On the Role of Global Warming on the Frequency of Record-Temperature Events

collaborator: Mark Petersen (LANL)

Empirics: Philadelphia temperature *(why Philadelphia?)*

annual pattern & long-term trends

daily temperature distribution

Tutorial: Evolution of record temperature events

magnitude of successive records

time between successive records

Comparison with Philadelphia data

role of global warming

Summary & Outlook

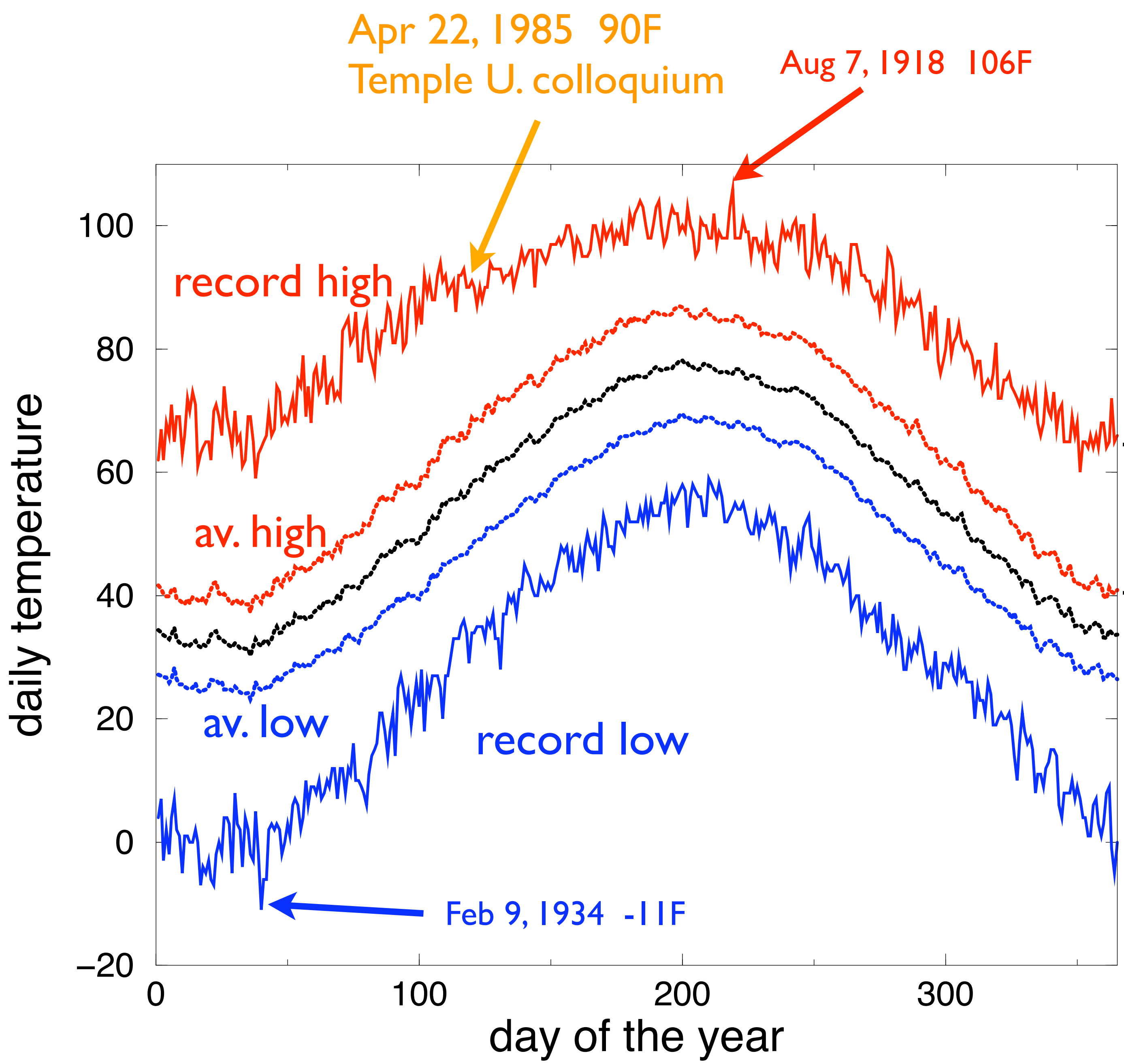
role of inter-day temperature correlations

seasonal effects

asymmetry between high & low temperatures

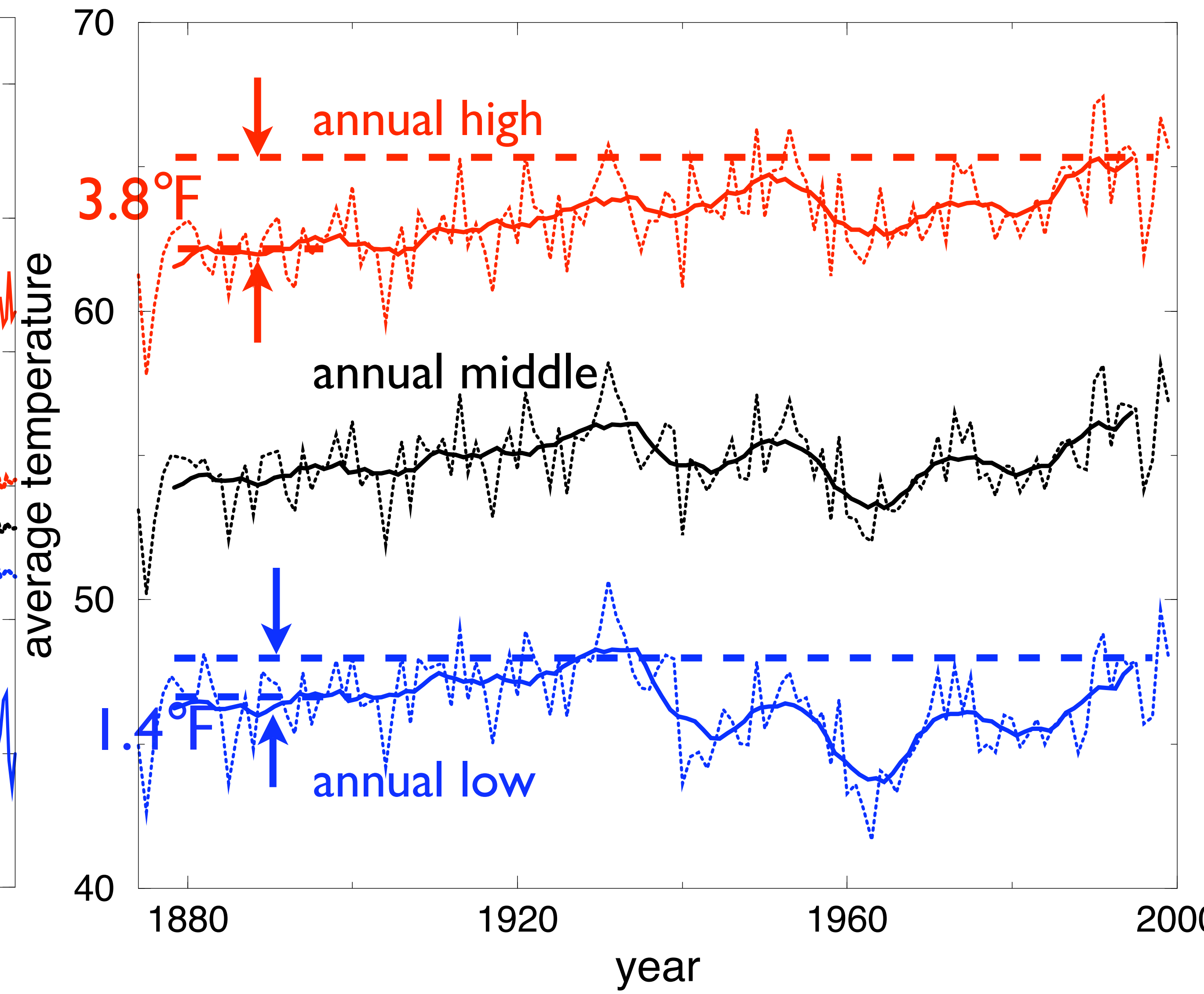
Philadelphia Climate

annual temperature pattern

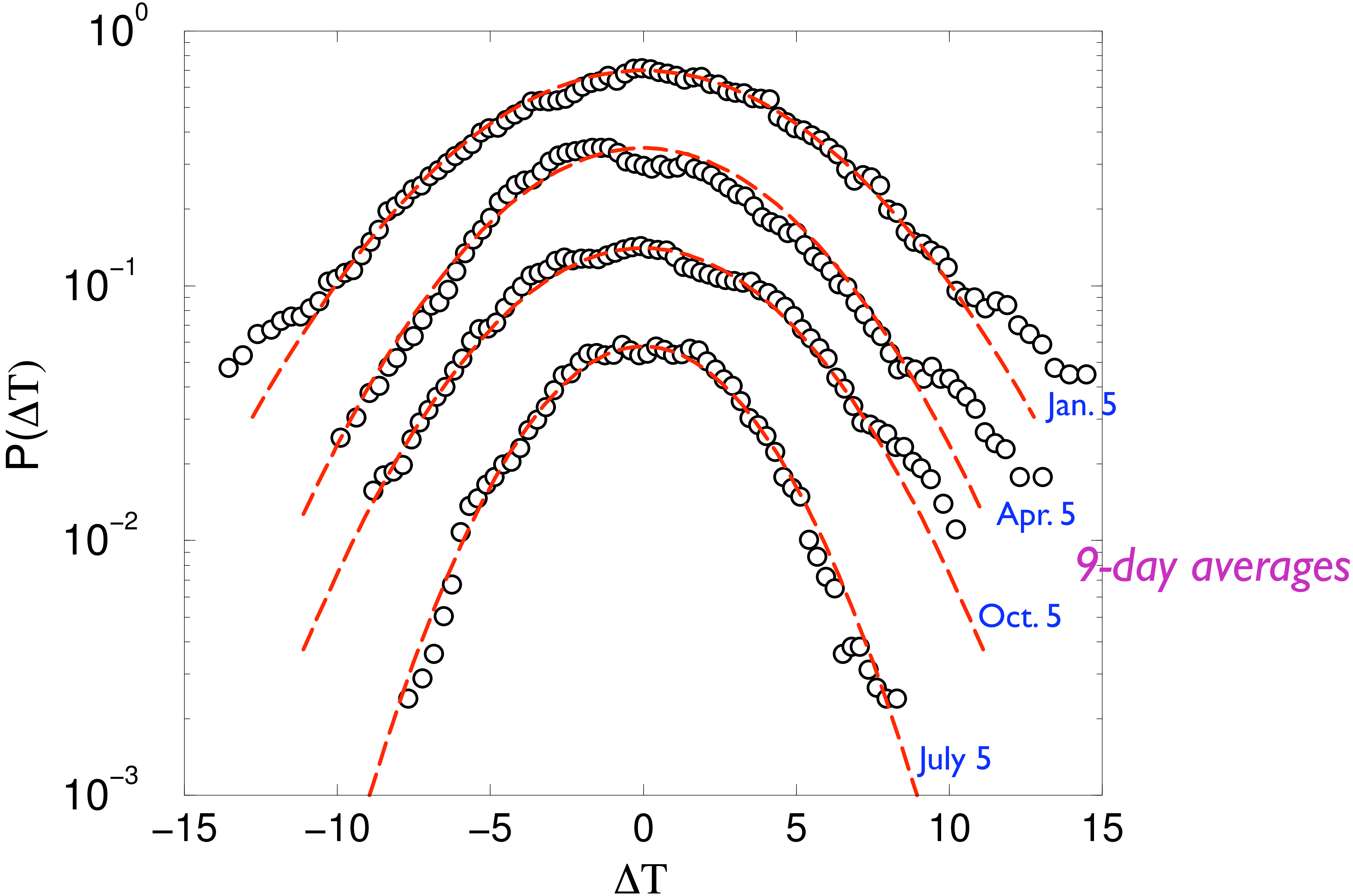


long-term temperature trends

IPCC 2001: global warming 0.6°F over past century



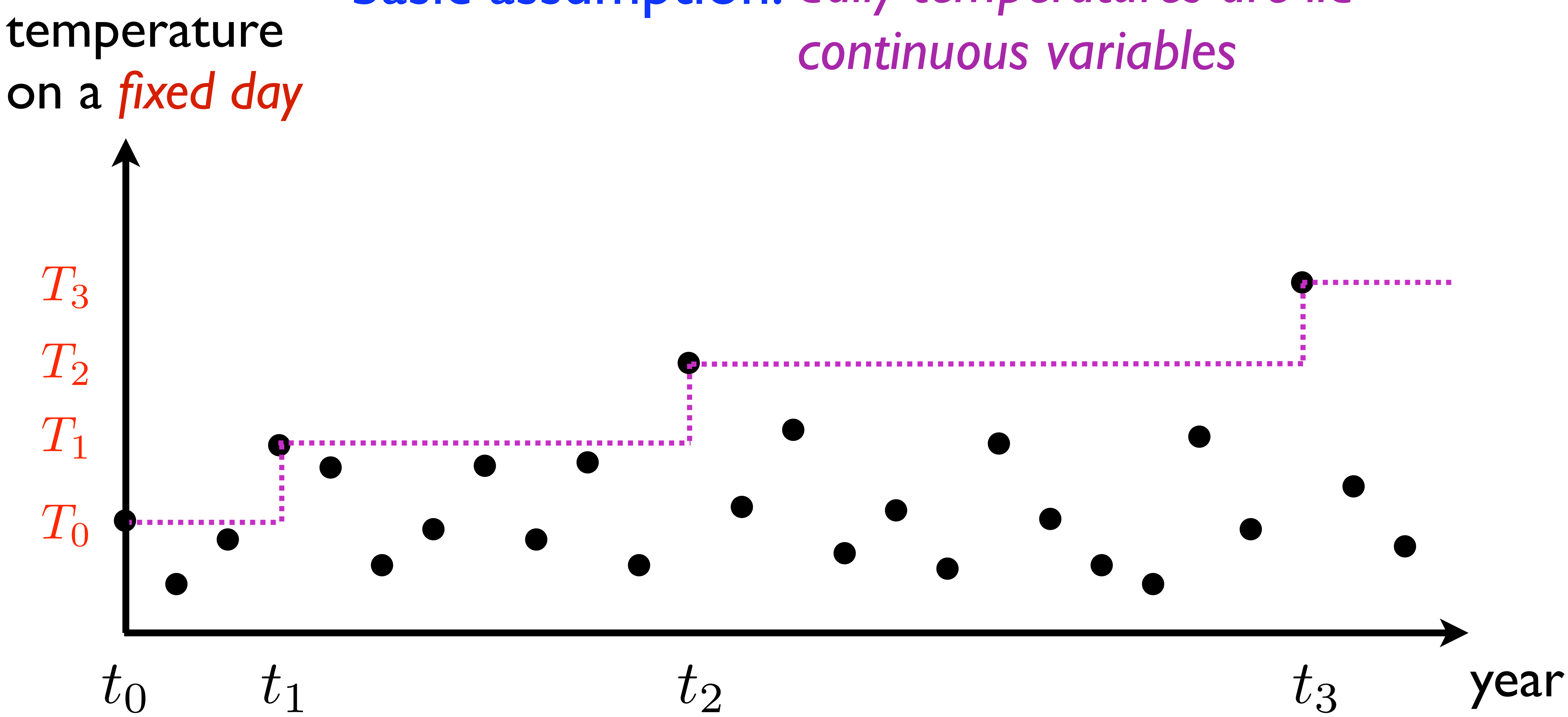
Daily Temperature Distribution



Evolution of Temperature Records

(Arnold et al., *Records*, 1998)

basic assumption: *daily temperatures are iid continuous variables*



Goal: compute $T_k, \mathcal{P}_k(T)$ $t_k, p_k(t)$

distribution independent!

Disclaimers

record temperatures → most data discarded

data from a single station only

no control for urban heat island effect

only 126 years of data---climatologically puny

unknown data quality and accuracy:

- only daily high & low are reported
- reported accuracy of 1°F

few records → asymptotic analysis questionable

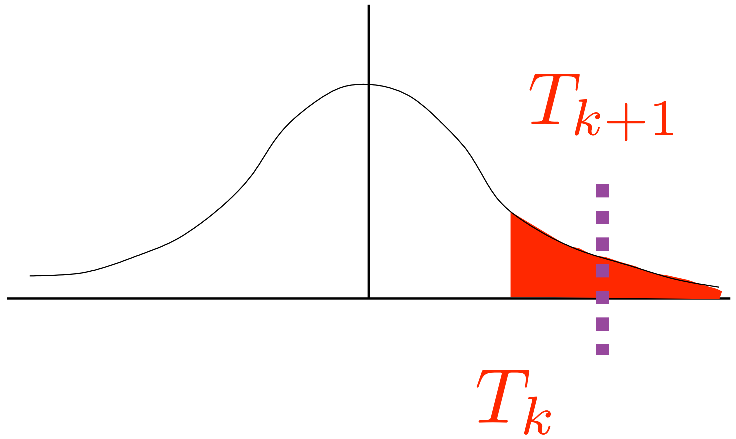
.....

Evolution of *Typical* Record Temperature

$p(T) \equiv \text{daily temperature distribution}$

$$T_0 = \int_0^\infty T\, p(T)\, dT$$

$$T_{k+1} = \frac{\displaystyle\int_{T_k}^\infty T\, p(T)\, dT}{\displaystyle\int_{T_k}^\infty p(T)\, dT}$$



Record Temperature Distributions

prob. temperature > T

$$p_>(T) \equiv \int_T^\infty p(T')\,dT'$$

prob. temperature < T

$$p_<(T) = 1 - p_>(T)$$

probability distribution of $k^{\rm th}$ record

previous record T' < T

next n temperatures < T'

last temperature = T

$$\begin{aligned}\mathcal{P}_k(T) &= \left(\int_0^T \mathcal{P}_{k-1}(T') \sum_{n=0}^\infty [p_<(T')]^n \, dT'\right) p(T)\,, \\ &= \left(\int_0^T \frac{\mathcal{P}_{k-1}(T')}{p_>(T')}\, dT'\right) p(T)\,.\end{aligned}$$

Record Time Evolution

$q_n(T_k) \equiv$ prob. $(k+1)^{\mathrm{st}}$ record n years after k^{th} at T_k

$$= p_<(T_k)^{n-1}\, p_>(T_k)$$

n-1 non-records record

expected time between k^{th} and $(k+1)^{\mathrm{st}}$ records at T_k:

$$t_{k+1} - t_k = \sum_{n=1}^{\infty} n\, p_<^{n-1}\; p_> = \frac{1}{p_>(T_k)}$$

finite for given T_k

waiting time distribution for k^{th} record: averaged over T_k

$$Q_n(k) \equiv \int_0^\infty \mathcal{P}_k(T)\, q_n(T)\, dT$$

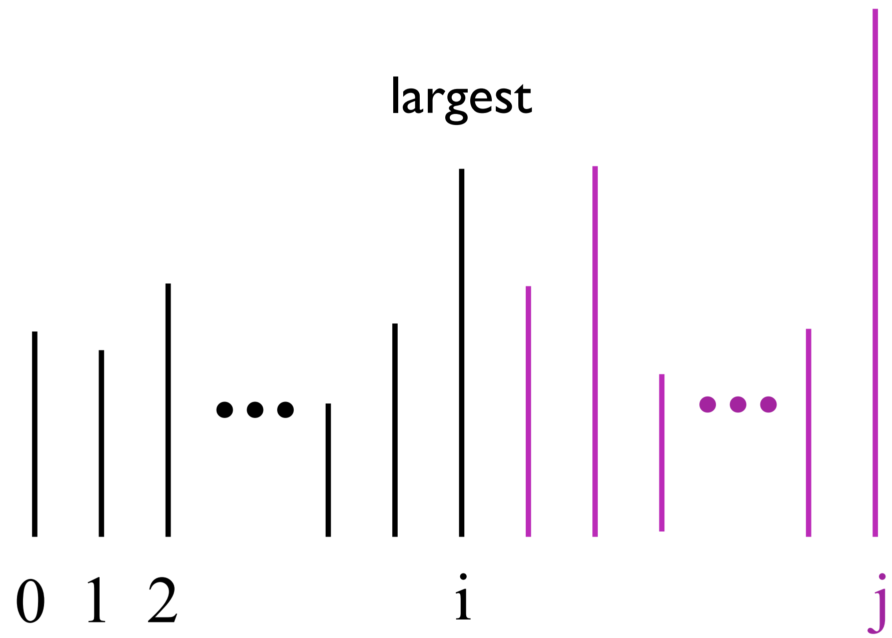
$$= \int_0^\infty \mathcal{P}_k(T)\, p_<(T)^{n-1}\, p_>(T)\, dT$$

in general ***infinite*** waiting time

Record Time Statistics

(Glick 1978; Sibani et al 1997, Krug & Jain 05, Majumdar)

define $\sigma_i = \begin{cases} 1 & \text{if record in } i^{\text{th}} \text{ year} \\ 0 & \text{otherwise} \end{cases}$



then $\langle \sigma_i \rangle = \dfrac{1}{i+1}$ $\quad \langle \sigma_i \sigma_j \rangle = \langle \sigma_i \rangle \langle \sigma_j \rangle$

therefore $\langle n(t) \rangle = \sum_{i=1}^{t} \langle \sigma_i \rangle \sim \ln t$ $\quad P(n) = \dfrac{(\ln t)^n}{n!} e^{-\ln t}$

probability that current record is broken in ith year:

$$= \frac{1}{i(i+1)} \rightarrow \text{time until next record} = \infty$$

Records with Gaussian Temperature Distribution

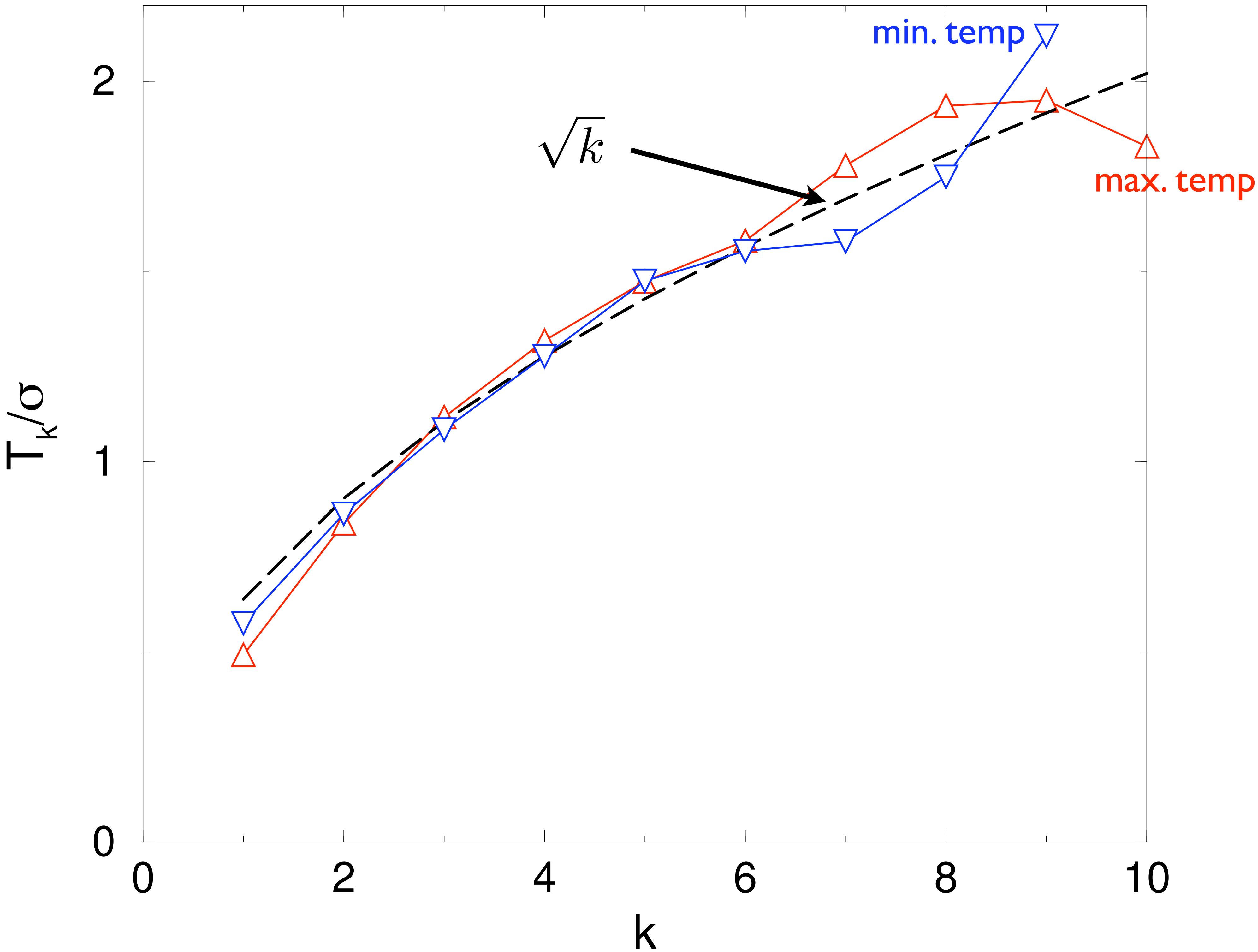
$$p(T) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-T^2/2\sigma^2} \qquad\qquad T_{k+1} = \frac{\int_{T_k}^{\infty} T\, p(T)\, dT}{\int_{T_k}^{\infty} p(T)\, dT}$$

$$T_0 = 0 \qquad\qquad T_1 = \sqrt{\frac{2}{\pi}}\,\sigma$$

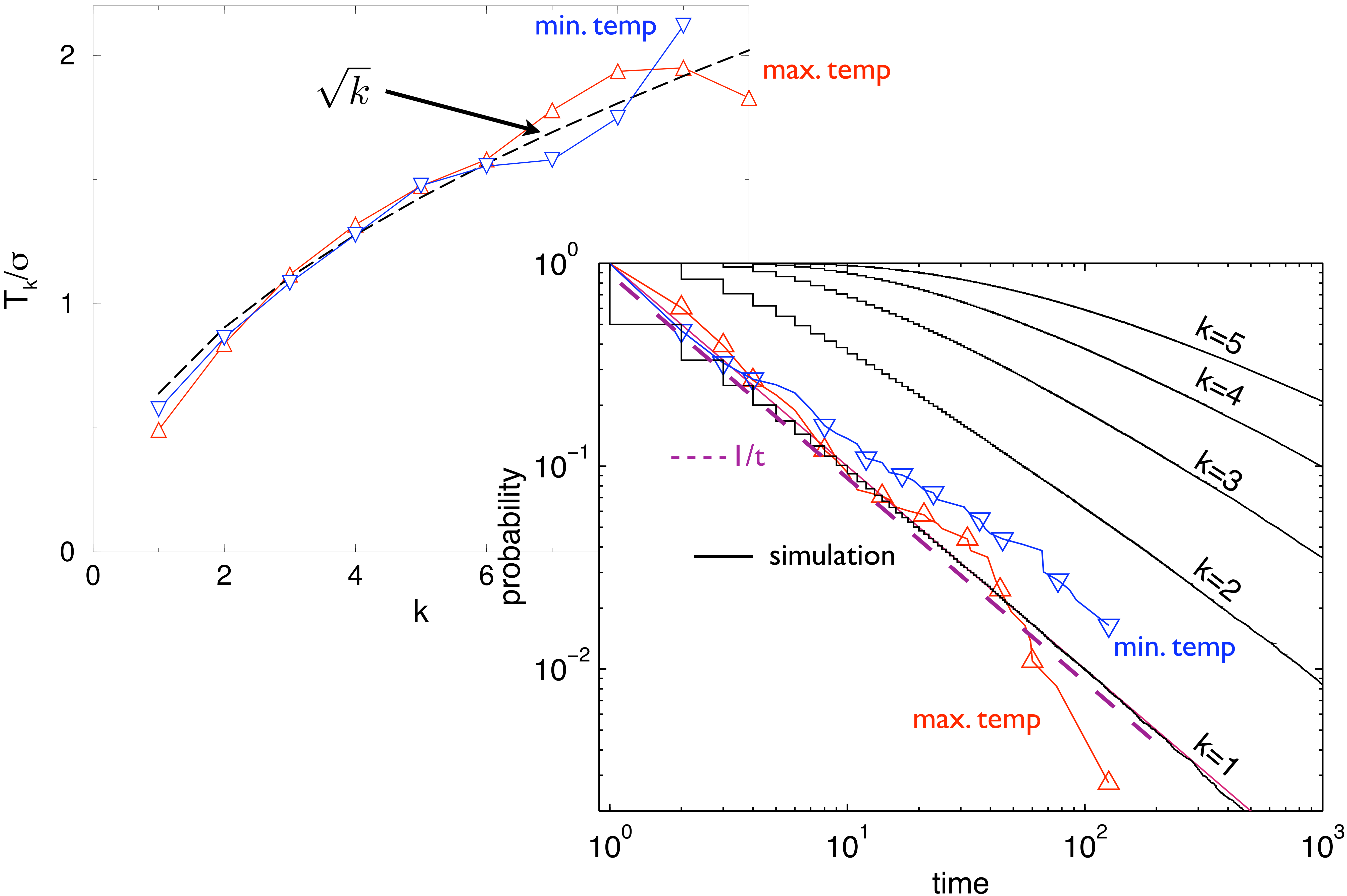
$$\begin{aligned} T_{k+1} \quad &= \quad \frac{\int_{T_k}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}}\, T\, e^{-T^2/2\sigma^2}\, dT}{\int_{T_k}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-T^2/2\sigma^2}\, dT} \\ &= \quad \frac{T_1\, e^{-T_k^2/2\sigma^2}}{\operatorname{erfc}\left(T_k/\sqrt{2\sigma^2}\right)} \\ &\sim \quad T_k \left(1 + \frac{\sigma^2}{T_k^2}\right) \qquad\qquad k \to \infty \end{aligned}$$

$$\frac{\partial T}{\partial k} \sim \frac{\sigma^2}{T} \qquad \to \qquad T_k \sim \sqrt{2k\sigma^2}$$

Philadelphia Record Temperature Values . . .



. . . and Record Times



Records If Global Warming Is Occurring

assume daily temperature distribution

$$p(T;t) = \begin{cases} e^{-(T-vt)} & T > vt \\ 0 & T < vt \end{cases}$$

as a soluble example only

exceedance probability

$$p_>(T_k; t_k + j) = \int_{T_k}^{\infty} e^{-[T-v(t_k+j)]}\, dT = e^{-(T_k - vt_k)}\, e^{jv}$$

prob of record at year n

$$q_n(T_k) \equiv p_>(T_k)\, p_<(T_k)^{n-1} \rightarrow e^{nv}\, X \prod_{j=1}^{n-1} (1 - e^{jv}\, X)$$

overestimate of record time

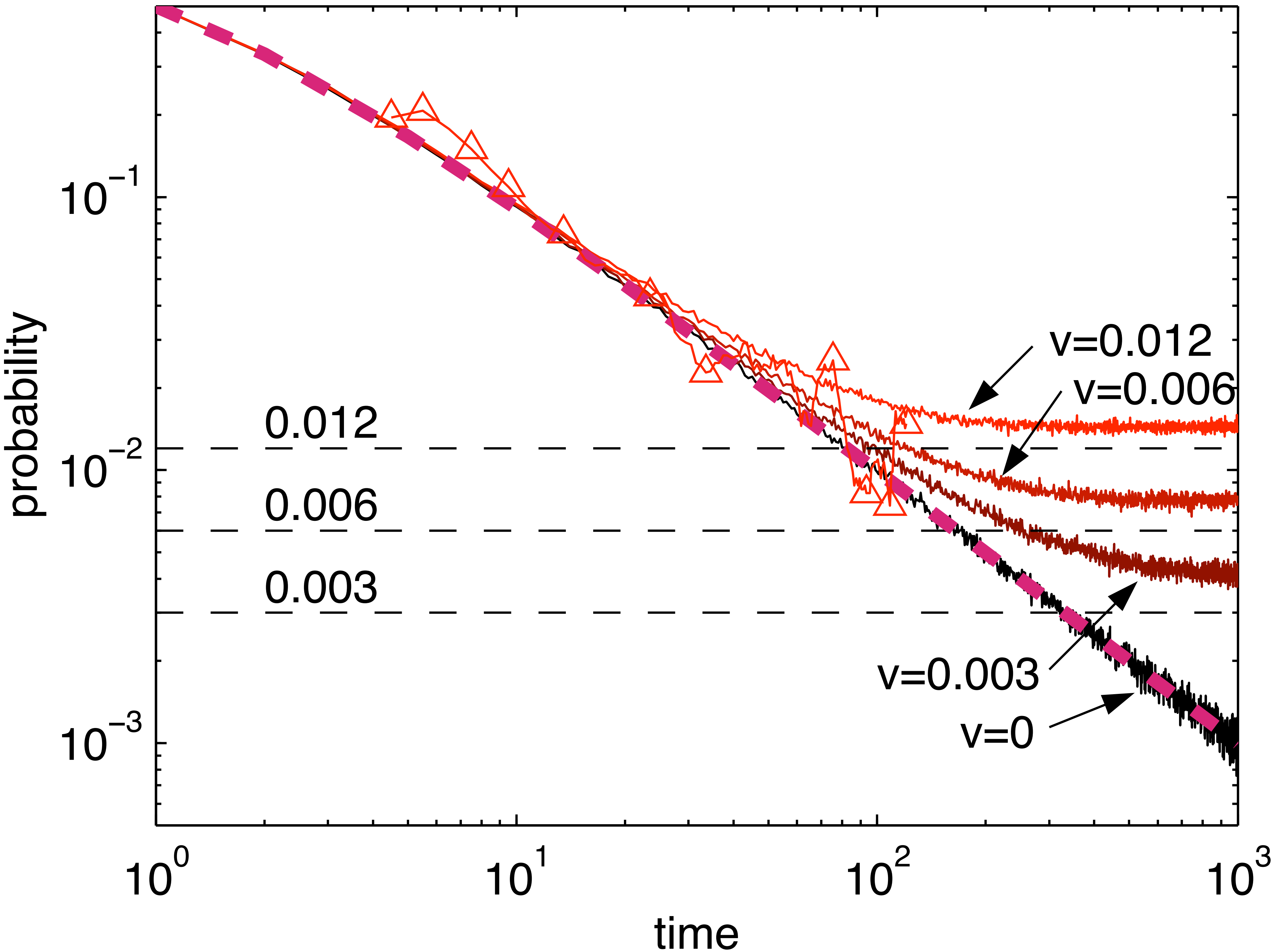
$$e^{jv} X = 1 \rightarrow (t_{k+1} - t_k)v = T_k - vt_k$$

$$\rightarrow t_k \sim \frac{k}{v}$$

records ultimately occur at constant rate

(Ballerini & Resnick, 85; Borokov, 99)

Frequency of Record High Temperatures

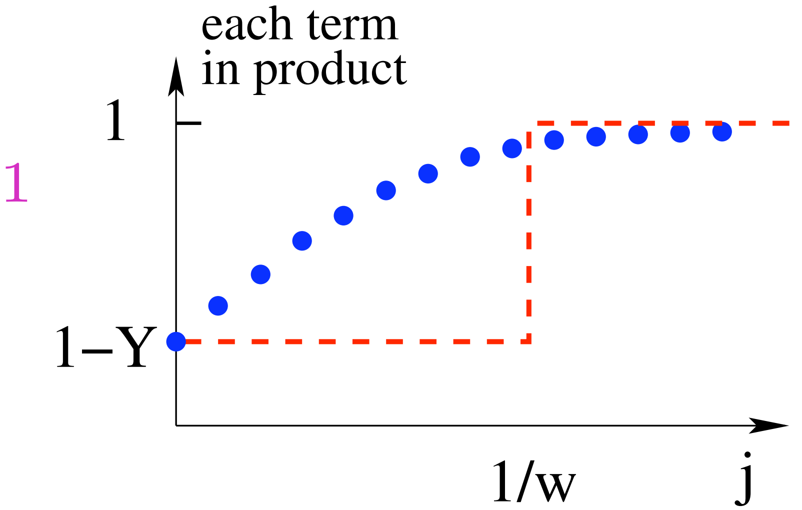


Global Cooling *(low temperature records in warming)*

$$q_n(T_k) \equiv p_>(T_k)\, p_<(T_k)^{n-1}$$
$$\to e^{-nw}\, Y \prod_{j=1}^{n-1} (1 - e^{-jw}\, Y)$$

with $w = |v| > 0$

$$Y = e^{-(T_k + w t_k)}$$



$$q_n(T_k) \sim \begin{cases} (1-Y)^n\, e^{-nw}\, Y & n < n* \sim w^{-1} \\ (1-Y)^{1/w}\, e^{-nw}\, Y & n > n* \end{cases}$$

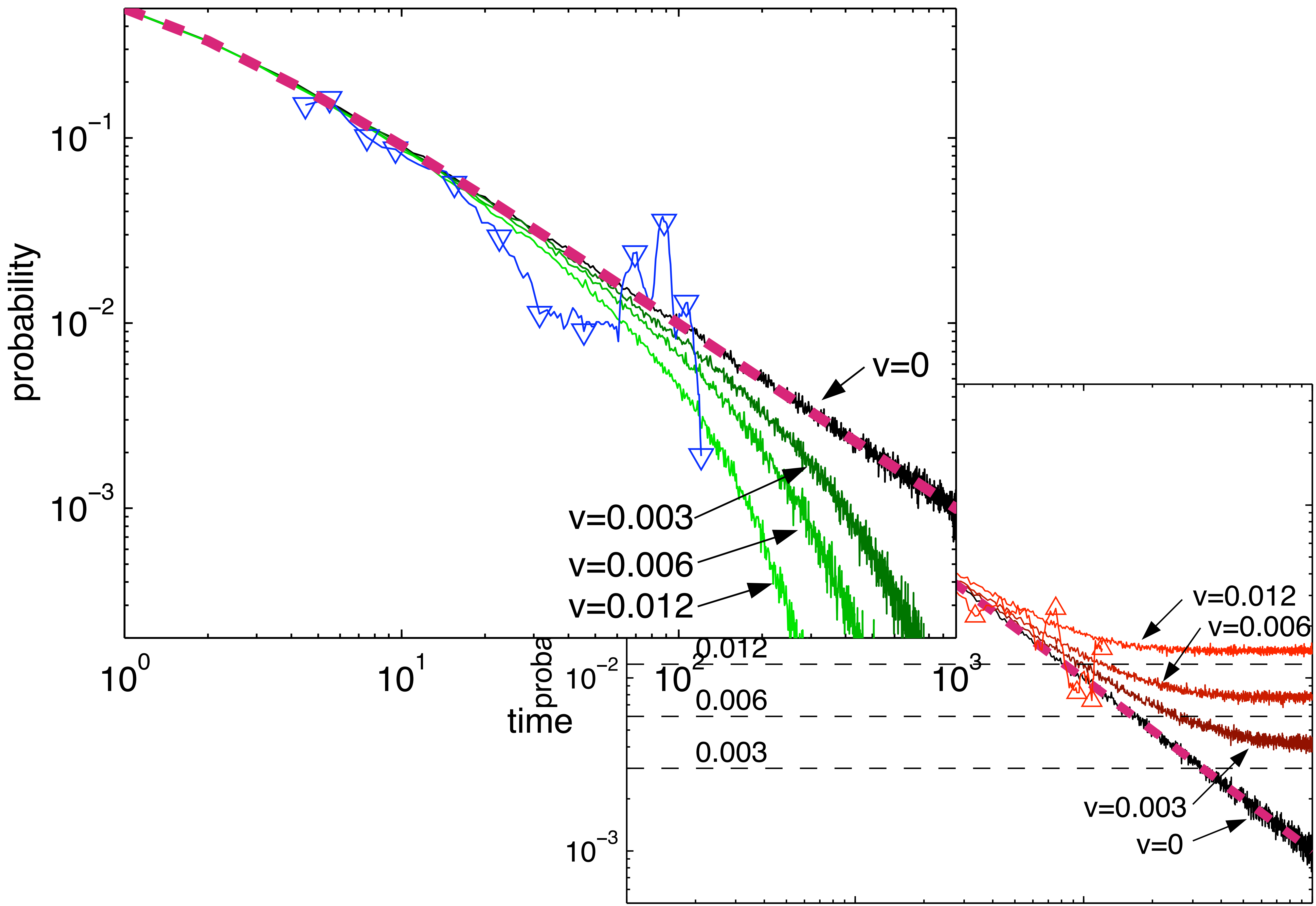
$$\to t_{k+1} - t_k \sim e^{[T_k + w t_k]} \qquad (e^{T_k} \text{ for } w = 0)$$

continuum limit: $\dfrac{\partial t}{\partial k} \sim e^{k + wt}$

$$\underbrace{(1 - e^{-wt})}_{<0} = w(e^k - 1) \approx wt$$

no solution for $t_k > \frac{1}{w}$

Frequency of Record Low Temperatures



Summary & Outlook

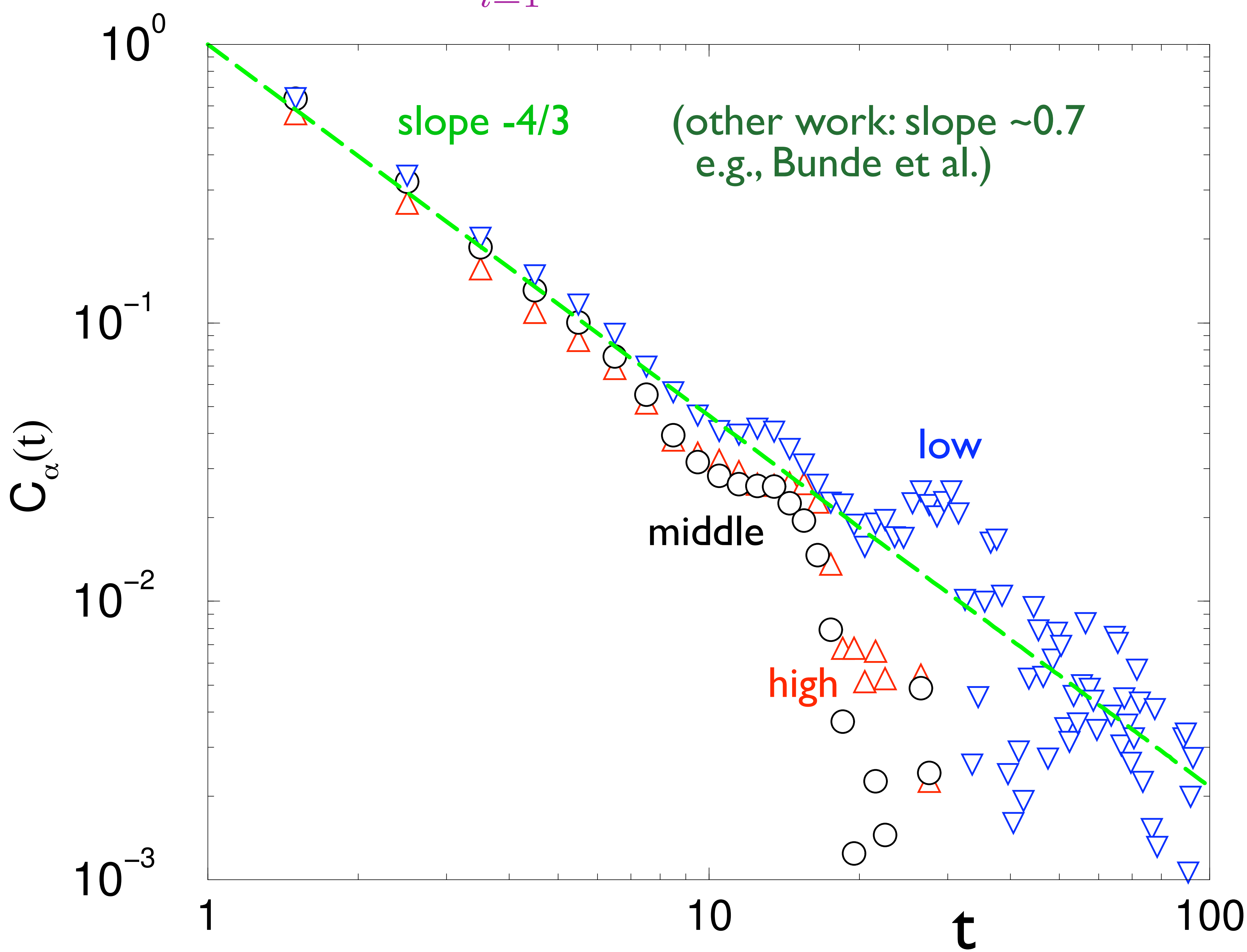
Record temperature and time statistics are obtainable from basic extreme value considerations

Current global warming rate does not yet significantly affect record temperature statistics

Inter-day temperature correlations also do not affect record temperature statistics

Inter-day Temperature Correlations

$$C_\alpha(t) = \frac{1}{365}\sum_{i=1}^{365} \frac{\langle T_i T_{i+t}\rangle - \langle T_i\rangle\langle T_{i+t}\rangle}{\langle T_i^2\rangle - \langle T_i\rangle^2} \quad \alpha = \text{low, middle, high}$$



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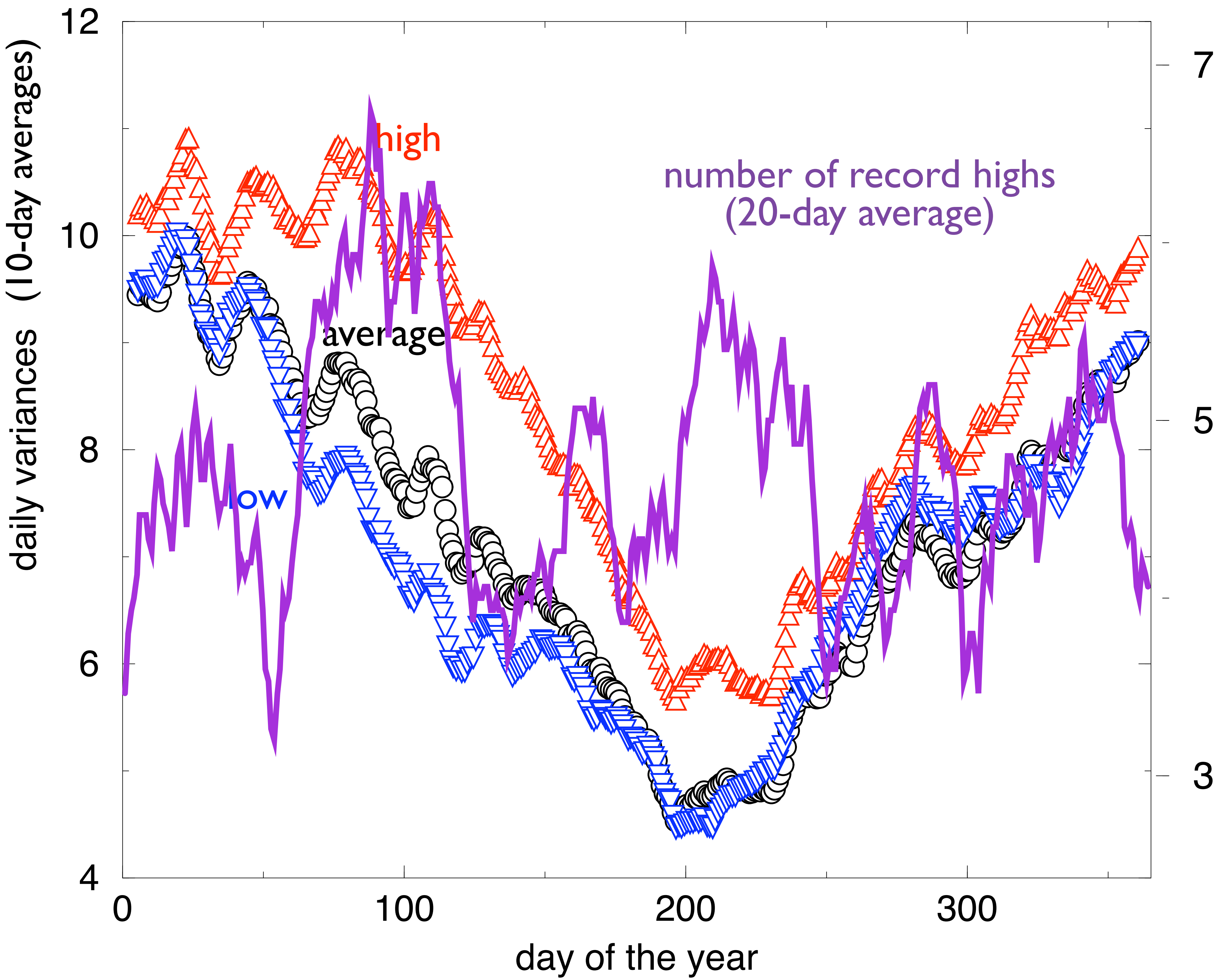
Open issues: Difference between high & low records

1705 record highs; 1346 record lows

Seasonal effects

more record highs in spring

Seasonal Variance & Record Numbers



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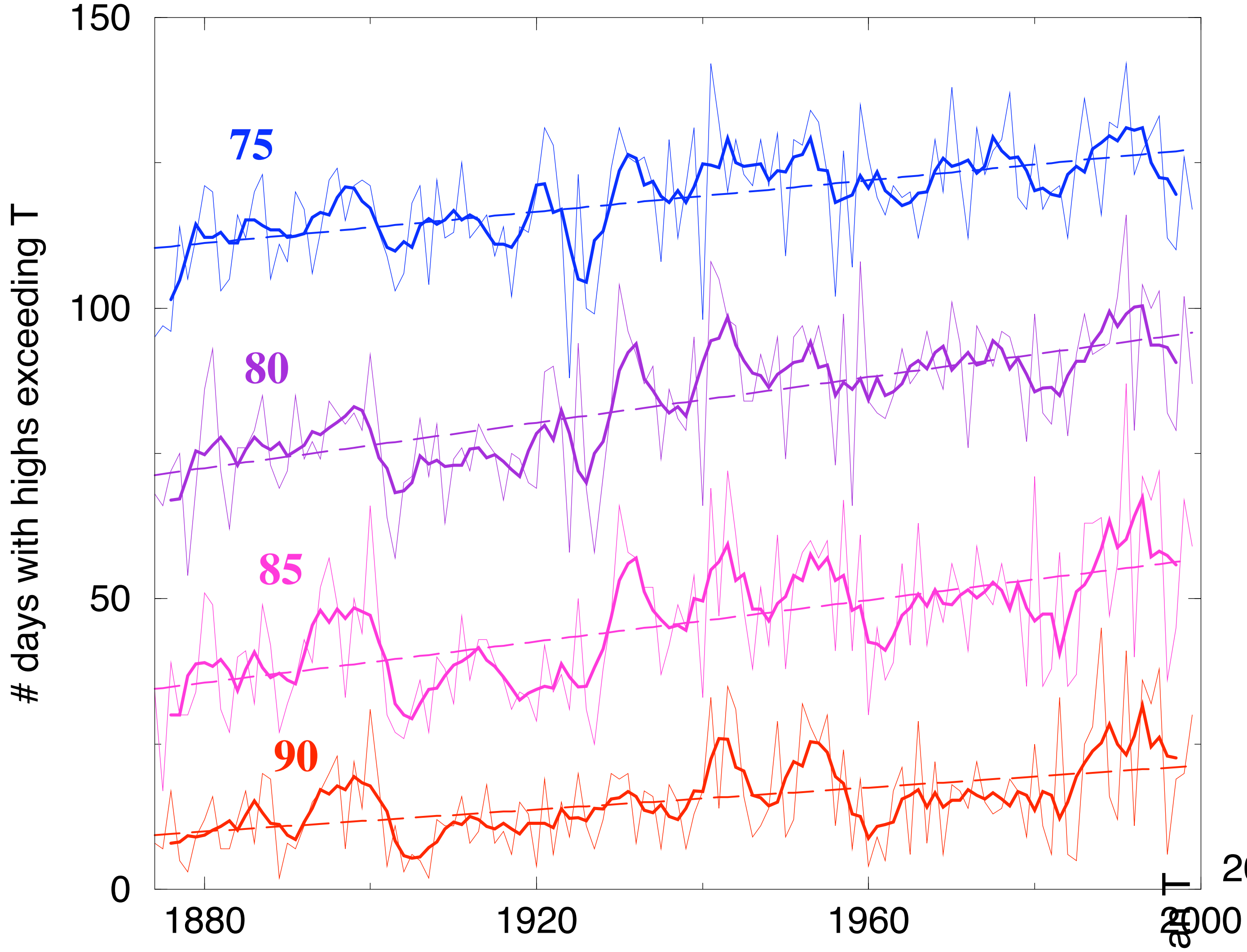
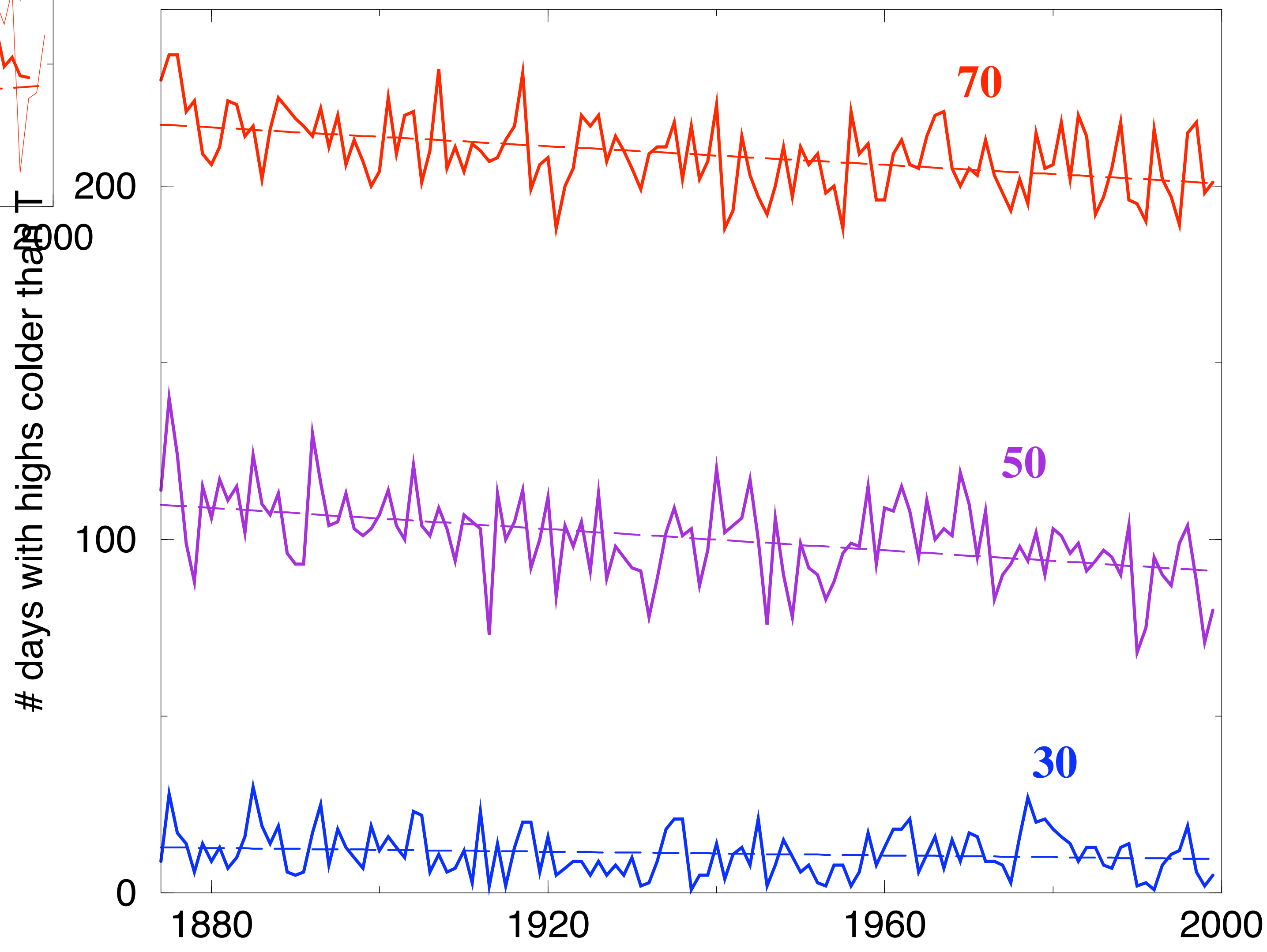
  - *1705 record highs; 1346 record lows*
- Seasonal effects
  - *more record highs in spring*
- Day/night asymmetry

more hot days....



and fewer cold days....



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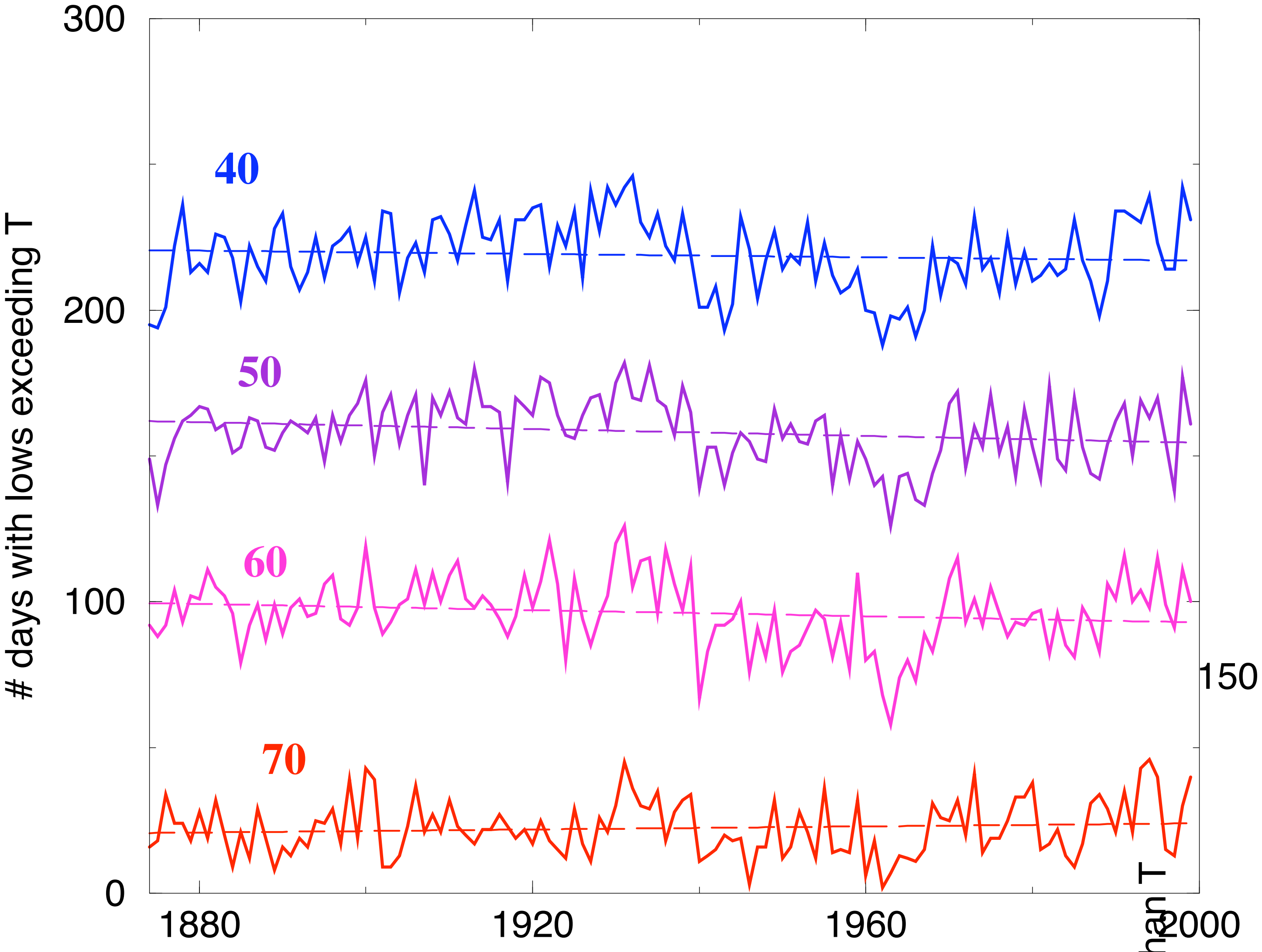

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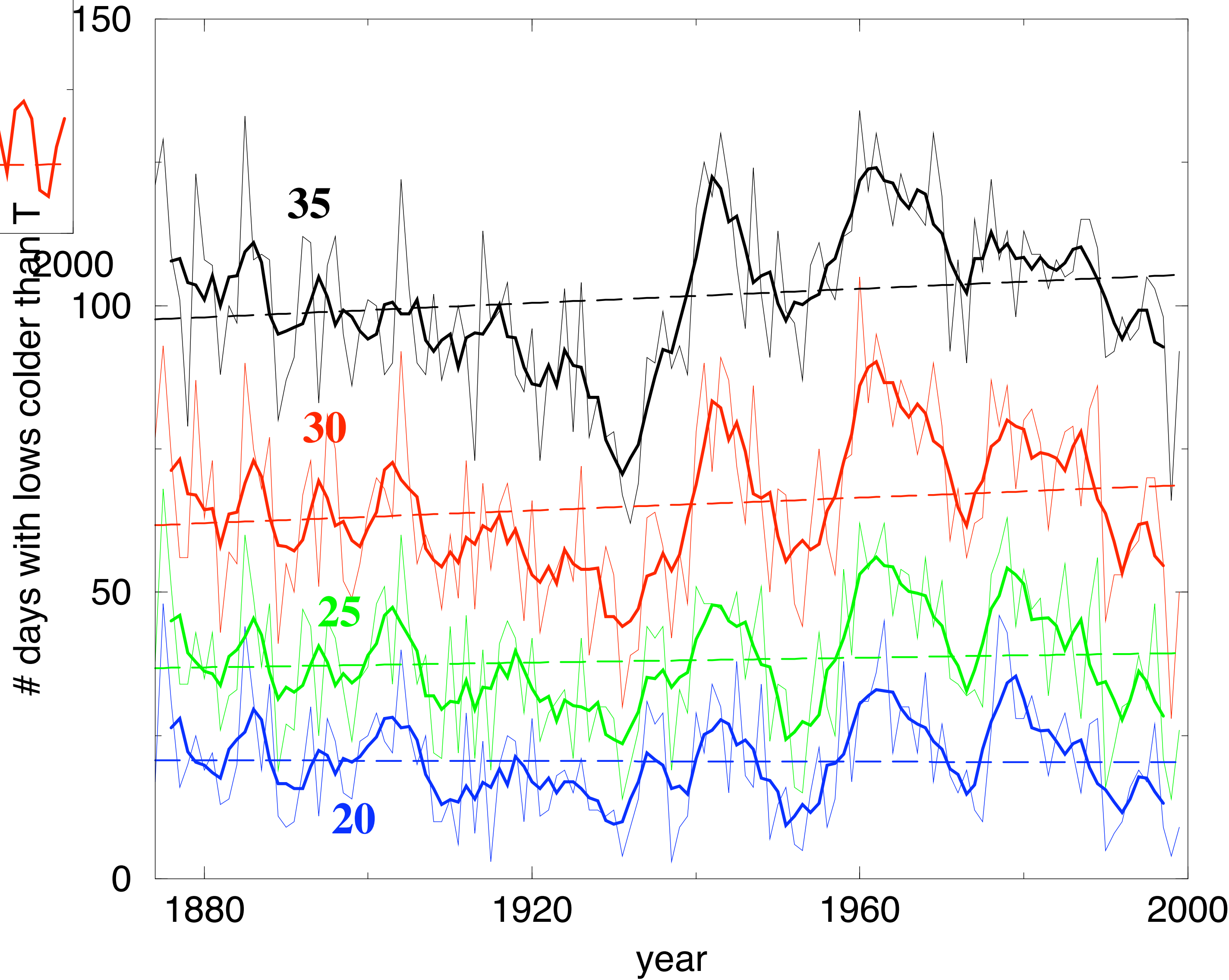

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  - *1705 record highs; 1346 record lows*
- Seasonal effects
  - *more record highs in spring*
- Day/night asymmetry
- Role of changing variability
- Serious data collection & model validation

Thank you Newton Institute!

Thank you for participating!

Have a safe trip home!

I HOPE TO SEE YOU AGAIN!