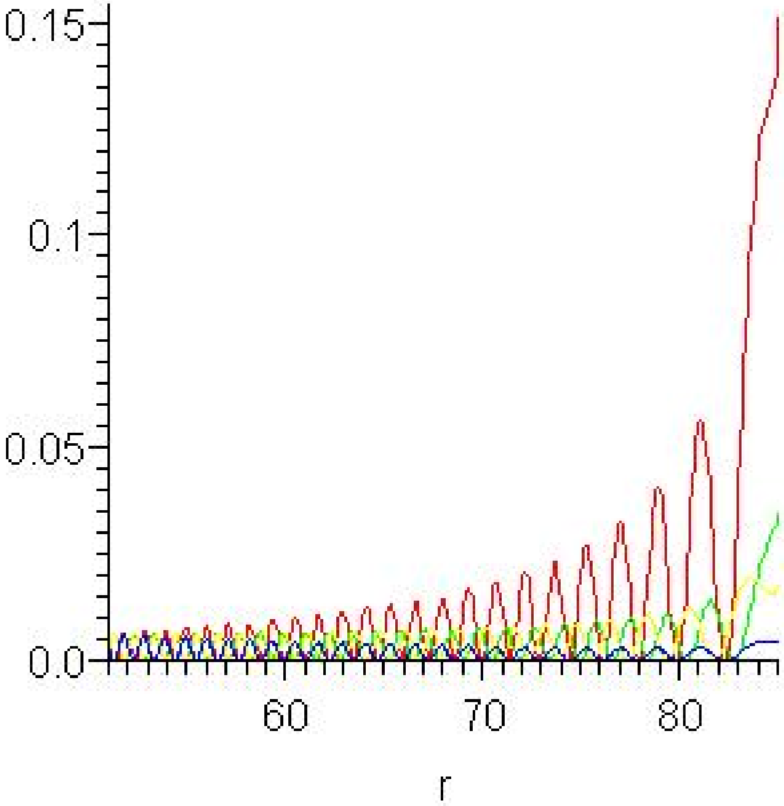
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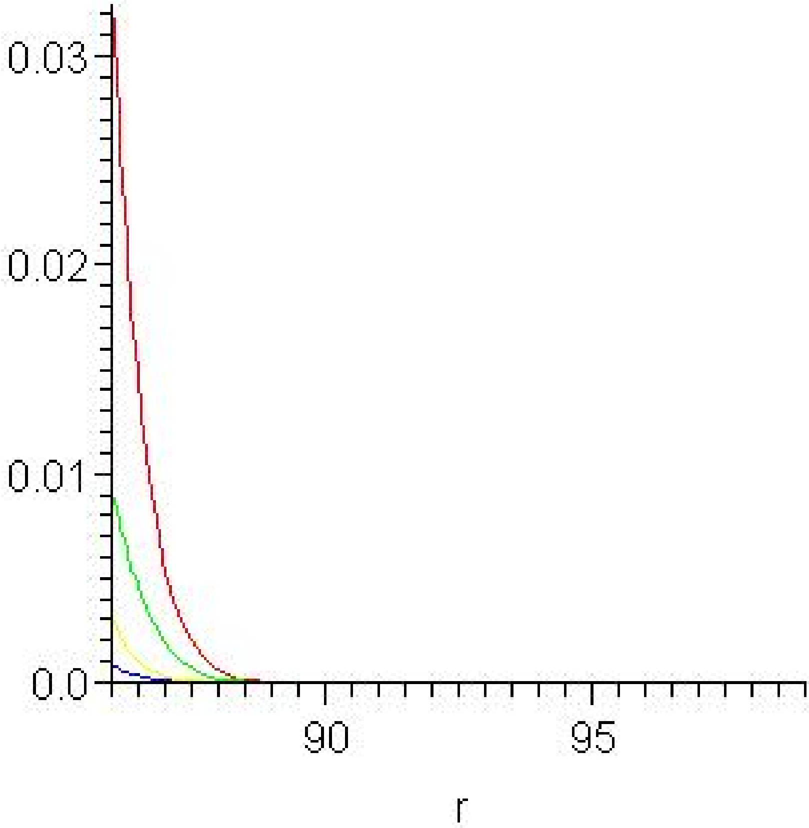
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