

Semiparametric Bayesian Latent Trajectory Models

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- ▶ *Modifiable risk factor*

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- ▶ Optimal for other outcomes?
- ▶ Recommends normal weight women gain 25-35 pounds (more gain recommended for underweight women and less gain recommended for overweight and obese women)
 - ▶ Should pregnant woman who has gained 35 pounds 5 weeks before her due date give up the pleasure of eating ice cream at the local dairy farm?

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- ▶ Birth weight recorded

Goal of Analysis

Determine whether *pattern* and *amount* of pregnancy weight gain are related to birth weight (and in particular, risk of small-for-gestational age births).

Here, our outcome, birth weight, is univariate, and our predictor, weight gain, is longitudinal.

Complication: how to characterize pattern of pregnancy weight gain? Pregnancy weight measures are an example of a *functional predictor*: a random curve that varies over time, space, or some other domain, with observations at every point in the domain that may only be measured at a finite set of points (indexed by j).

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 - ▶ y_i = response variable (infant birth weight)

Latent class trajectory model

Group-based trajectory models are used to identify clusters of subjects following similar trajectories over time.

While we may not believe that each subject's weight gain measures in pregnancy exactly follow one of C curves, this may be a very useful summary of the data.

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- ▶ Number of latent classes unknown - BIC criteria may be poor

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- ▶ Allow response (birth weight) distribution to vary nonparametrically across clusters
- ▶ Conduct inferences on changes in quantiles of birth weight

Semiparametric Hierarchical Model

$$\begin{aligned}y_i &\sim \mathsf{N}(\mu_i, \tau_y^{-1}), \quad i = 1, \dots, n, \\x_i(t_{ij}) &\sim \mathsf{N}(f_i(t_{ij}), \tau_x^{-1}), \quad j = 1, \dots, n_i, \\(f_i, \mu_i) &\sim Q,\end{aligned}$$

- ▶ Dependence between functional predictor $f_i \in \Omega$ & response $y_i \in \mathfrak{R}$ characterized through Q
- ▶ Q =random probability measure

Hierarchical Basis Function Representation

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$$f_i(t) = \beta_{i1}t + \beta_{i2}t^2 + \beta_{i3}t^3 + \beta_{i4}(t - 13)_+^3 + \beta_{i5}(t - 26)_+^3$$

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- ▶ $DP(\alpha P_0) =$ denotes the Dirichlet process with precision α & base probability measure P_0
- ▶ This DP prior induces the following Blackwell & MacQueen (1973) rule:

$$(\theta_i | \theta_1, \dots, \theta_{i-1}) \sim \left(\frac{\alpha}{\alpha + i - 1} \right) P_0 + \sum_{j=1}^{i-1} \left(\frac{1}{\alpha + i - 1} \right) \delta_{\theta_j},$$

δ_{θ} = measure concentrated at θ .

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- ▶ Marginal density of y is a mixture of normals - flexible
- ▶ Within predictor cluster, density of y is a single normal - inflexible!
- ▶ Does not allow flexible inferences on changes in risk of low and high birth weight (quantiles) over weight gain clusters (dictated by shape of normal distribution)

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- ▶ It is appealing to consider alternatives that allow separate, but dependent clustering

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- ▶ We need a joint distribution on the basis coefficients, characterized by the probability measure P
 - ▶ We cluster individuals into unique groups, with each group h characterized by unique values of the basis coefficients β_h^*

Predictor Component Model

- ▶ We start with the following model for the basis coefficients:

$$\beta_i \sim P_1 = \sum_{h=1}^H \pi_h \delta_{\beta_h^*}, \quad \beta_h^* \stackrel{iid}{\sim} P_{01},$$

$$\boldsymbol{\pi} = (\pi_1, \dots, \pi_H)' \sim \text{Diri}\left(\frac{\alpha}{H}, \dots, \frac{\alpha}{H}\right),$$

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- ▶ Weights π have a Dirichlet prior, so that $\lim_{H \rightarrow \infty} P_1 \sim DP(\alpha P_{01})$.

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- ▶ $\gamma_i \in \{1, \dots, H\}$ = predictor cluster index for subject i , so $\gamma_i = h$ with probability π_h

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- ▶ The drawback is that we wouldn't be borrowing information about the distribution of birth weights across weight gain clusters (in some sense we would be forcing differences in the response distribution)
- ▶ In order to have cluster-specific response distributions while also borrowing information across clusters, we use a mixture specification with both global and local components

Response Component Model

- ▶ To allow response distribution to change flexibly across predictor clusters, instead of using a single Dirichlet prior, we use a mixture of local and global components

$$(\mu_i | \gamma_i = h) \sim P_{2h} = \sum_{l=1}^L \nu_{hl} \delta_{\mu_l^*}, \quad \mu_l^* \stackrel{iid}{\sim} P_{02},$$

$$\boldsymbol{\nu}_h = (\nu_{h1}, \dots, \nu_{hL})' \sim (1 - \psi) \delta_{\boldsymbol{\nu}_0^*} + \psi \delta_{\boldsymbol{\nu}_h^*},$$

$$\boldsymbol{\nu}_h^* \stackrel{iid}{\sim} \text{Diri}\left(\frac{\lambda}{L}, \dots, \frac{\lambda}{L}\right), \quad h = 0, 1, \dots, H,$$

- ▶ Implies $\lim_{L \rightarrow \infty} P_{2h} \sim (1 - \psi) P_{20}^* + \psi P_{2h}^*$, with $P_{2h}^* \sim DP(\lambda P_{02})$ independently for $h = 0, 1, \dots, H$.
- ▶ $P_{20}^* = \text{global}$ component (allows borrowing of information)
- ▶ $P_{2h}^* = \text{local}$ component specific to predictor cluster h

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- ▶ In limit as $\psi \rightarrow 0$, predictor and response clustering occur independently (global component dominant and no association between weight gain and birth weight)
- ▶ In limit as $\psi \rightarrow 1$, $\Pr(\mu_i = \mu_j) = 0$ if i and j are not in same predictor cluster (local component dominant so that there are very strong differences in birth weight among weight gain clusters)

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- ▶ **Fast alternatives are needed**

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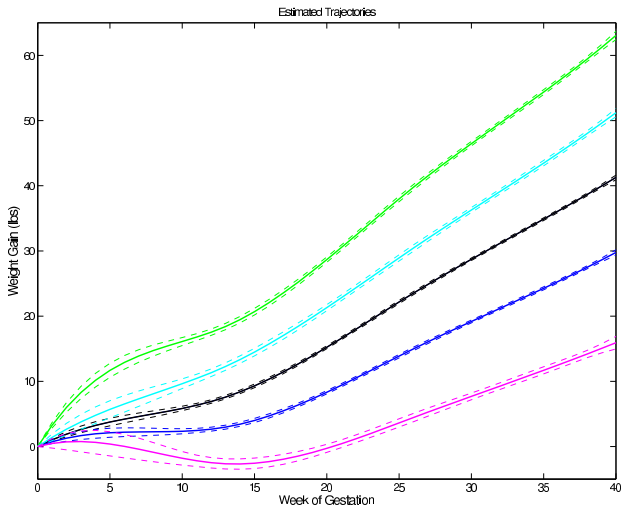
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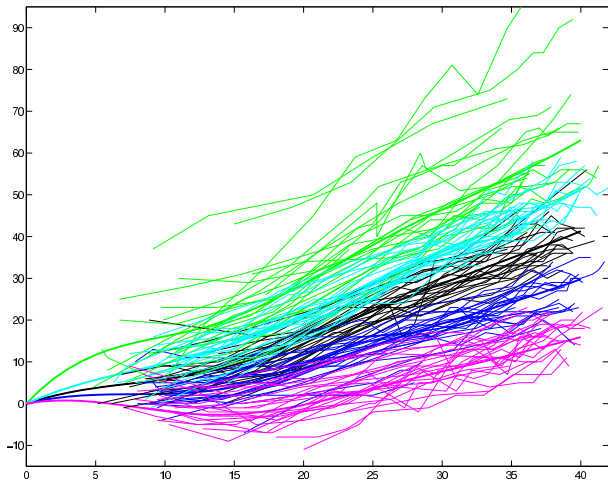
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- ▶ **Avoids label ambiguity and is quite efficient**

Estimated trajectories



Estimated and observed trajectories

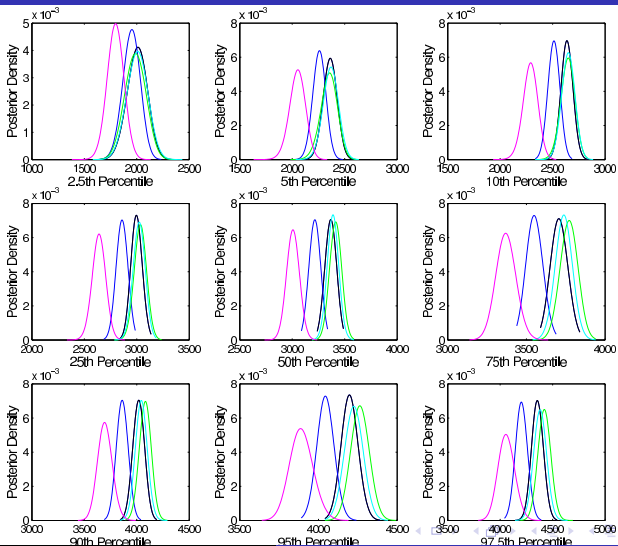


Birth weight results

Next we present plots of the quantiles of the birth weight distribution for each cluster. The posterior distribution of each quantile in the group $\{2.5, 5, 10, 25, 50, 75, 90, 95, 97.5\}$ is plotted by cluster (adjusted for gestational age at delivery).

Note that using normality within a cluster would imply that the 9 plots on the following page should be identical.

Quantile plots



Association between weight gain and low birth weight

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- ▶ Women in higher weight gain clusters had 9.3, 6.8, 6.6, and 6.8% low birth weight babies, respectively

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- ▶ Allows response distribution to vary nonparametrically with functional predictors
- ▶ Approach useful in other applications - e.g., joint modeling of images & disease status