Multi-Stratum Experiments: Session 2

Steven Gilmour
Queen Mary, University of London

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There is a nested unit structure $U_1/\cdots/U_s$, or $\cdots$/Superblocks/Blocks/Units.

Some treatment factors must have the same level in all experimental units within the same category of U_i for some $i < s$.

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Randomized-not-reset factors are very common in practice (and probably will continue to be). There are unexplored problems here, but these probably have little to do with multi-stratum designs.

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We will concentrate on regular designs in which all treatment effects are either completely confounded with or orthogonal to blocks.

Application: Seven factors which can affect the efficiency of a ball mill: motor speed (X_1), feed mode (X_2), feed sizing (X_3), material type (X_4), gain (X_5), screen angle (X_6), screen vibration level (X_7) (Montgomery, 1993).

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- Most authors try to use the minimum possible (statistically) number of blocks;
- I assume that we have negotiated the maximum possible (practically) number of blocks.

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If two effects are confounded, their generalized interaction is also confounded, e.g. if $X_1X_2X_3$ and $X_1X_4X_5$ are confounded, so is $X_2X_3X_4X_5$.

Confounding of Easy-to-Set Factor Effects

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We are just choosing confounded designs in which we are forced to confound the main effects of hard-to-set factors.

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Given the same unit structure, why should we be prepared to use inter-block information to estimate main effects, but be reluctant to do so to estimate interactions?

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with the interaction of X_2 and X_3, etc., i.e. for $\hat{\beta}_1$ estimated from the main effects only model, $E(\hat{\beta}_1) = \beta_1 - \beta_{23}$ under the full model, etc.]

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- The interaction between two positive main effects is more likely to be negative than positive (and so on for higher-order interactions).
- It is better to have two aliased terms exaggerating each other's effects than cancelling out.

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- How would it change the optimality criteria?

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A General Theory of Regular Designs

Chen and Cheng (1999) solved the problem by formally considering a 2^{q-r} fraction in 2^m blocks as a $2^{(q+m)-(r+m)}$ fraction, with m two-level pseudo-factors $b_1, \ldots, b_m$ used to define the blocks.

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In the above example, the defining contrasts are

$$I \equiv -X_1X_2X_3X_4X_5 \equiv X_1X_2b_1 \equiv -X_3X_4X_5b_1 \equiv$$

$$X_3X_4b_2 \equiv -X_1X_2X_5b_2 \equiv X_1X_2X_3X_4b_1b_2 \equiv -X_5b_1b_2$$

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Research has continued in this area.

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I would always choose the best fraction, then block it as well as possible, i.e. get as much information as possible, then allocate it to strata as well as possible.

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There is no choice here. Although we are choosing a confounded fractional factorial design, the randomization restriction means that once we have chosen the fraction there is nothing else to do.

Restrictions in Both Strata

If there are too few blocks to have even a single replicate of all combinations of the hard-to-set factors *and* the number of easy-to-set factors is too great to allow a complete set of combinations in each block, then we have a new problem.

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They used the standard definitions of resolution and aberration, applied to the whole design, but at the same time minimising the aliasing of easy-to-set factor effects with hard-to-set factor effects.

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Others have studied secondary estimation capacity (Yang *et al.*, 2006), weak minimum aberration (Yang, Zhang and Liu, 2007), etc.

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Unfortunately, details on the factors are confidential.

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Also, it generalizes to more general multi-stratum structures. See also Bingham *et al.* (2008) for different generalizations.

Graphical Analysis

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If effects are estimated with different standard errors, only a Bayesian analysis will give meaningful information.

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- reduce the variance between experimental units (thus contradicting the basic assumption of the randomization approach);
- reduce the variance between observational units within experimental units.