Lagrangian acceleration in confined 2d turbulent flow

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OUTLINE

- Motivation
- Complex geometries and volume penalization
- Discretization of the penalized Navier-Stokes equations
- Invariants of the inviscid flow
- Eulerian statistics
- Lagrangian statistics
- Conclusions

MOTIVATION

2D turbulence in bounded domains plays an important role in oceanography, e.g. vortex formation in coastal currents.

Many experiments in rotating tanks (e.g. *Swinney et al., 2002*), resulting in quasi 2D geostrophic flows, have shown the formation of long-lived coherent vortices.

Few numerical studies of 2D turbulence in bounded domains, e.g. *Li, Montgomery et al. 1996/97* using a spectral method with Bessel functions of the first kind, i.e. circular analogues of the Chandrasekhar-Reid functions.
Limited to low resolution and low Reynolds numbers, i.e. Re < 1000 due to the numerical complexity.

Numerical simulations of forced 2D turbulence in circular geometry for Reynolds numbers up to 3500 using a Tchebycheff-Fourier discretization have been presented in *Clercx et al., 2001*.

Aim of the present talk:

Study the influence of the geometry on the flow dynamics and
in particular on the Eulerian and Lagrangian statistics.

Complex Geometries

cartesian geometry

spectral/wavelet solvers
for Navier-Stokes equ.

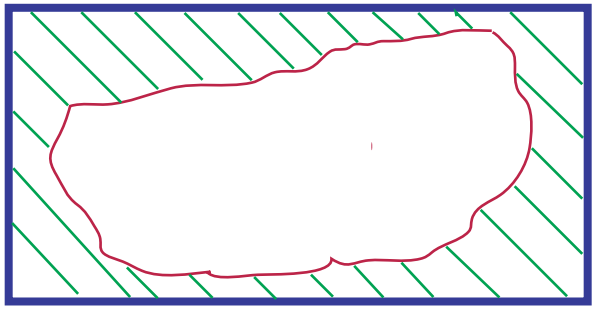
+

complex geometry

???

=

penalty approach



use solver in cartesian
geometry
+ penality term

Different strategies:
- surface penalisation, Peskin, '70ies
- volume penalisation, Caltagirone, '80ies
- imbedding methods, Glowinski, '80ies

Volume penalisation method

$$\partial_t \vec{u}_\eta + \vec{u}_\eta \cdot \nabla \vec{u}_\eta + \nabla p_\eta - \nu \nabla^2 \vec{u}_\eta + \frac{1}{\eta} \chi_{\Omega_s} \vec{u}_\eta = \vec{f} \quad \text{and} \quad \nabla \cdot \vec{u}_\eta = 0$$

$$\chi_{\Omega_s}(\vec{x}) = \begin{cases} 1 & \text{for} \quad \vec{x} \in \bar{\Omega}_s, \\ 0 & \text{elsewhere} \end{cases}$$

Force on the obstacle:

$$\vec{F} = -\lim_{\eta \to 0} \frac{1}{\eta} \int_{\Omega_s} \vec{u}_\eta \, dx = \int_{\partial \Omega_s} \sigma(\vec{u}, p) \cdot \vec{n}_f \, d\gamma$$

with the stress tensor $\sigma(\vec{u}, p) = \frac{1}{2\nu} (\nabla \vec{u} + (\nabla \vec{u})^t) - pI$.

For the vorticity we get

$$\partial_t \omega_\eta + \vec{u}_\eta \cdot \nabla \omega_\eta - \nu \nabla^2 \omega_\eta + \nabla \times (\frac{1}{\eta} \chi_{\Omega_s} \vec{u}_\eta) = \nabla \times \vec{f}$$

Ref.:
- Arquis and Caltagirone, CRAS '84
- Angot, Bruneau and Fabrie, Num. Math. '99

Discretization of the penalized Navier-Stokes equations (I)

Navier–Stokes equations are solved in a square domain of size $L = 2\pi$ using vorticity–velocity formulation. The circular container Ω of radius $R = 2.8$ is imbedded in the square domain and the no–slip boundary conditions on the wall $\partial\Omega$ are imposed using a volume penalisation method (*Arquis & Caltagirone* 1984, *Angot et al.* 1999).

The resulting governing equation is,

$$\partial_t \omega + \vec{u} \cdot \nabla \omega - \nu \nabla^2 \omega + \nabla \times \left(\frac{1}{\eta} \chi \vec{u} \right) = 0$$

where $\vec{u}$ is velocity field, with$\nabla \cdot \vec{u} = 0$, $\omega = \nabla \times \vec{u}$ the vorticity, ν the kinematic viscosity and $\chi(\vec{x})$ a mask function which is 0 inside the fluid, *i.e.* $\vec{x} \in \Omega$, and 1 inside the solid wall. The penalisation parameter η is chosen to be sufficiently small ($\eta = 10^{-3}$).

Discretization of the penalized Navier-Stokes equations (II)

- Fourier pseudospectral method

$$\omega(\vec{x},t) = \sum_{\vec{k}\in\mathbb{Z}^2} \widehat{\omega}(\vec{k},t)\ \exp(i\vec{k}\cdot\vec{x})$$

- Dealiasing using the 2/3 rule

- Semi-implicit time disretization with adaptive time stepping

- Code validation in (*KS, Computers & Fluids*, 34, 2005)

Invariants of the inviscid flow (I)

Different invariants of the flow, *i.e.* quantities which are conserved by the flow dynamics for inviscid flows, can be derived (*Kraichnan & Montgomery*, 1980),

- the circulation Γ (total vorticity) is defined as

$$\Gamma = \int_\Omega \omega d\vec{x} = \oint_{\partial\Omega} \vec{u} \cdot ds, \tag{1}$$

- energy E, enstrophy Z and palinstrophy P as

$$E = \frac{1}{2}\int_\Omega |\vec{u}|^2 d\vec{x}\,,\; Z = \frac{1}{2}\int_\Omega |\omega|^2 d\vec{x}\,,\; P = \frac{1}{2}\int_\Omega |\nabla\omega|^2 d\vec{x}, \tag{2}$$

  respectively.

Invariants of the inviscid flow (II)

- the energy dissipation is given by $d_t E = -2\nu Z$ and the enstrophy dissipation by

$$d_t Z = -2\nu P + \nu \oint_{\partial\Omega} \omega(\vec{n} \cdot \nabla\omega) ds, \tag{3}$$

where $\vec{n}$ denotes the outer normal vector with respect to $\partial\Omega$. The surface integral reflects the enstrophy production at the wall involving the vorticity and its gradients.

- the angular momentum M of the flow with respect to the center of the domain is

$$M = 2 \int_\Omega \psi d\vec{x} \tag{4}$$

where $\psi = \nabla^{-2}\omega$ denotes the stream–function.

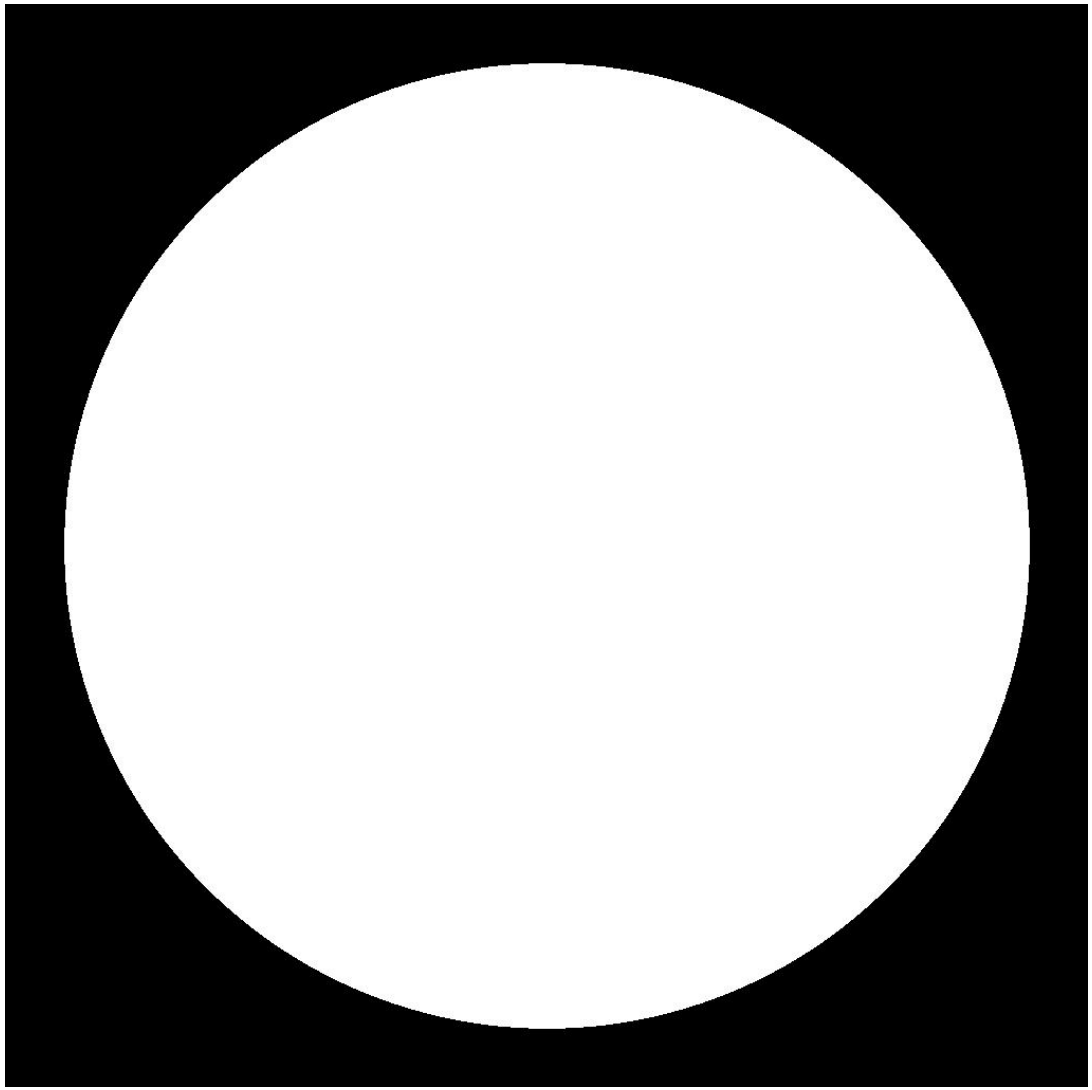
2D decaying turbulence in a circular domain

Mask:

Resolution

N = 1024x1024

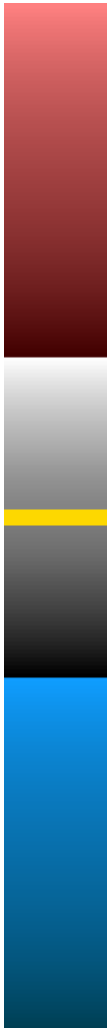
Re = 50000



2D decaying turbulence in a circular domain

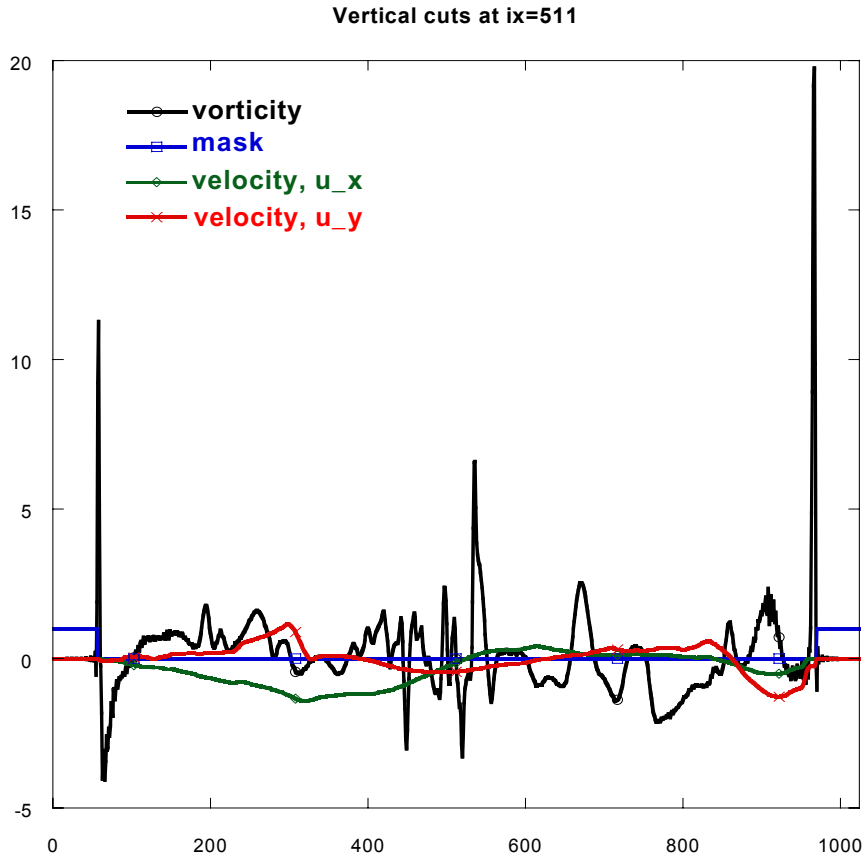
KS, M. Farge, *Phys. Rev. Lett., 95 (24), 2005.*

Vorticity:



2D decaying turbulence in a circular domain

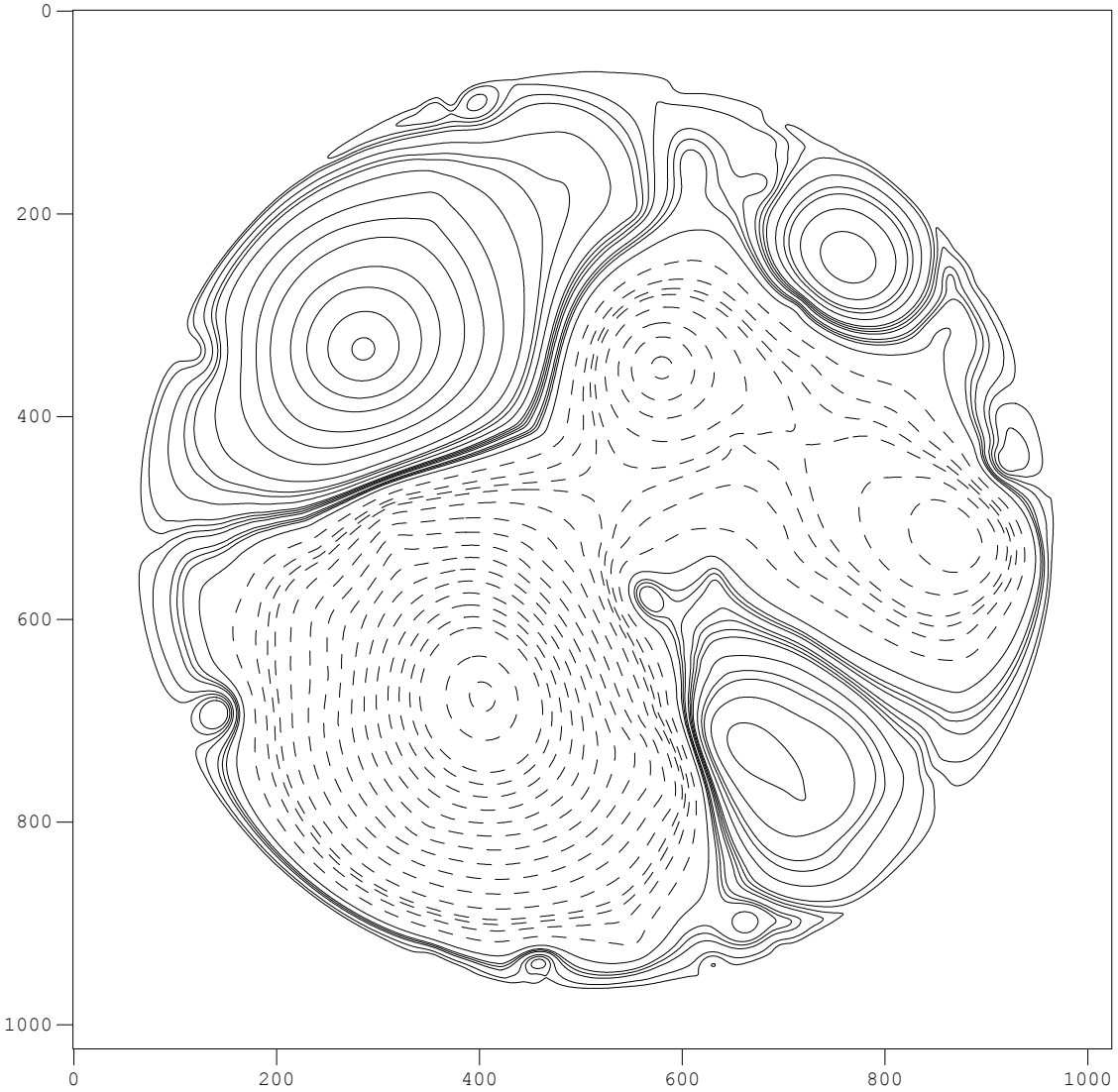
Cuts at t=320



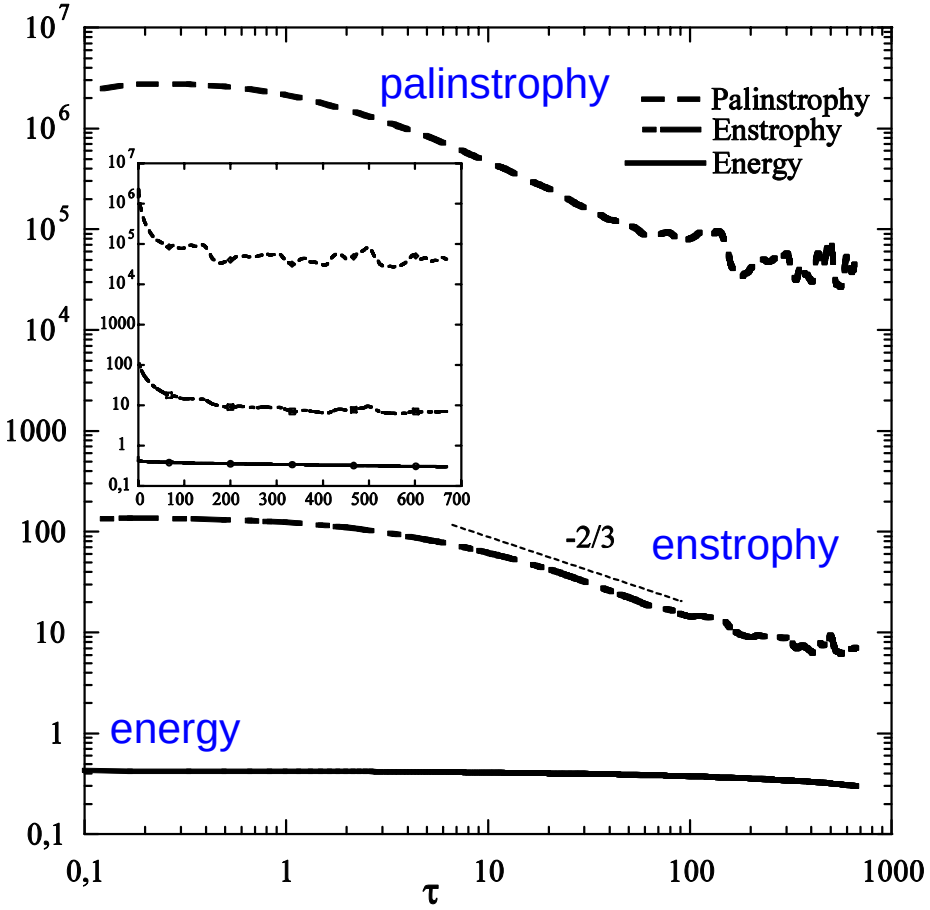
Vertical cuts of vorticity, and the velocity components together with the mask function

2D decaying turbulence in a circular domain

Streamfunction:
t=320

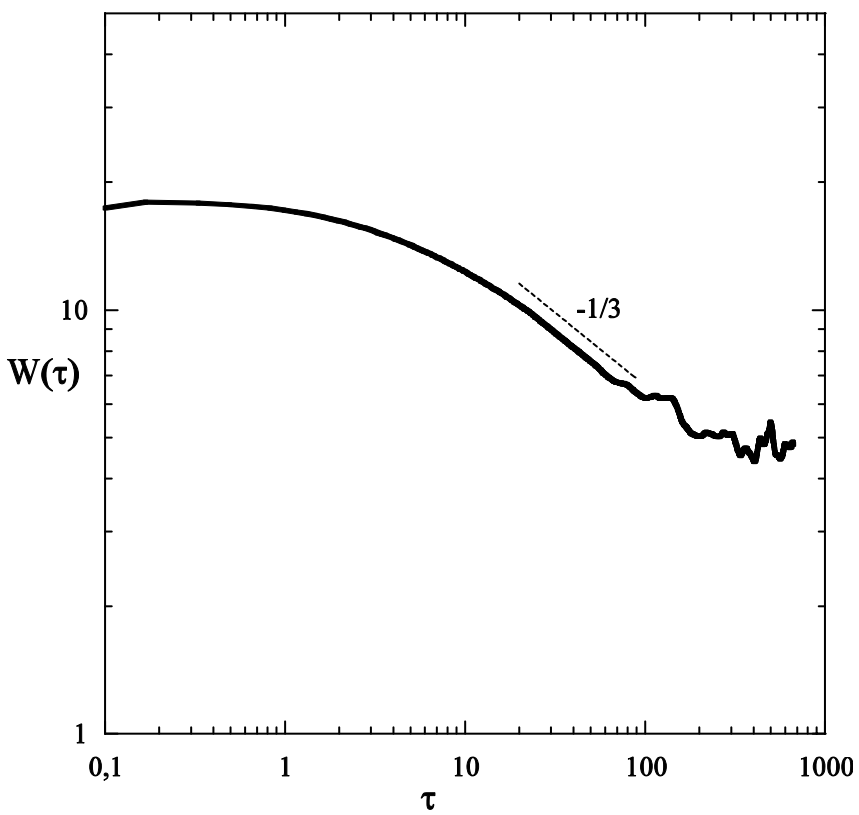


2D decaying turbulence in a circular domain

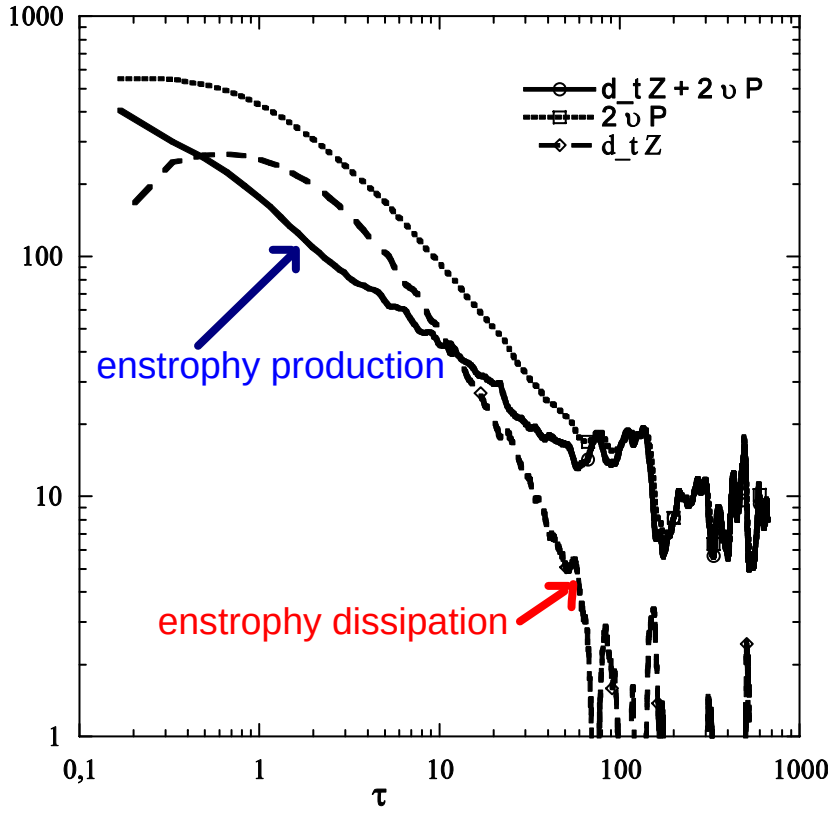


Time evolution of energy E, enstrophy Z and palinstrophy P.

2D decaying turbulence in a circular domain

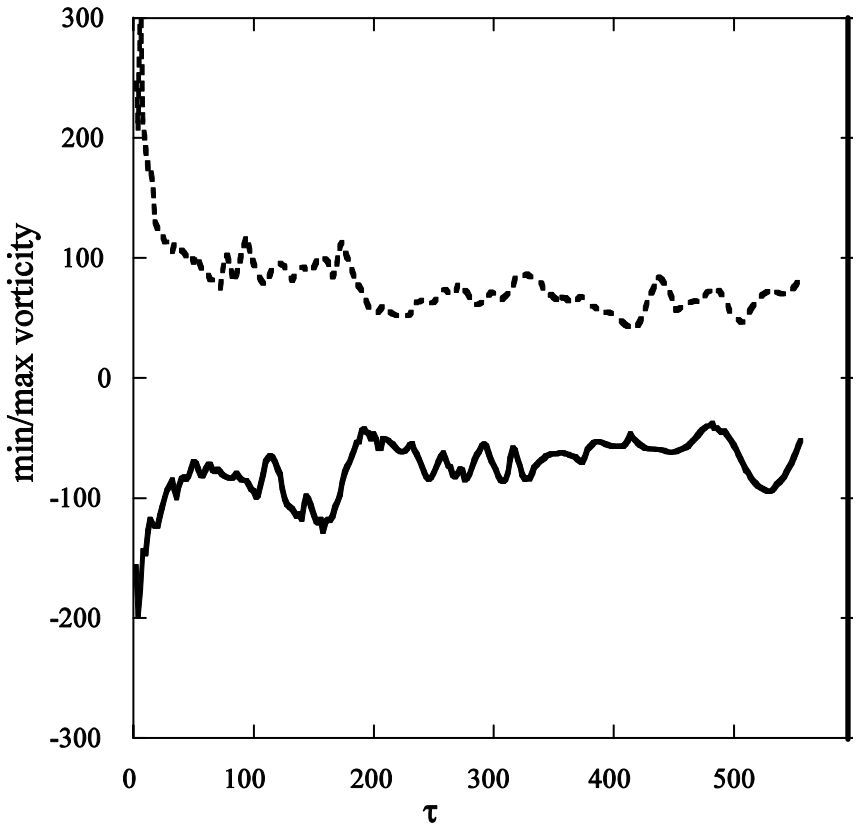


Evolution of the average wavenumber W = sqrt(Z/W)

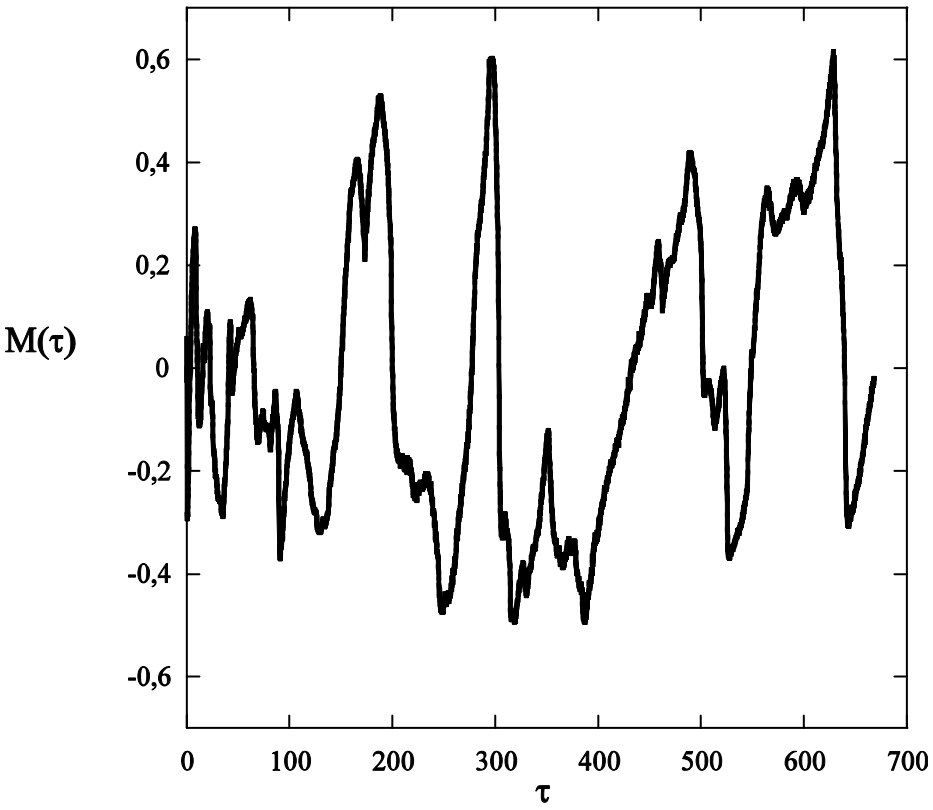


Evolution of d_t Z, 2 υ P, and d_t Z + 2 υ P

2D decaying turbulence in a circular domain

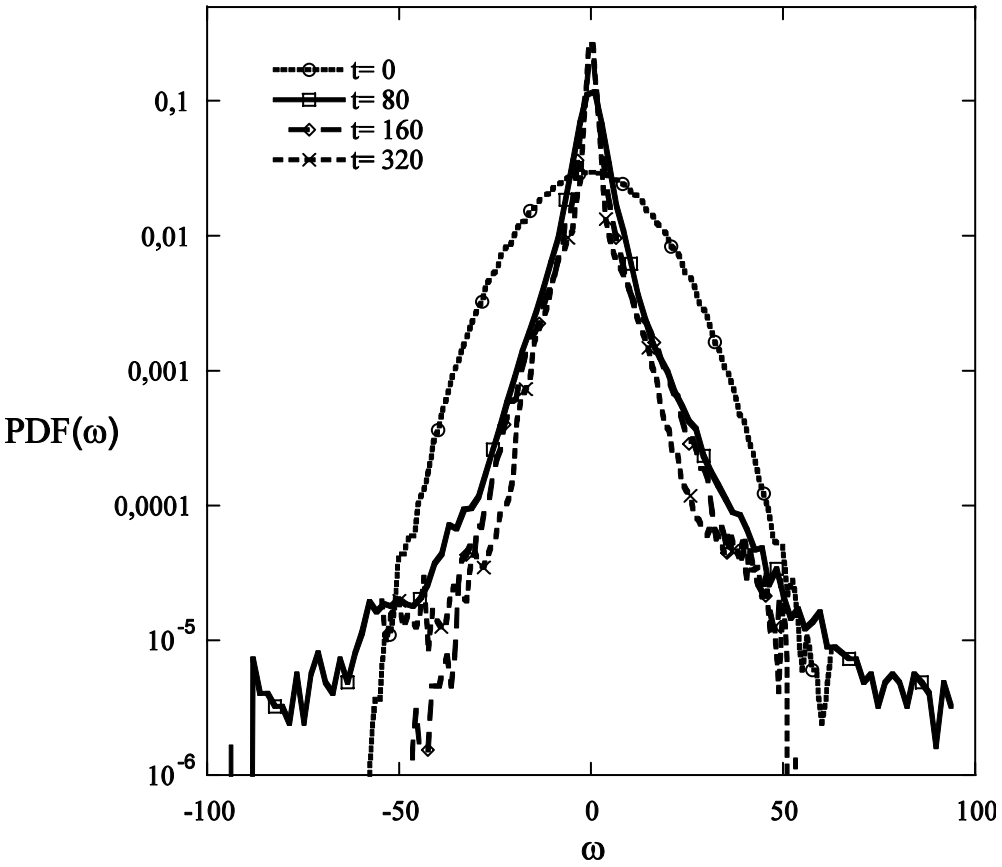


Evolution of min/max vorticity ω.

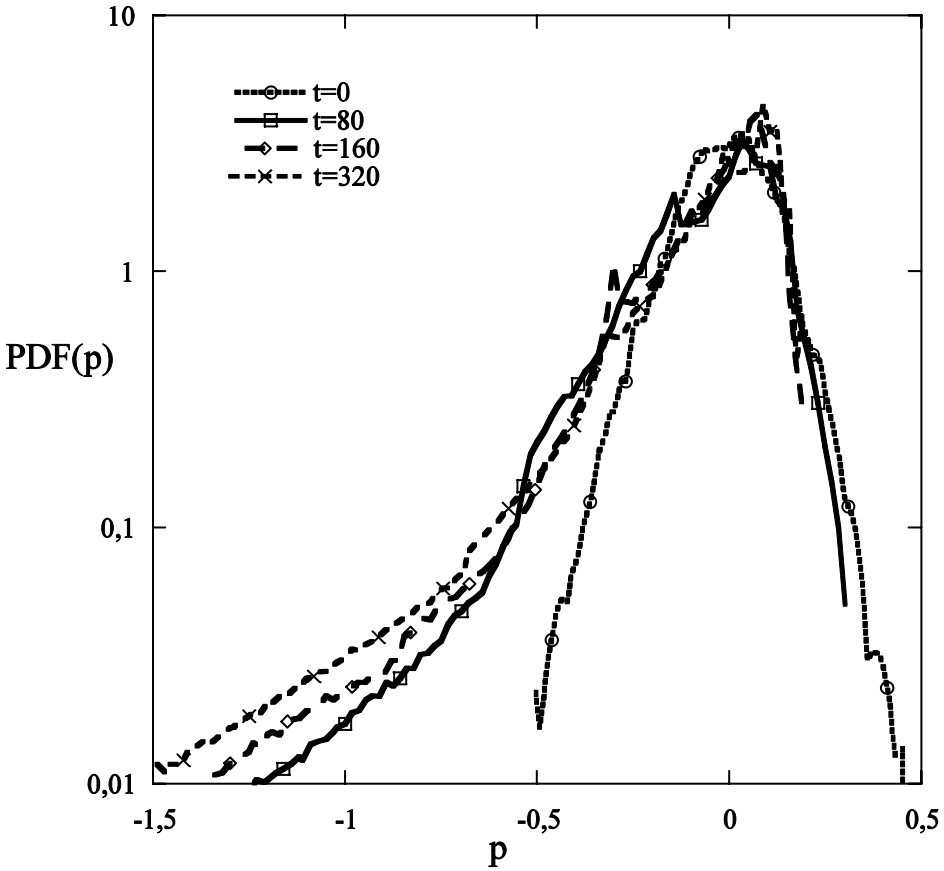


Evolution of angular momentum M.

2D decaying turbulence in a circular domain



PDF of vorticity ω.



PDF of pressure p.

Half-time CONCLUSIONS

DNS in circular geometry using a pseudospectral method with volume penalization.

No-slip boundaries play a crucial role for decaying turbulent flows.

At early times: decay of the flow which leads to self-organisation and the emergence of vortices in the bulk flow, similarly to flows in double periodic boxes.

At later times: production of coherent vortices at the boundary compensates the enstrophy dissipation and the flow decay is drastically reduced. This is reflected in the time evolution of enstrophy and palinstrophy which decay in a non monotonous way.

The pressure PDF is strongly skewed with an exponential shape for negative values due to the presence of coherent vortices.

Lagrangian statistics

Lagrangian quantities

* **Lagrangian acceleration :** $\vec{a}_L = -\nabla p + \nu \nabla^2 \vec{u}$.
* 1020 Trajectories
* Decaying turbulence $\longrightarrow$ **need to make the statistics stationary** :

  Lagrangian quantities $L(t)$ are divided by their instantaneous standard deviation computed from all particles at each time : $L(t)/\sigma_L(t)$

  P.K. Yeung *Annu. Rev. Fluid Mech.* (2002)

(a) Periodic geometry

(b) Circular geometry

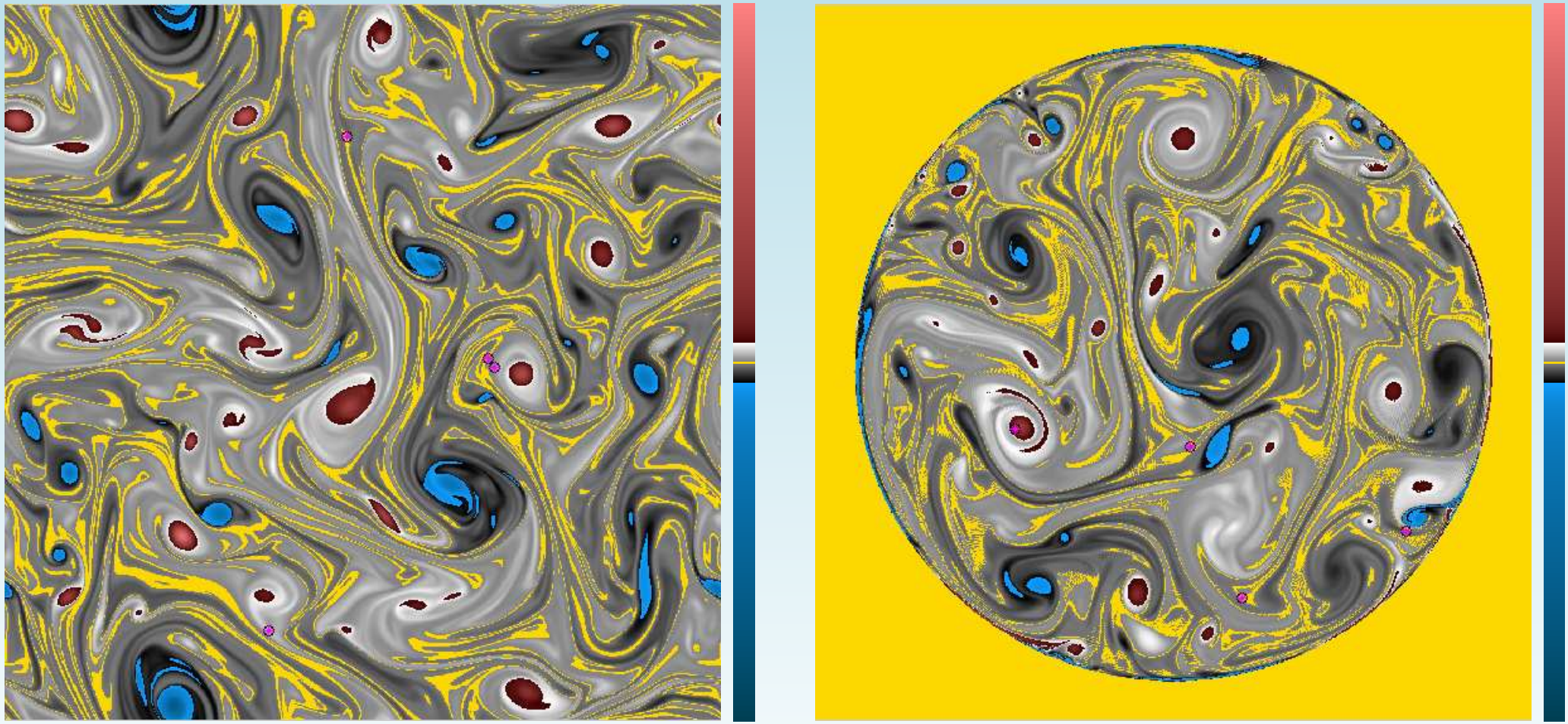


FIG. 1: Snapshots of vorticity fields.

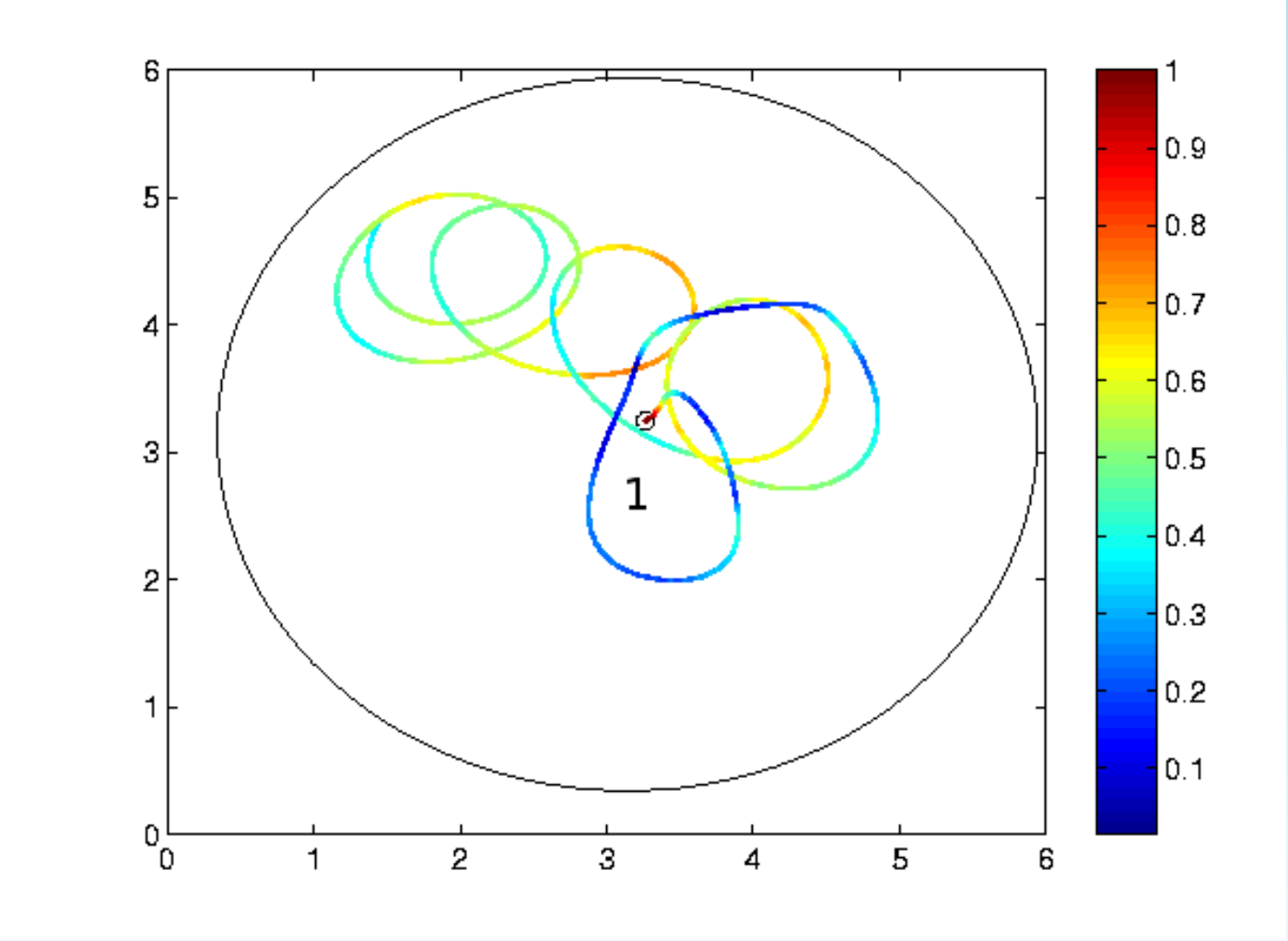


FIG. 2: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

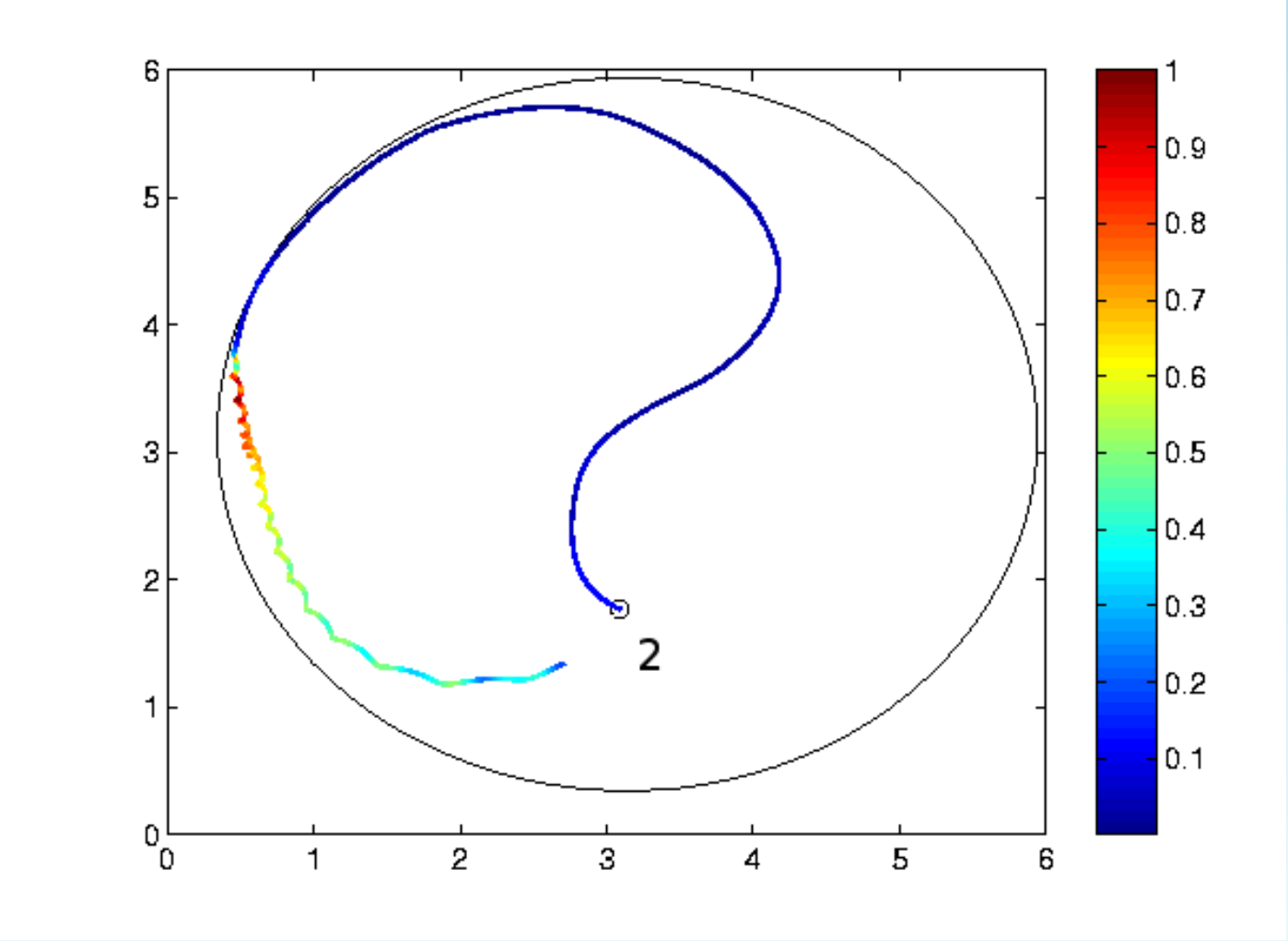


FIG. 3: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

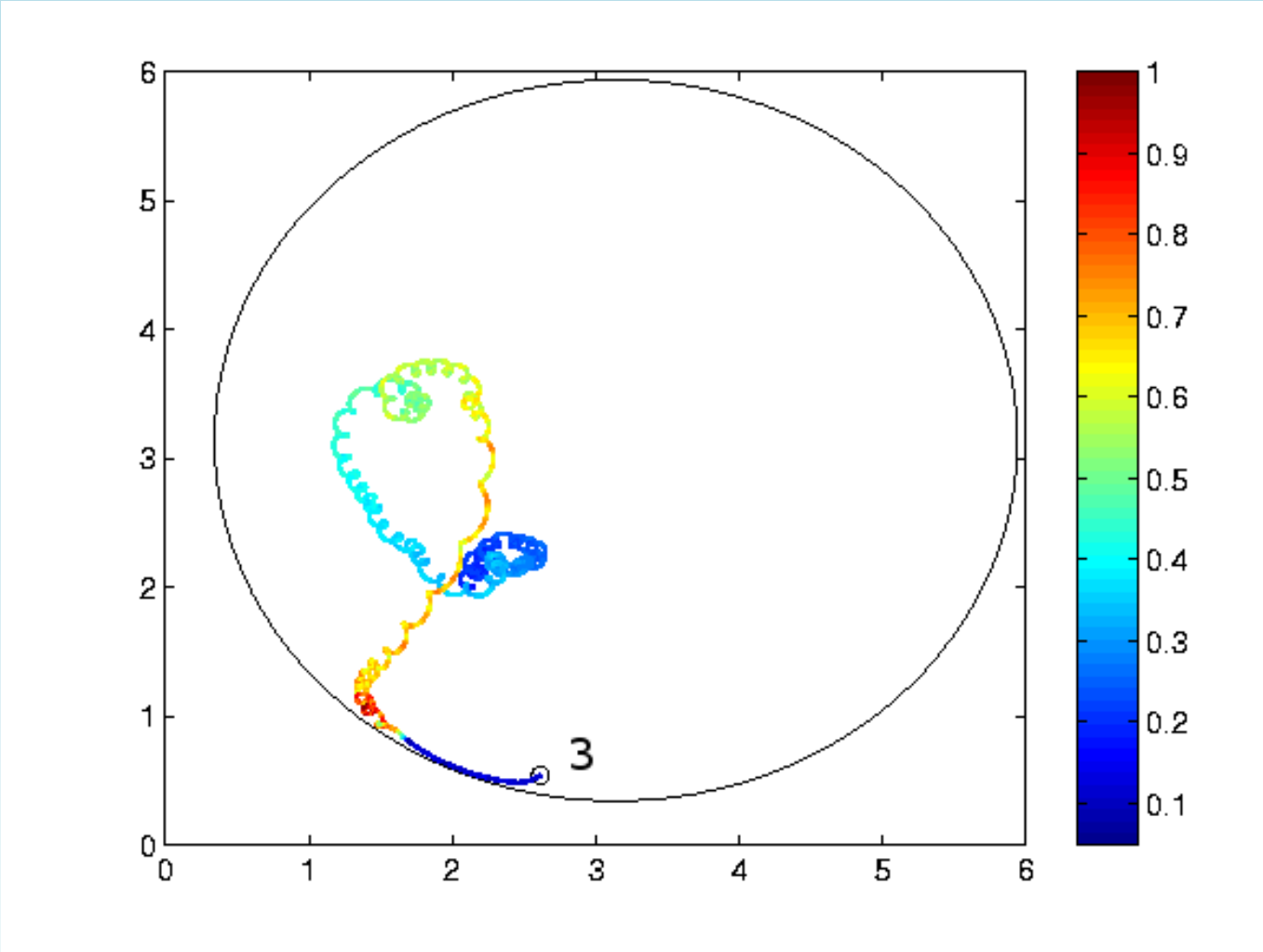


FIG. 4: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

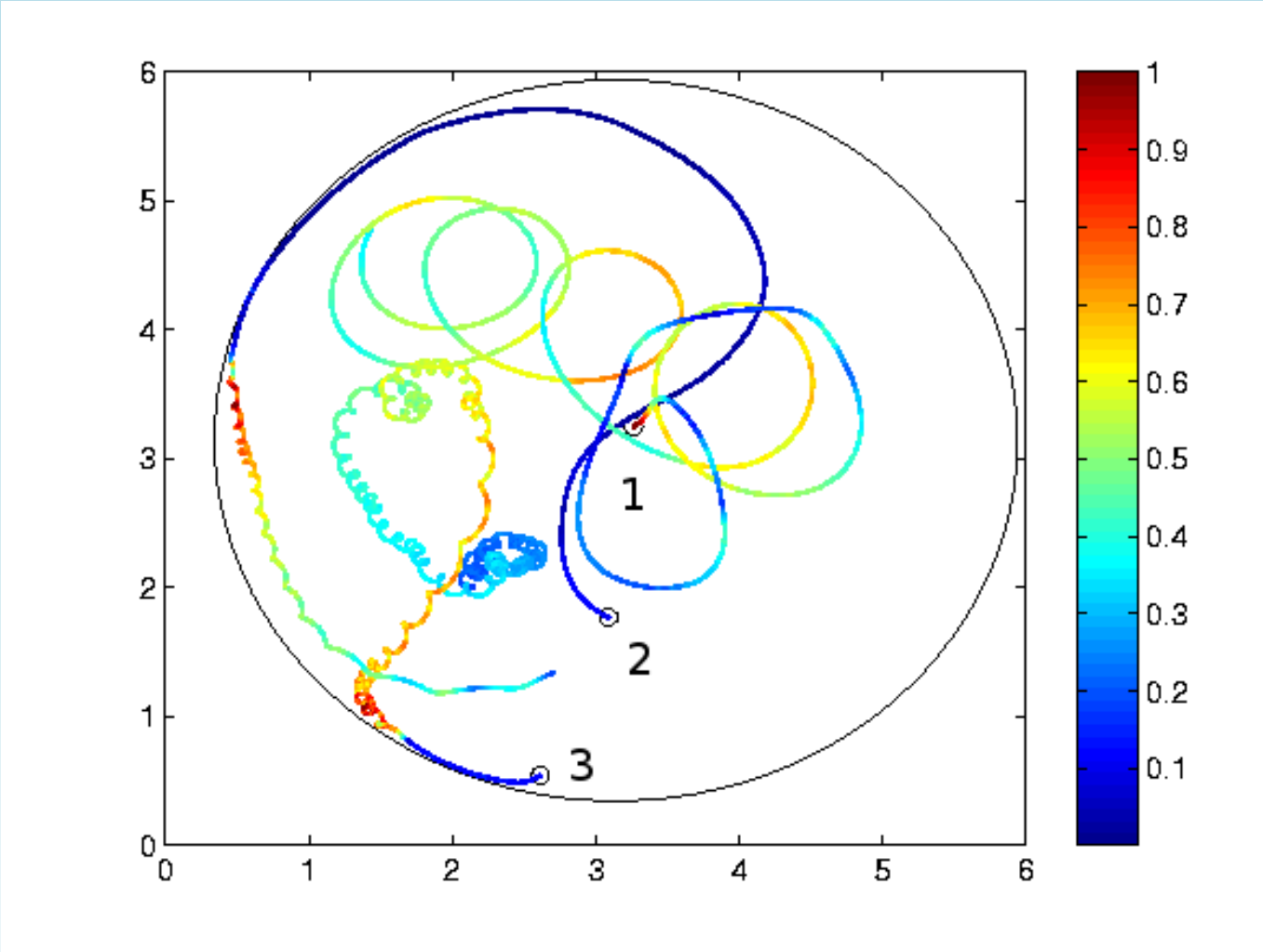


FIG. 5: Trajectory colored with $|\vec{a}_L(t)|/max(|\vec{a}_L(t)|)$, where $max|\vec{a}_1| = 3.6$, $max|\vec{a}_2| = 11.7$ and $max|\vec{a}_3| = 33.3$ for the particles 1, 2 and 3, respectively.

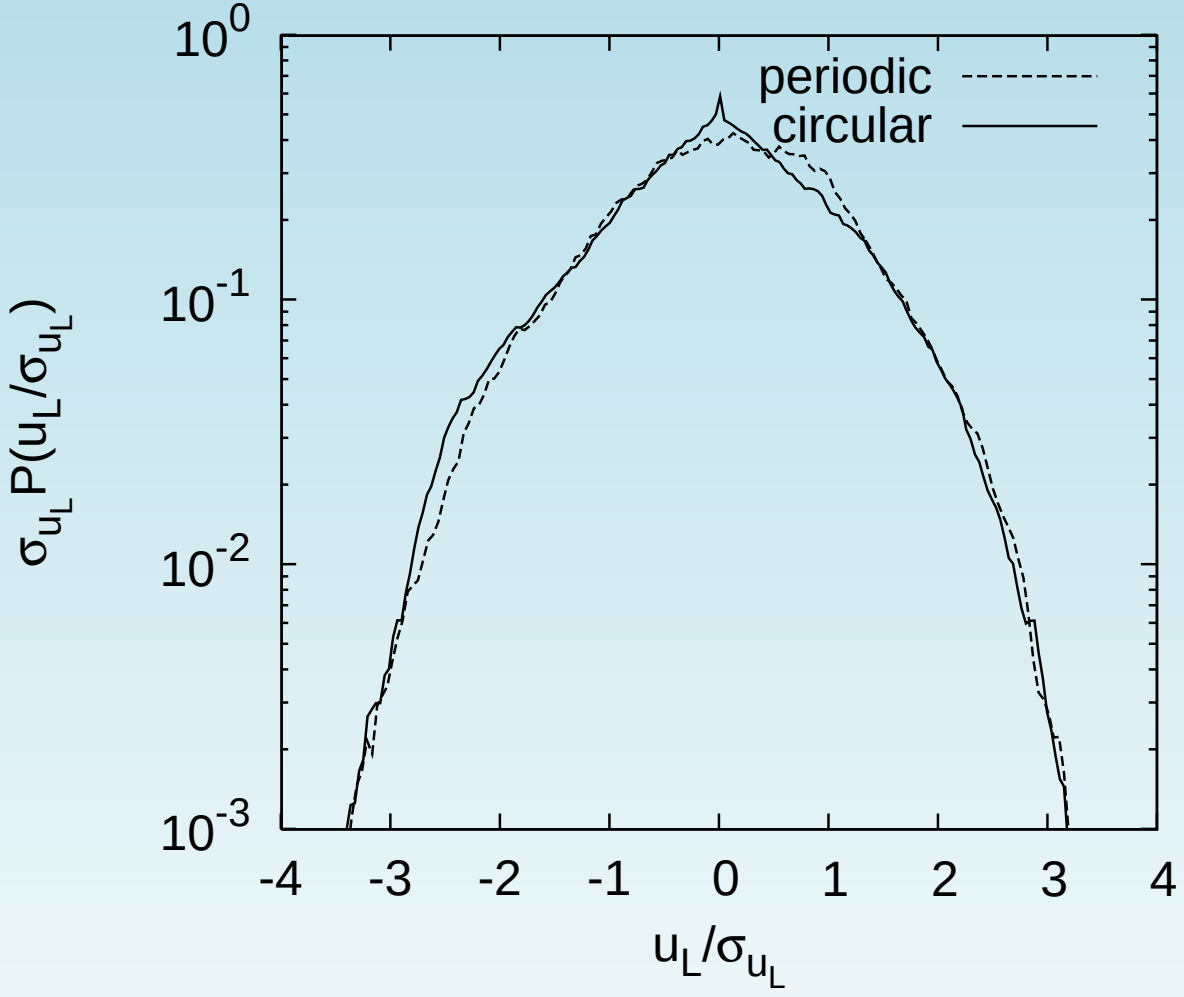


FIG. 6: PDFs of normalized Lagrangian velocities u_L/σ_{u_L} where $\sigma_{u_L} = \langle u_L^2 \rangle^{1/2}$ ($\langle \cdot \rangle$ denotes the ensemble average), for the periodic geometry and for the circular geometry.

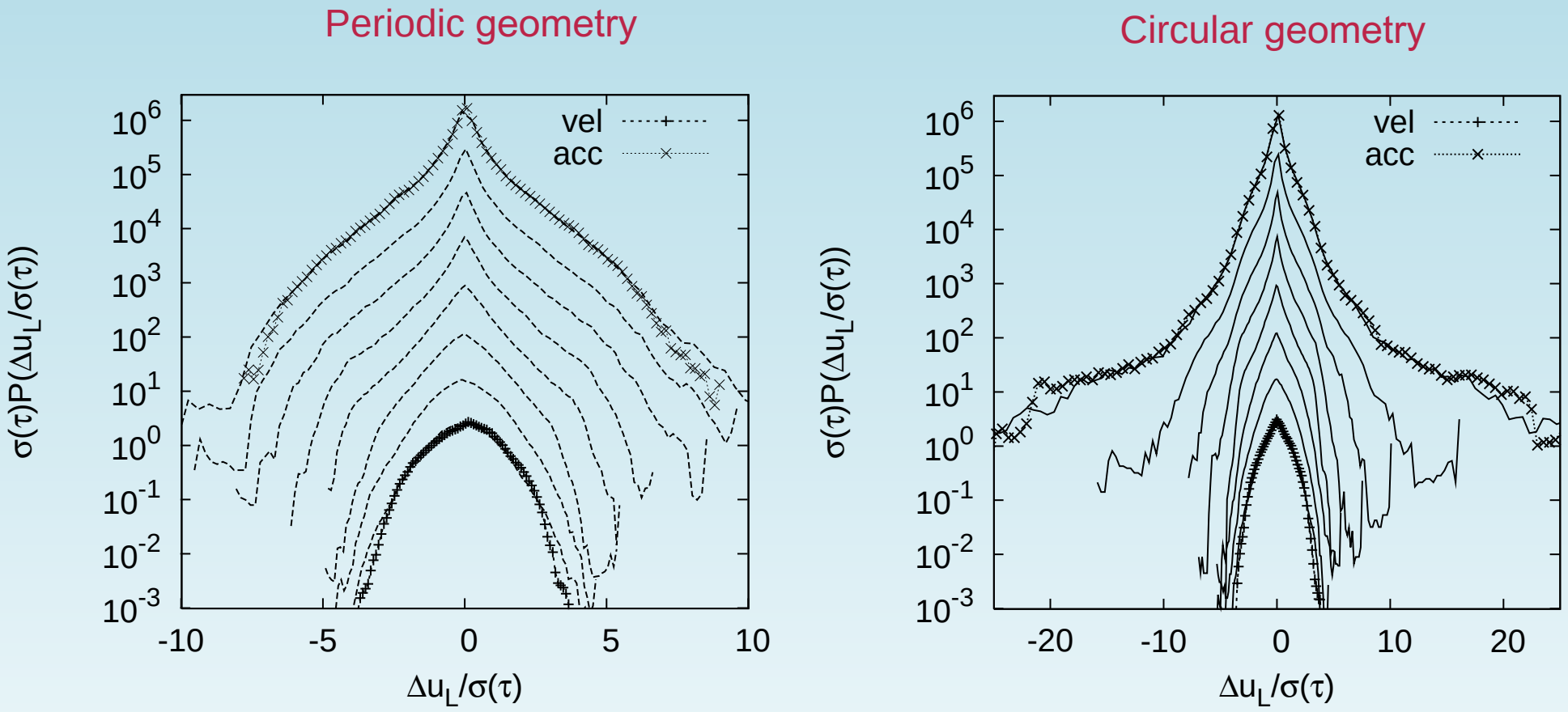


FIG. 7: PDFs of normalized Lagrangian velocity increments $\Delta u_L(\tau)/\sigma(\tau)$ where $\sigma(\tau) = \langle (\Delta u_L(\tau))^2 \rangle^{1/2}$, for periodic (left) and circular geometry (right).

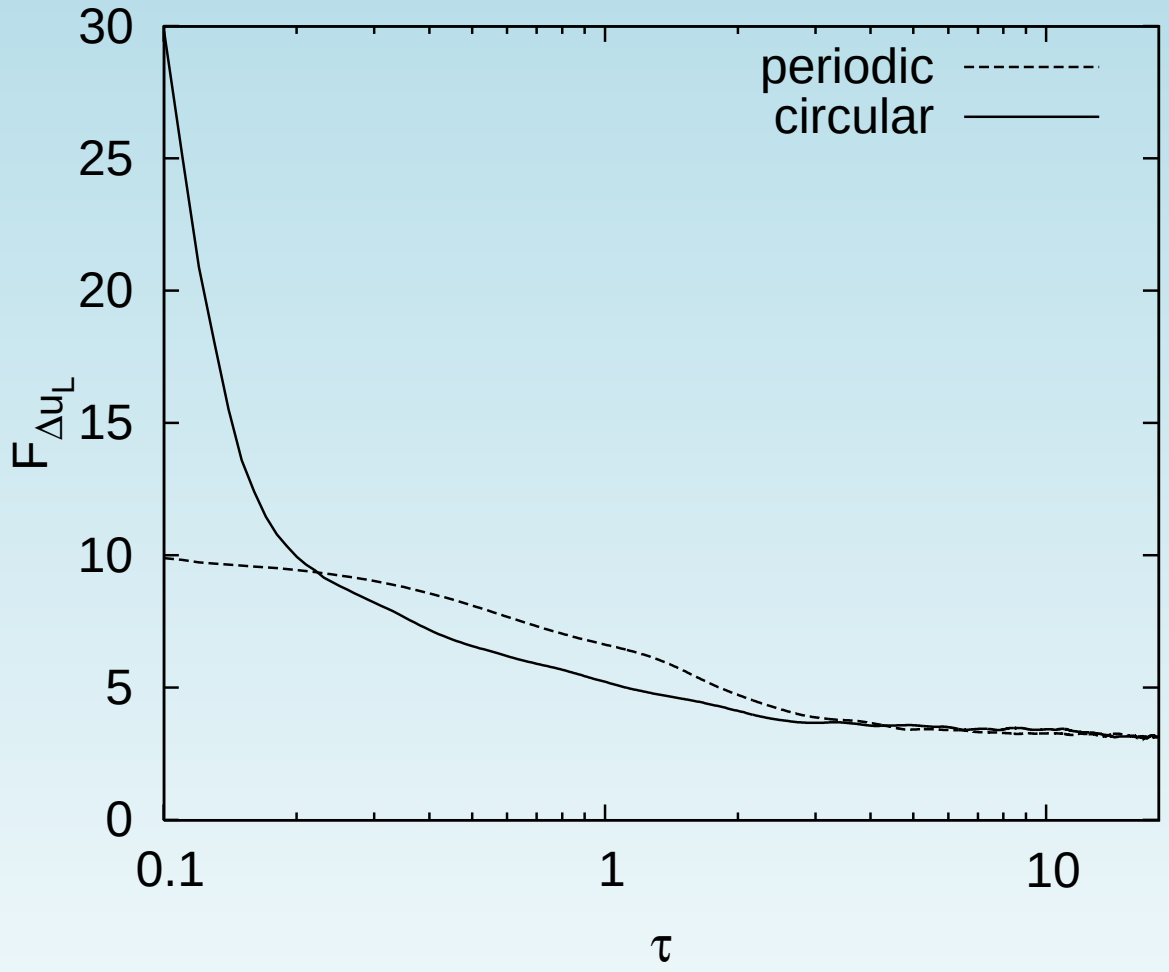


FIG. 8: Flatness of the Lagrangian velocity increments as a function of τ for the periodic and circular geometry.

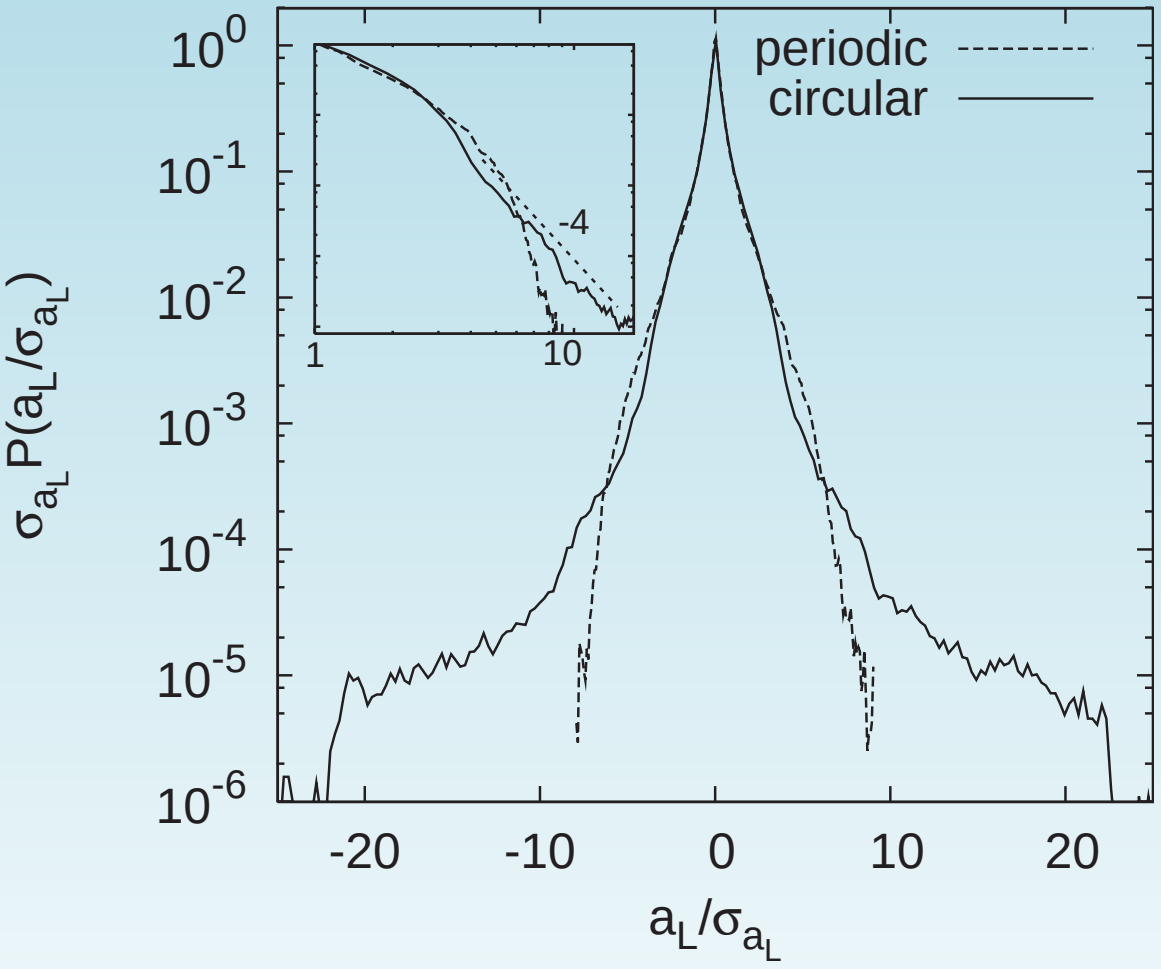


FIG. 7: PDFs of the normalized Lagrangian acceleration a_L/σ_{a_L} where $\sigma_{a_L} = \langle a_L^2 \rangle^{1/2}$ for both cases. Inset : PDFs of the normalized Lagrangian acceleration in double logarithmic scale

Conditional statistics

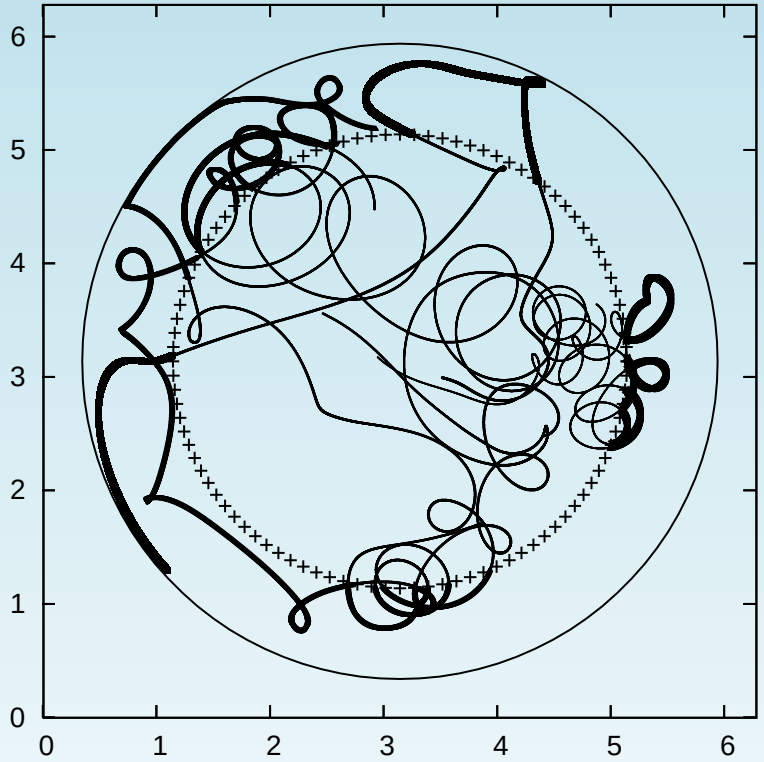


FIG. 8: Trajectories in the circular geometry. The trajectories are divided into particles inside and outside the disk defined by the radius r_0 (circle in dotted line).

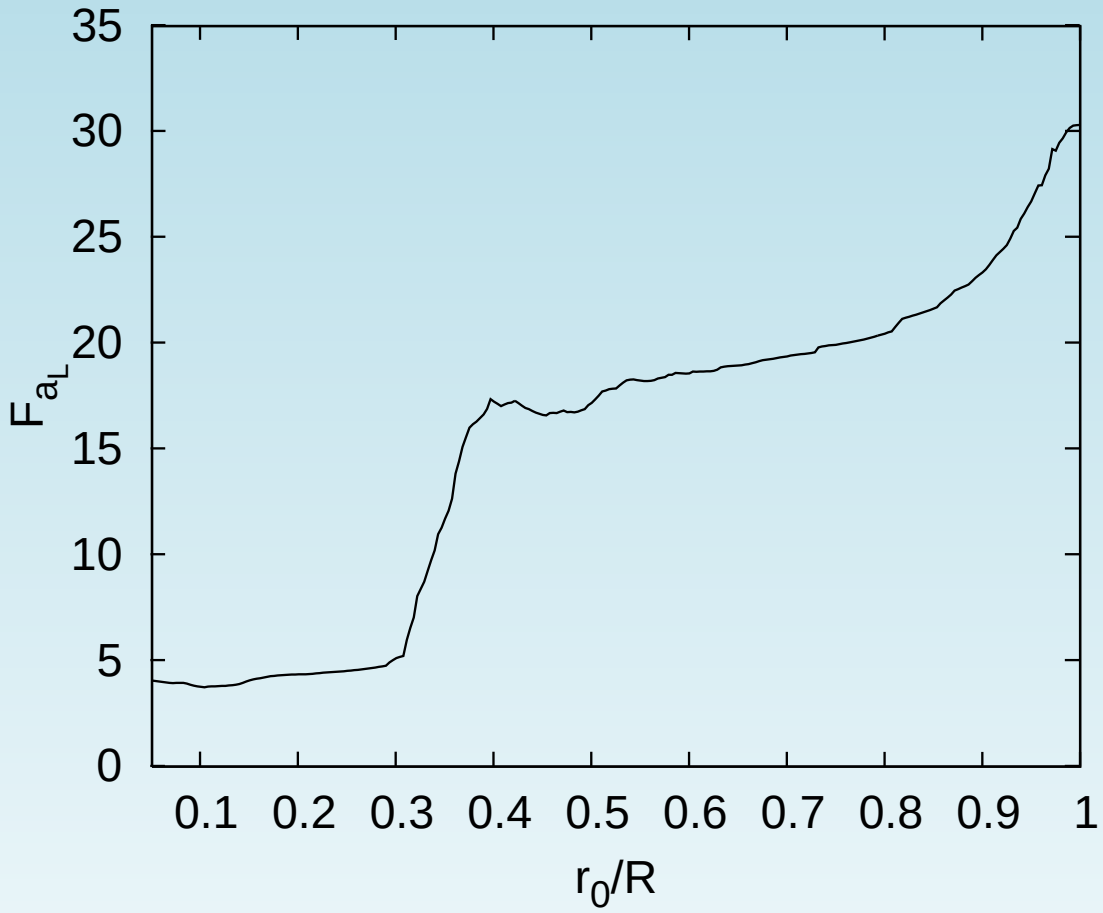


FIG. 9: Conditional flatness of the Lagrangian acceleration as a function of radius r_0/R.

$\longrightarrow$ *Lagrangian boundary layer thickness* δ_L defined by a **critical radius** $r_0/R = 0.3$.

Conclusions

* Influence of no-slip boundaries on Lagrangian velocity and acceleration :
  - **no significant influence on Lagrangian velocity** except the small cusp around zero in the PDF.
  - **heavy tails in the Lagrangian acceleration PDF** $\longrightarrow$ extreme values .
* Conditional statistics :
  - presence of a ***Lagrangian boundary layer thickness***.
  - Influence of the wall in approximatively 90% of the domain surface.
* B. Kadoch , W.J.T. Bos , K. Schneider *Phys. Rev. Lett.* 100, 2008.

Future work

- Influence of Reynolds number
- Comparison with Eulerian quantities

  Eulerian acceleration Vs Lagrangian acceleration
- 3D $\longrightarrow$ careful reassessment in experimental results

  influence of the wall with conditional statistics in experience ?