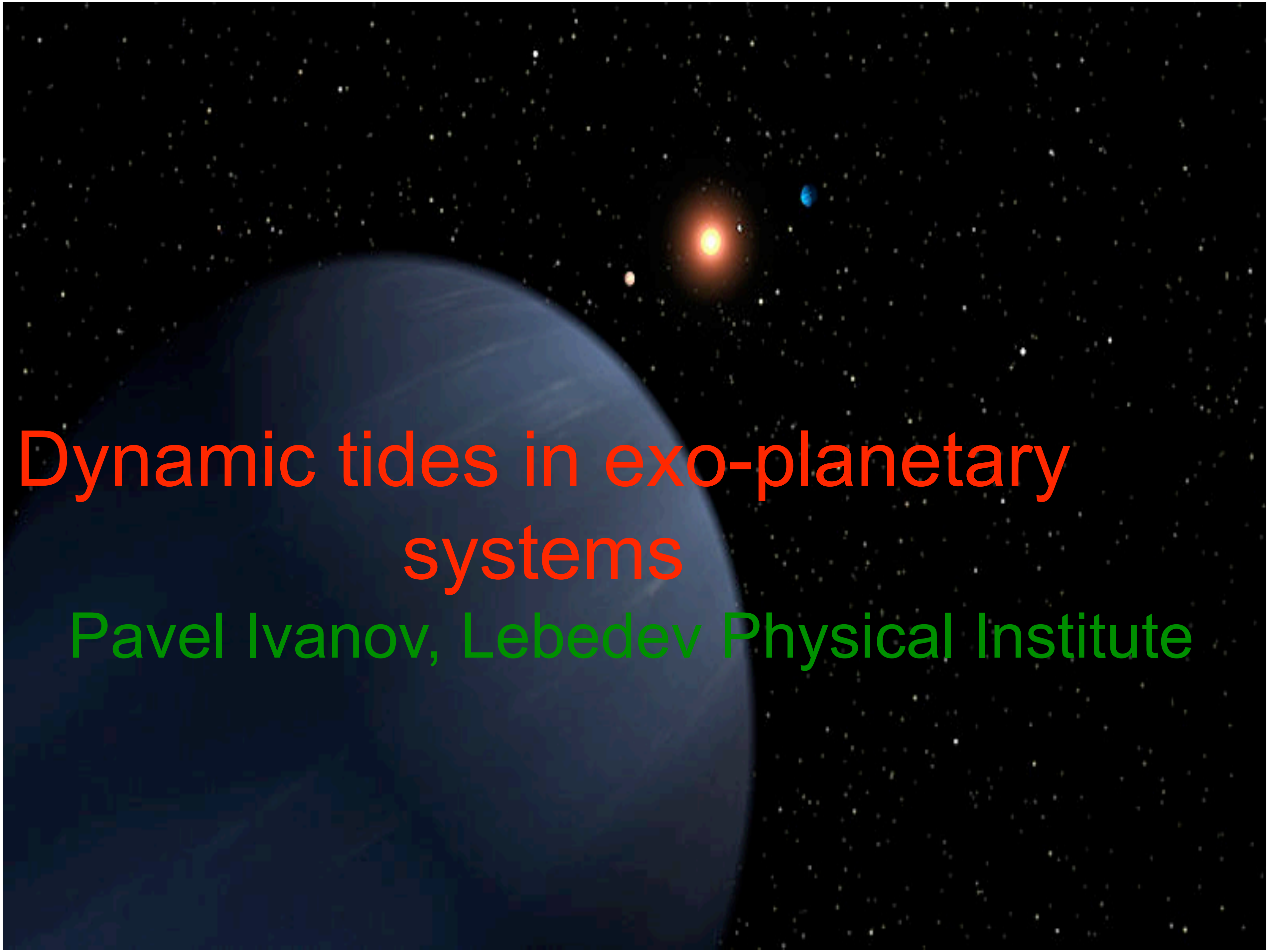

Dynamic tides in exo-planetary systems

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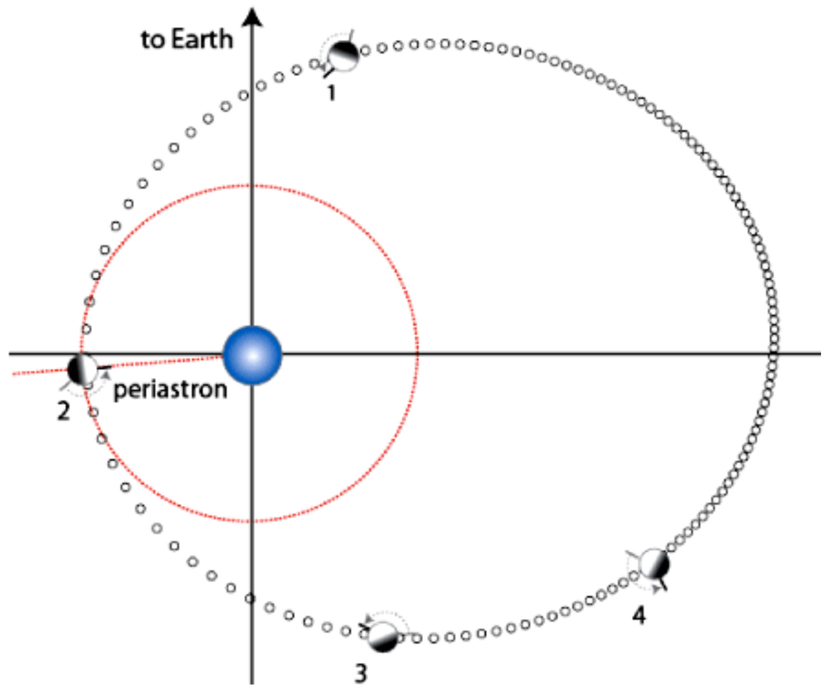
Basic definitions

e – orbital eccentricity, r_p – periastron distance, M_* - planet's mass, M – mass of the star. There are three important frequencies: Ω – angular velocity of the planet,



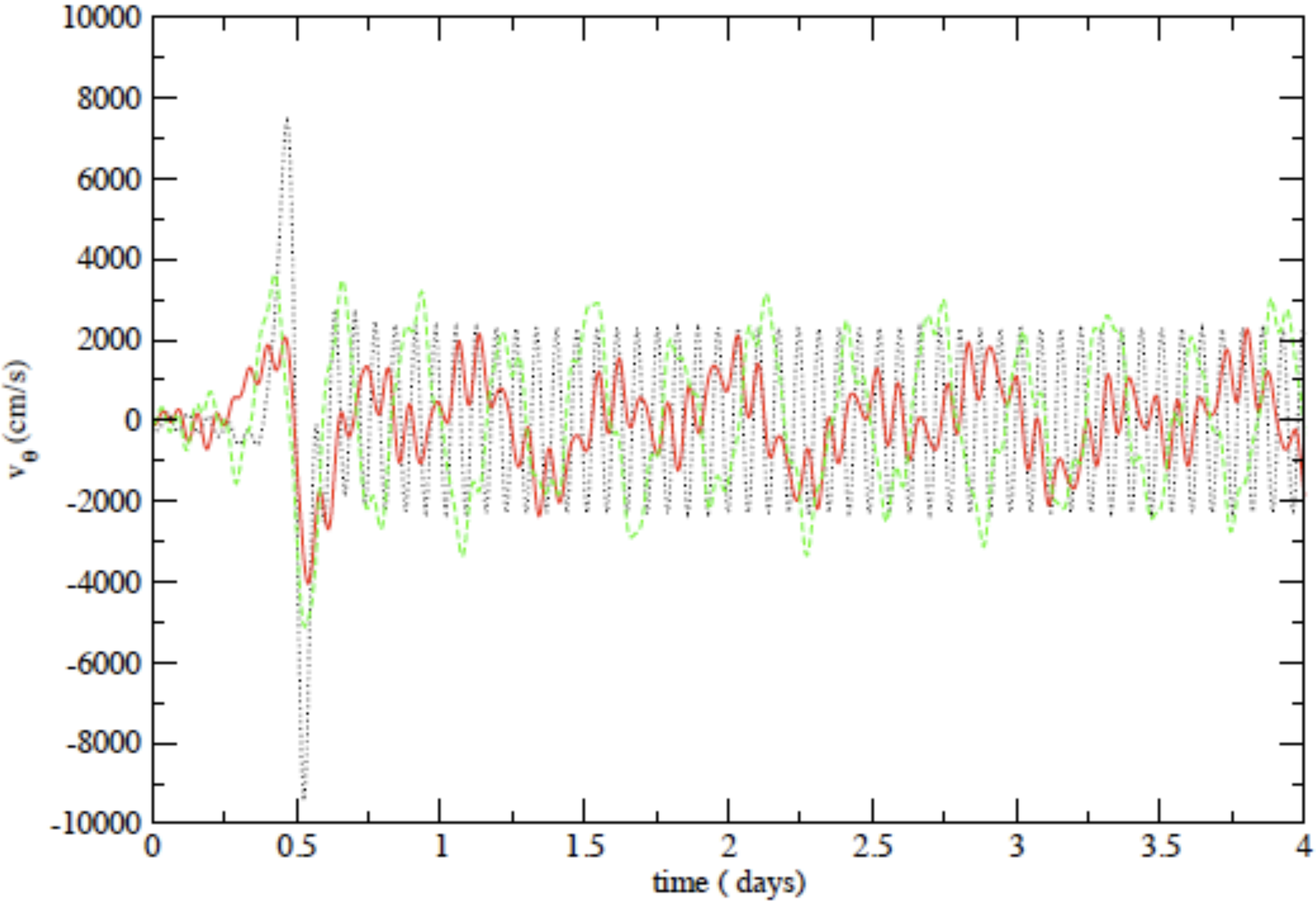
$$\Omega_{\rm p} \sim \sqrt{\frac{GM}{r_{\rm p}^3}} \quad \Omega_* = \sqrt{\frac{GM_*}{R_*^3}} : \Omega_{\rm p} \ll \Omega_*$$

$$\eta = \Omega_*/\Omega_{\rm p} = \sqrt{\frac{1}{(1+q)} \frac{M_*}{M} \frac{r_{\rm p}^3}{R_*^3}}.$$

Dynamic and quasi-static tides

- In principal there are two contributions due to tidal interactions, which can be easily separated in the case of an eccentric orbit:
- 1) quasi-static tides. The energy and angular
- momentum are transferred between the orbit
- and the planet on a time characteristic scale
- Ω_p^{-1} by viscous forces. The main uncertainty
- - the value of “tidal Q” defined as the ratio
- of energy stored in tidal bulge to the amount
- of energy dissipated per forcing period.

- 2) Dynamic tides. During the periastron passage oscillations are excited at frequencies corresponding to planet’s normal
- mode frequencies. These are: fundamental
- modes with $\sigma \sim \Omega_*$. Inertial waves with
- $\sigma \sim \Omega << \Omega_*$. In case of stably stratified planets (non considered) g-modes.
- Quasi-static tides are important for orbits with
- a moderate eccentricity. Dynamic tides are
- More important for orbits with $1-e <<1$.



10000
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
v_θ (cm/s)
0
0.5
1
1.5
2
2.5
3
3.5
4
time (days)

First passage problem

- Let us consider a highly eccentric (technically, parabolic) orbit. Assume that a rotating planet is not perturbed before the periastron passage.

- What would be its energy and angular momentum gain after the periastron passage?

Contribution of f-modes

- Since the f-modes have their frequencies
- of order of $\sim \Omega_*$, one can use perturbation theory in small parameter
- Ω/Ω_* . For non-rotating planet the results
- were obtained by Press and Teukolsky in
- 1997. The slowly rotating case was considered in Lai 1997 and Ivanov & Papaloizou 2004.
-

- The amount of energy transferred is shown below. It is expressed in units of $E_*=GM_*^2/R_*$.
- Q_f ~0.5 are the so-called “tidal overlap integrals”.
- The frequencies of normal modes, etc. are expressed in units of ~ Ω_*. Transfer of energy is maximal for non-rotating planet and minimal for the planet in state of “pseudo-synchronisation” with

$$\Omega_{\rm ps}^{\rm f} = \frac{3}{4\sqrt{2}\beta\eta}\ln\left(\sqrt{\frac{2}{3}}8\sigma_{\rm f}\eta\right).$$

$$E_{\rm f}^{\rm max} \approx \frac{16\sqrt{2}}{15}\pi^2\bar{\sigma}_{\rm f}^3 Q_{\rm f}^2\eta {\rm e}^{-\frac{4\sqrt{2}}{3}\bar{\sigma}_{\rm f}\eta}E_* \quad E_{\rm f}^{\rm min} \approx \frac{\pi^2}{5\sqrt{2}}\frac{Q_{\rm f}^2}{\eta}{\rm e}^{-\frac{4\sqrt{2}}{3}\bar{\sigma}_{\rm f}\eta}E_*.$$

Contribution of inertial waves

- The inertial waves have their eigen-frequencies of order of $\sim \Omega \ll \Omega_*$. For
- them the perturbation theory exploiting the fact that rotation is small is not possible. There is, however, an approach, which allows one to deal with
- inertial waves in a way, similar to that of
- pulsations of a non-rotating star (Papaloizou & Ivanov 2005, Ivanov & Papaloizou 2007.)

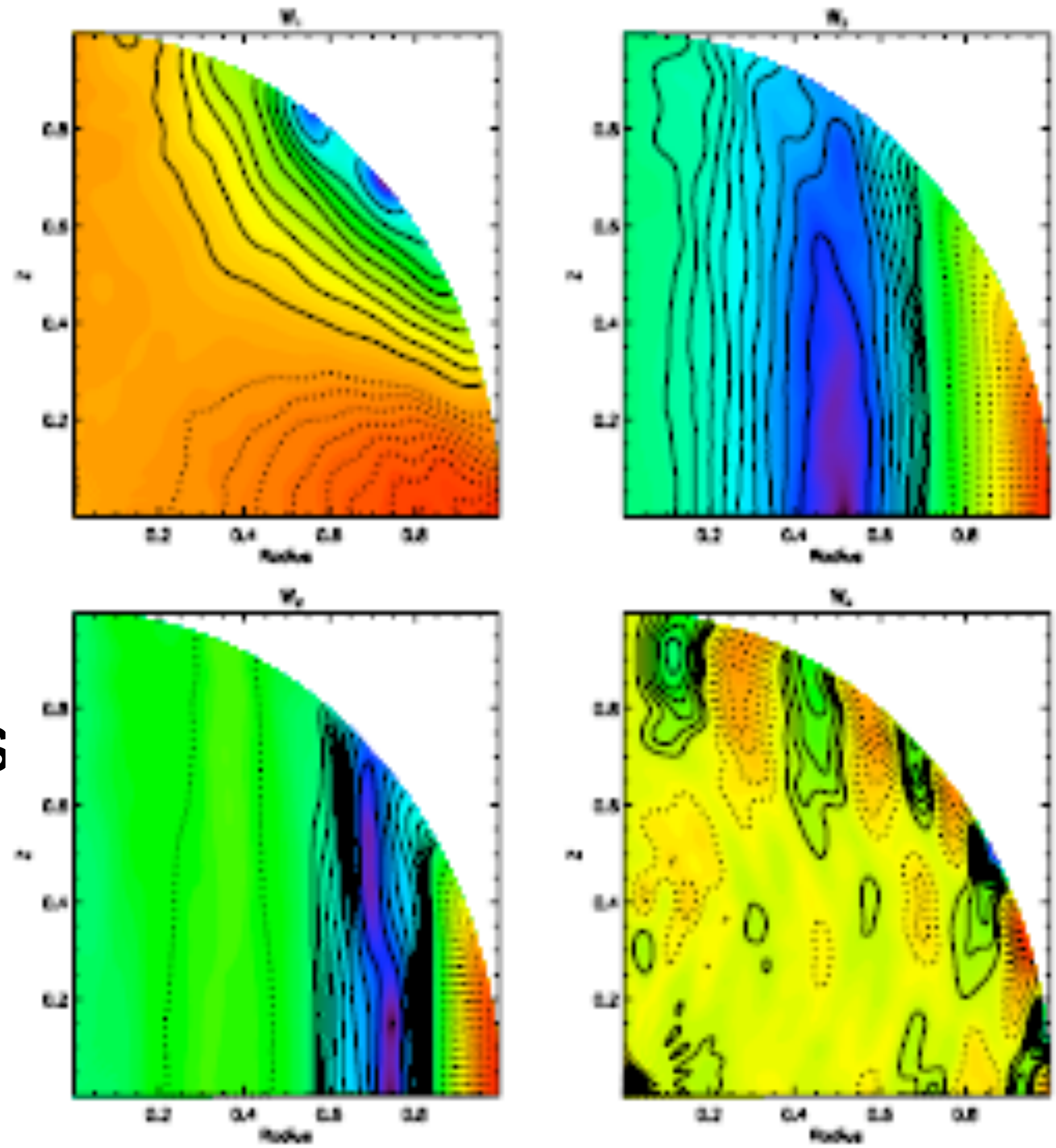
- In the leading order in small parameter
- $(\Omega/\Omega_*)^2$ it can be shown that the quantities

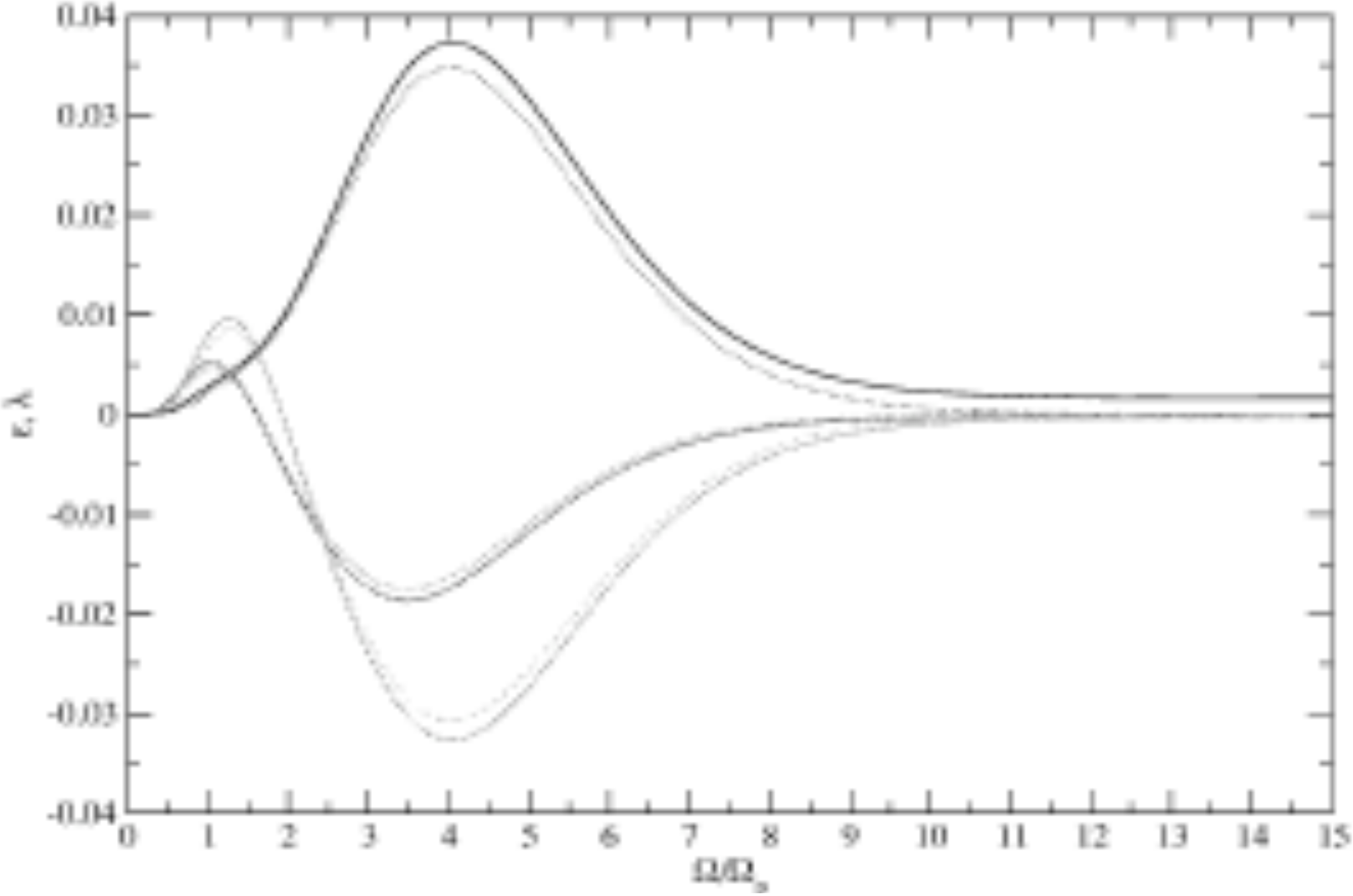
$$\epsilon_m = (1+q)^2\eta^6(\Delta E_m/E_*), \quad \lambda = (1+q)^2\eta^5(\Delta L_2/L_*)$$

where $E_*=GM_*^2/R_*$ and $L_* = M_*\sqrt{GM_*R_*}$, depend only on ratio Ω/Ω_p. Note that the energy transfer is given in the rotating frame. In the inertial frame the energy transfer is

$$E_1 = E + \Omega L.$$

- The quantities ε and λ contain summation series over an infinite number of inertial
- eigen modes. However, only two
- main global modes mainly determine energy/angular momentum transfer.





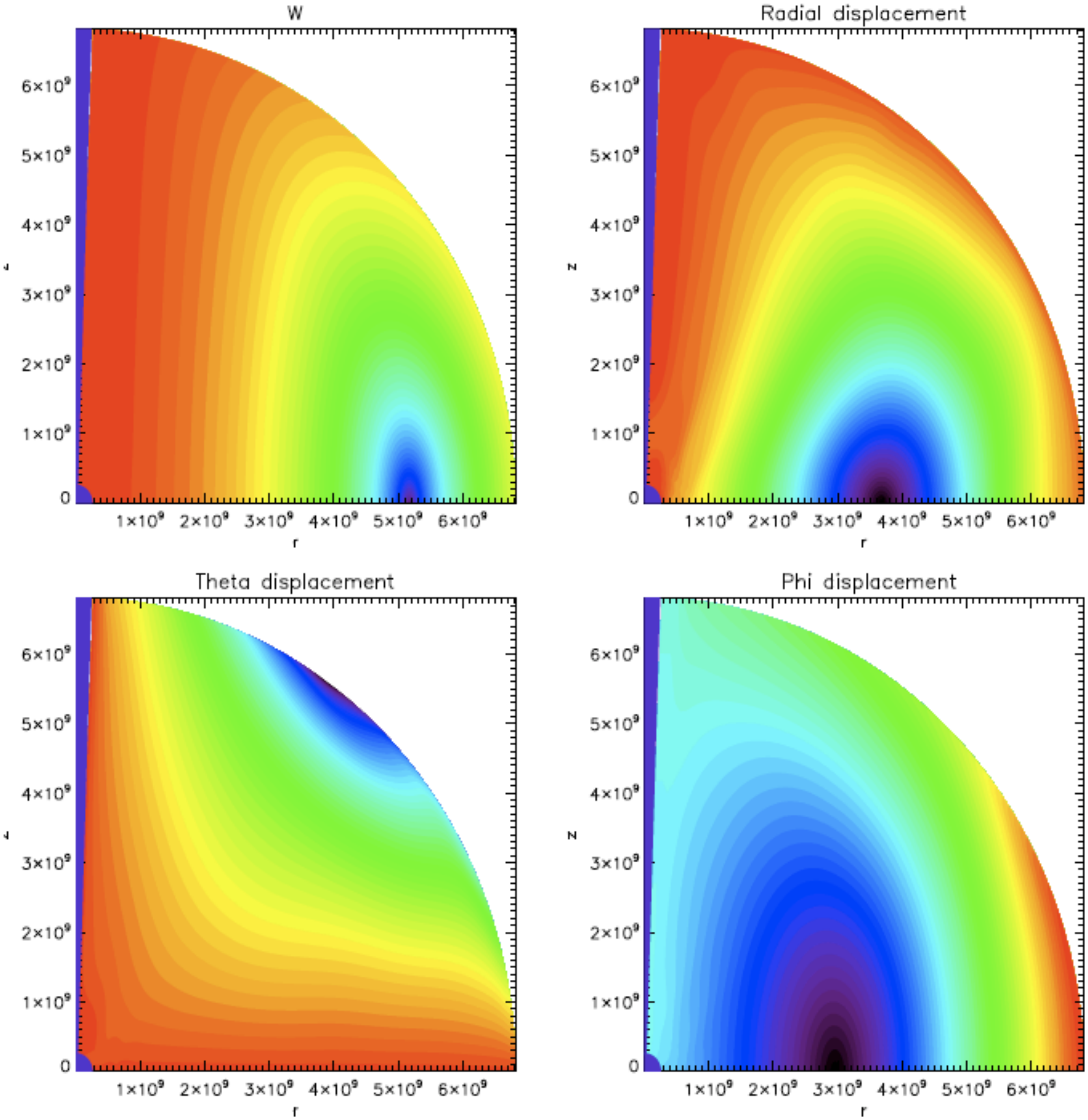
$$\Delta E_2 \approx \frac{0.0065}{(1+q)^2\eta^6} E_*.$$

Table 4. ϵ_* and $\hat{\Omega}_{eq}$ for the planet models with $M_* = M_J$.

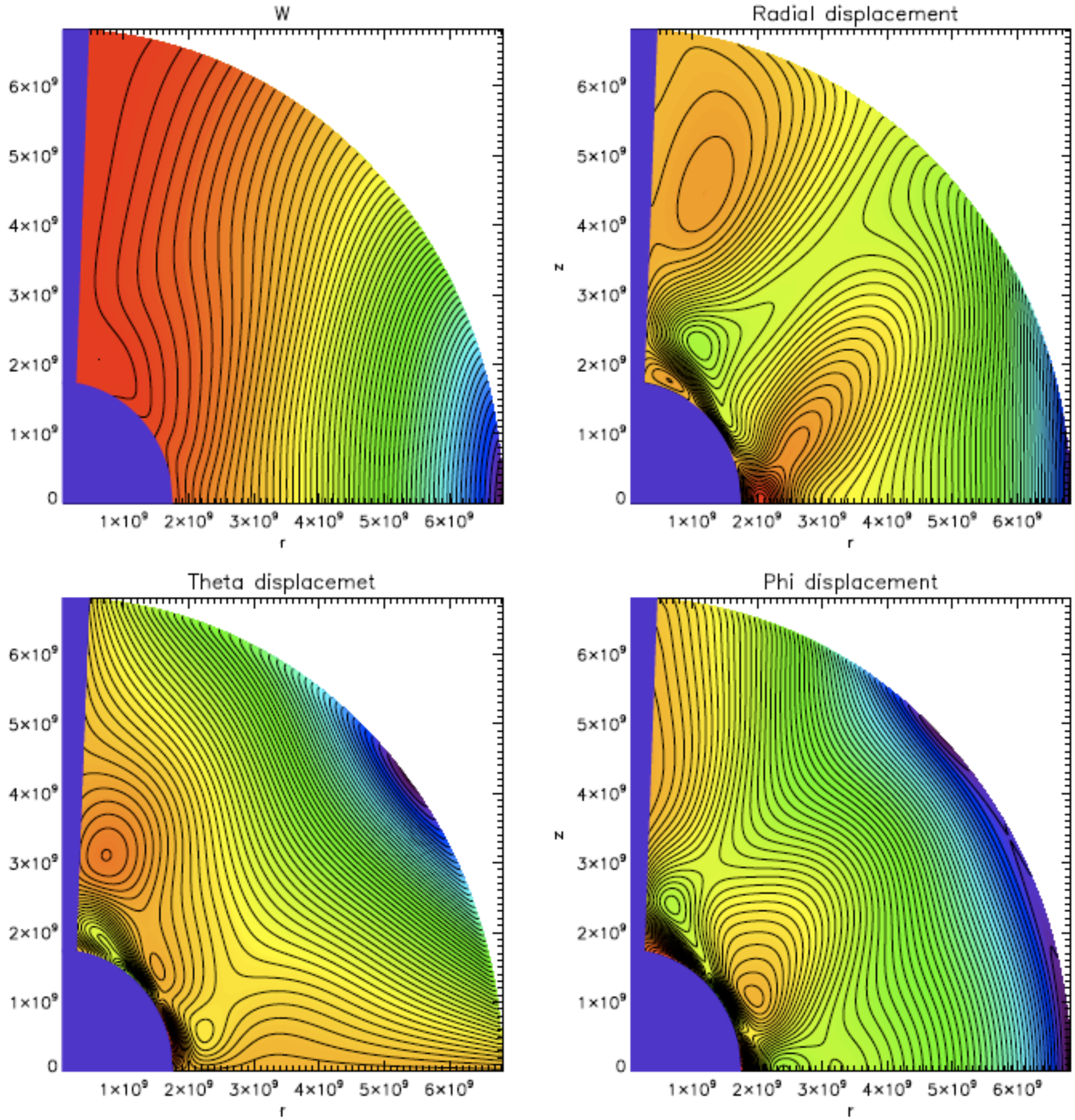
R_*/R_J	1	1.2	1.4	1.6	1.8	2
ϵ_*	5.3×10^{-3}	3.6×10^{-3}	3.45×10^{-3}	3.6×10^{-3}	3.7×10^{-3}	3.6×10^{-3}
$\hat{\Omega}_{eq}$	1.46	1.52	1.56	1.59	1.6	1.61

Table 5. ϵ_* and $\hat{\Omega}_{eq}$ for the planet models with $M_* = 5M_J$.

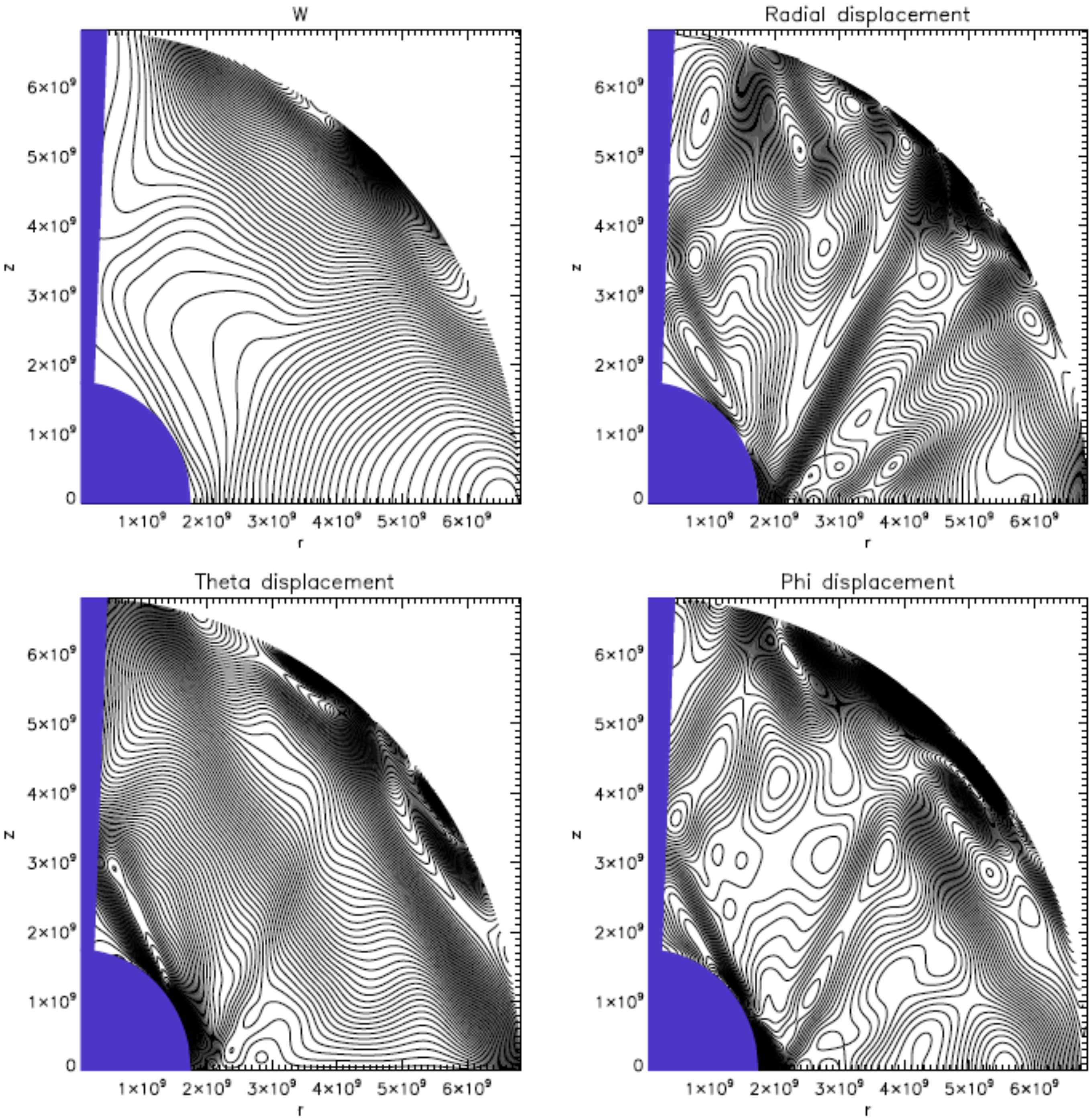
R_*/R_J	1.03	1.2	1.4	1.6	1.8	2
ϵ_*	3.6×10^{-3}	3.6×10^{-3}	4.2×10^{-3}	3.7×10^{-3}	3.55×10^{-3}	3.4×10^{-3}
$\hat{\Omega}_{eq}$	1.55	1.55	1.54	1.54	1.54	1.55



W
Radial displacement
Theta displacement
Phi displacement
z
r
0
1×10⁹
2×10⁹
3×10⁹
4×10⁹
5×10⁹
6×10⁹



W
Radial displacement
Theta displacemet
Phi displacement
r
z
0
1×10⁹
2×10⁹
3×10⁹
4×10⁹
5×10⁹
6×10⁹



W
Radial displacement
Theta displacement
Phi displacement
z
r
0
1×10⁹
2×10⁹
3×10⁹
4×10⁹
5×10⁹
6×10⁹

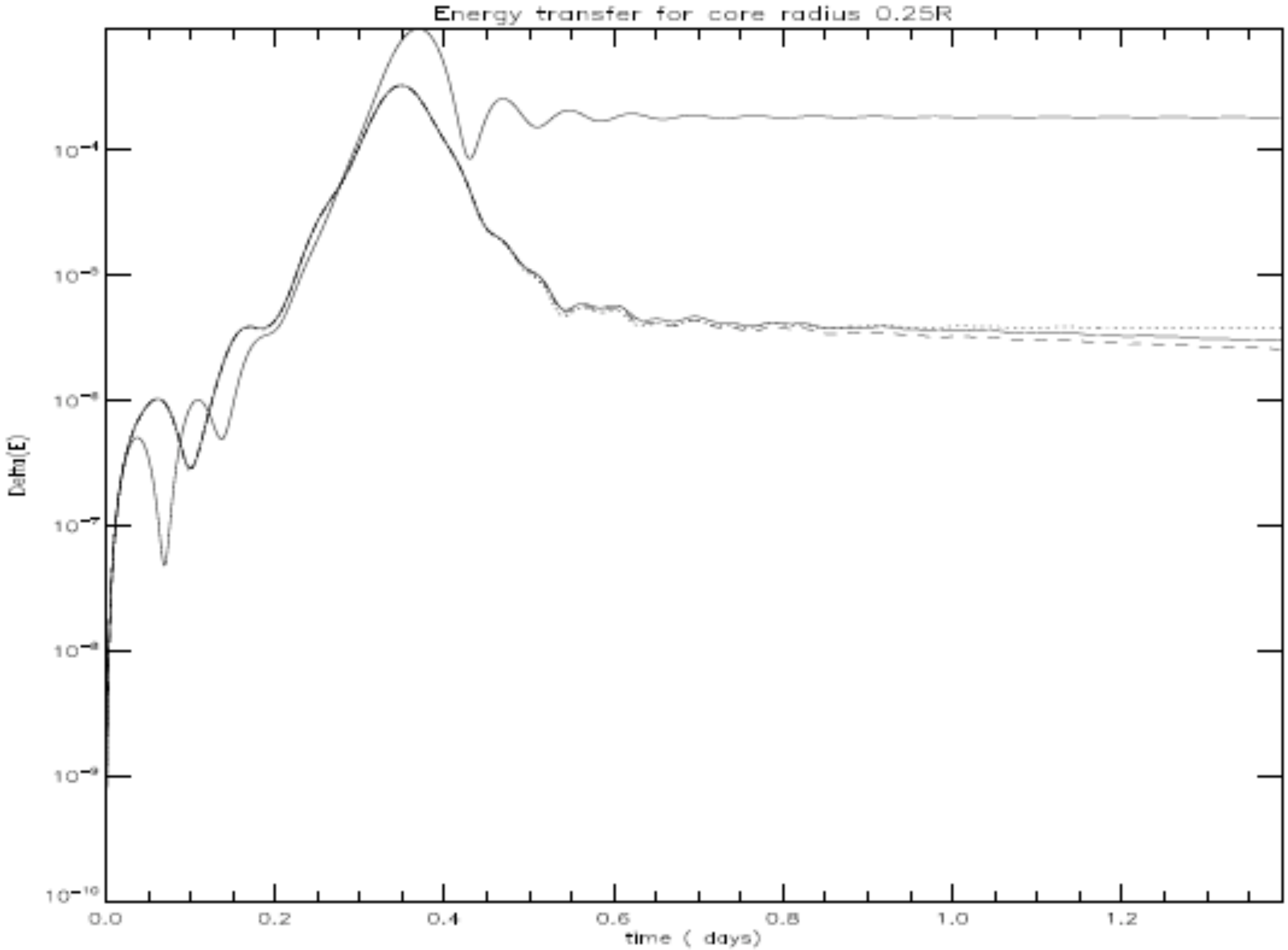


Figure 7. The energy transferred to the planet as a function of time due to the $m = 2$ tide is plotted for a model with a solid core of radius $0.25R_*$. The encounter is for $\eta = 4$ and $\Omega/\Omega_* = 0.8$. The curves from the lowermost to uppermost at large time are for a resolution of 200×200, a resolution of $400 * 400$, for comparison a model with no core and a model with no core and no rotation.

Inertial versus f-modes

- When η is small – f-modes win, in the
- opposite limit – inertial waves win.

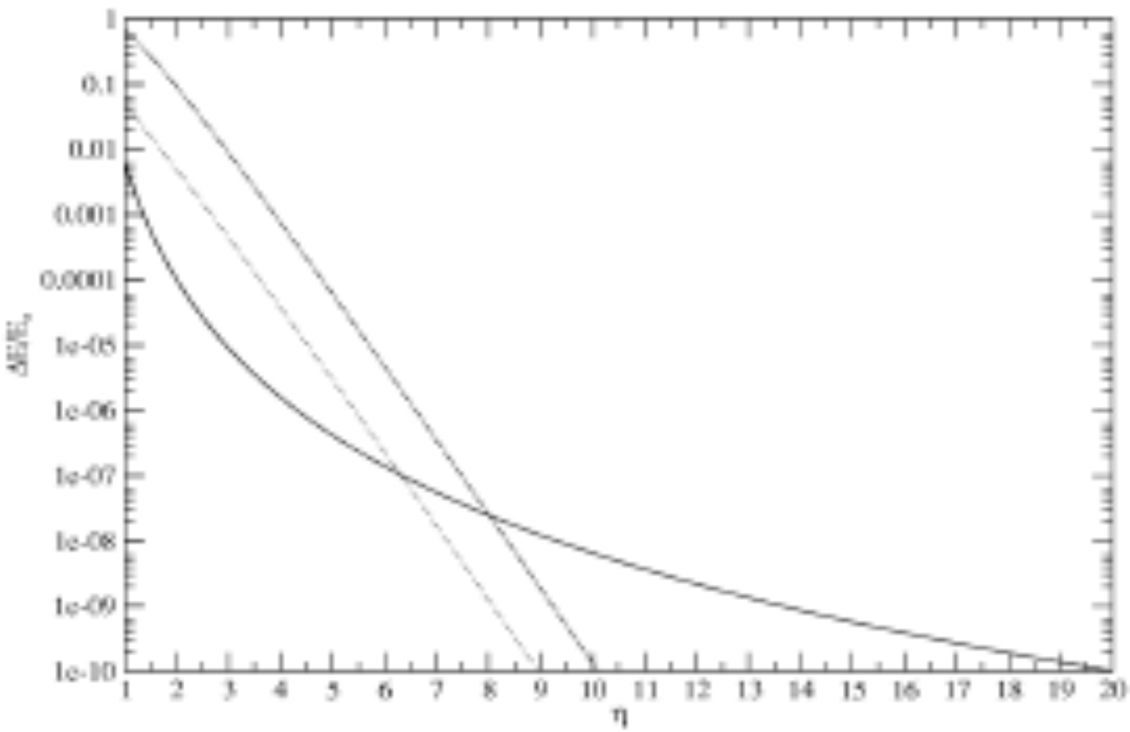
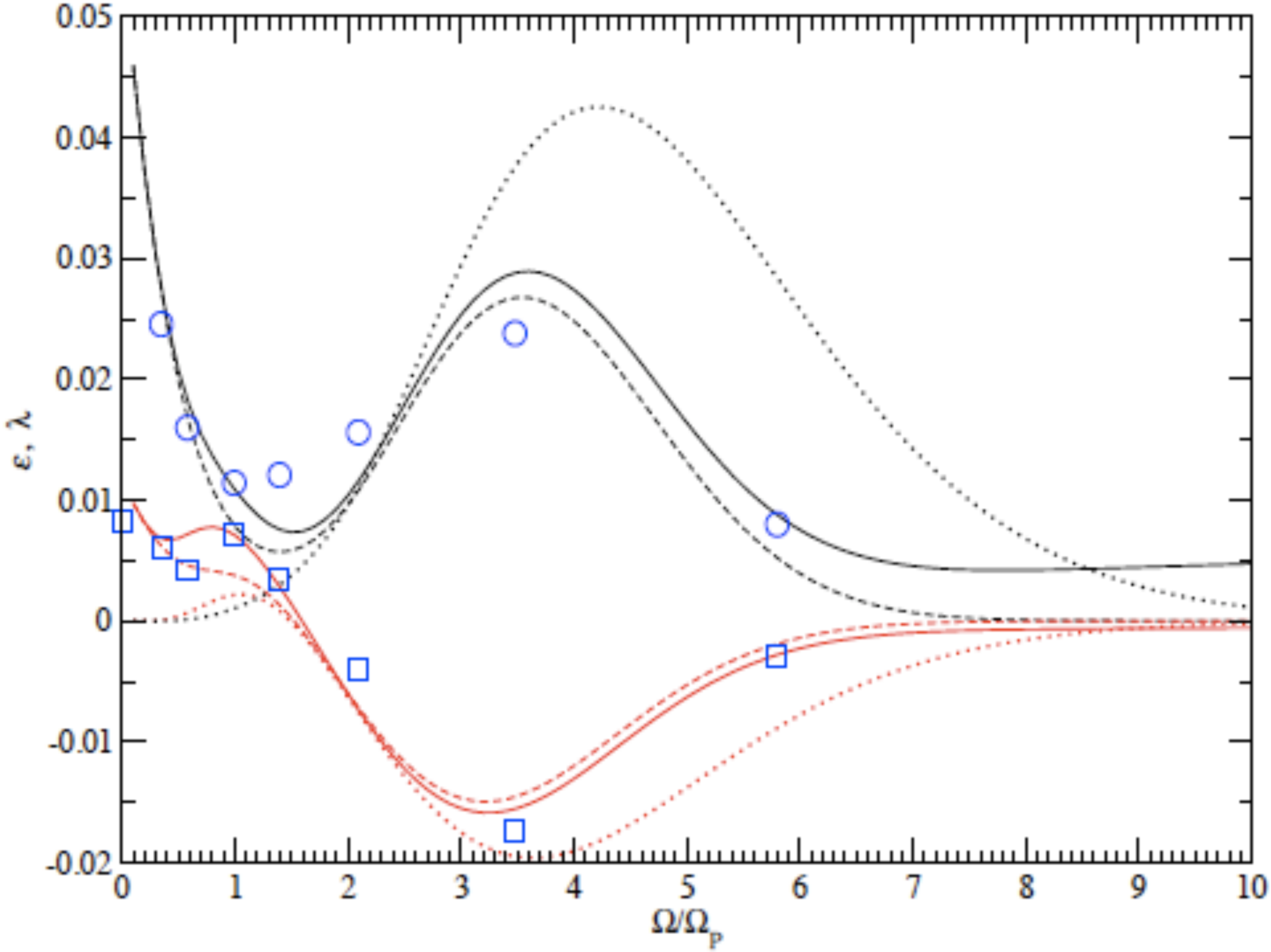
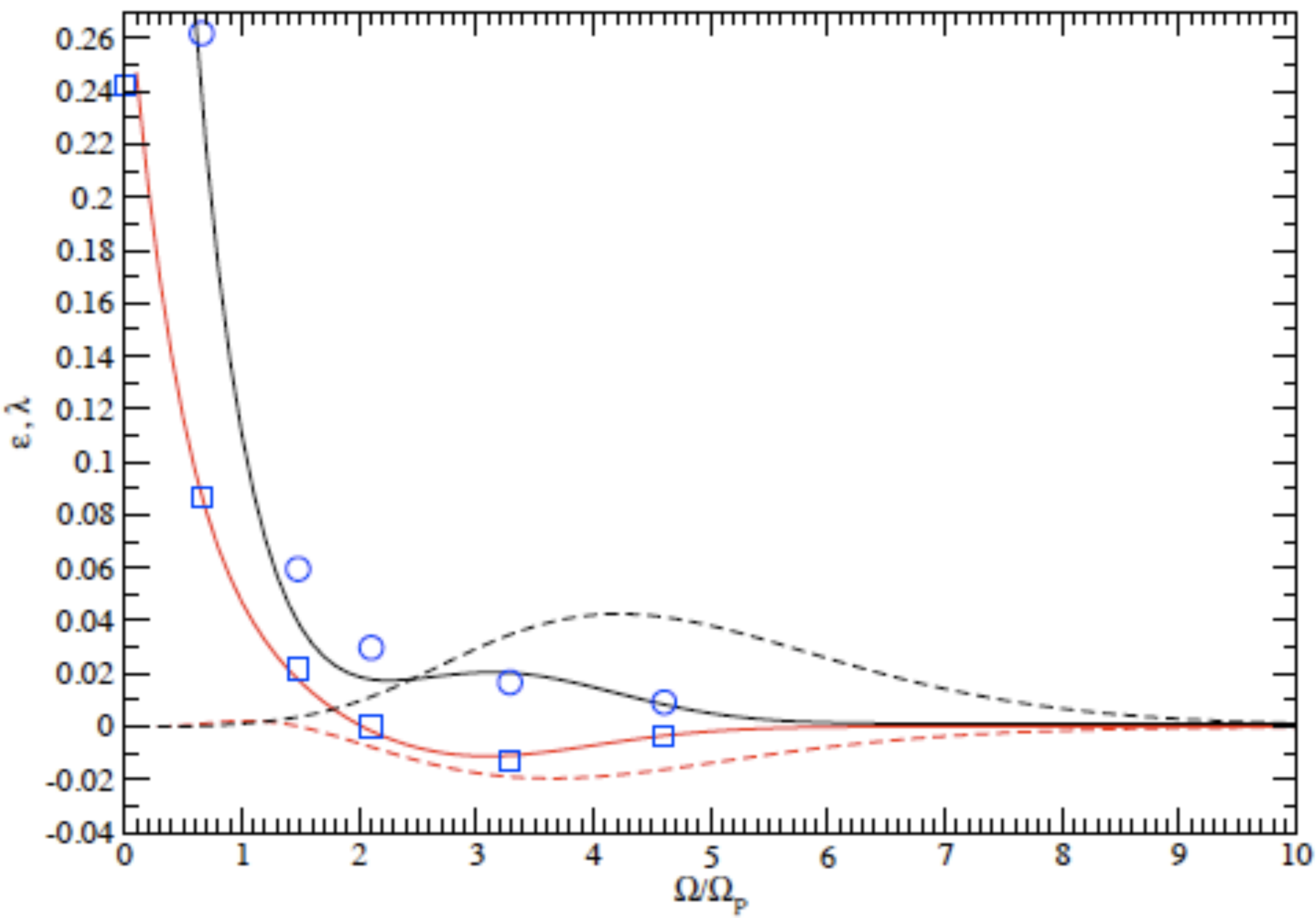


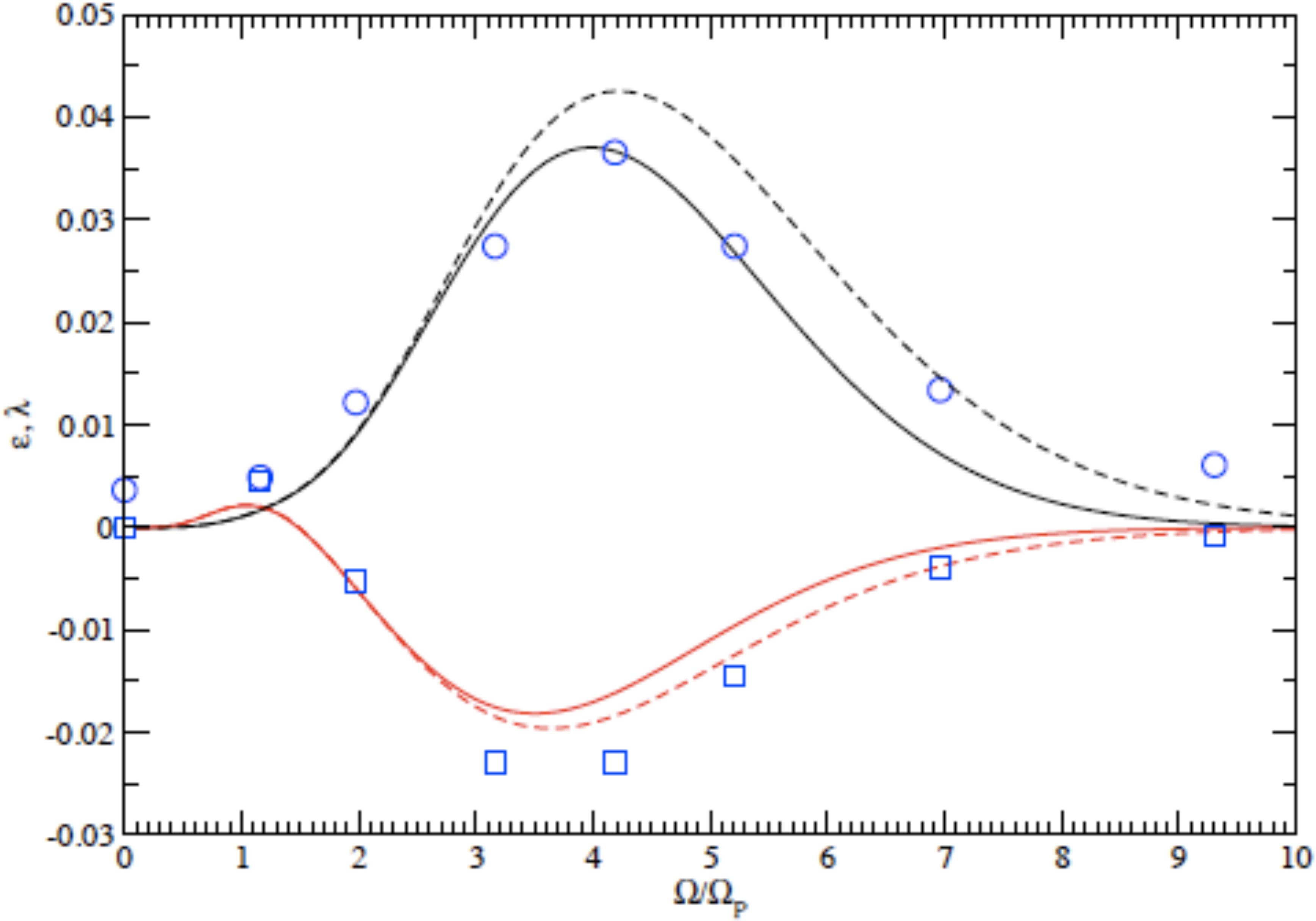
Figure 4. The dependence of the energy transfer on η. The black curve gives the contribution of the inertial waves; the red and green curves give the contribution of the fundamental mode. We assume that $q \ll 1$ and $\bar{\Omega} = \bar{\Omega}_{\rm crit} \approx 1.6$.



0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
ε, λ
0
1
2
3
4
5
6
7
8
9
10
Ω/Ω_P



0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
ε, λ
0
1
2
3
4
5
6
7
8
9
10
Ω/Ω_p



0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0
1
2
3
4
5
6
7
8
9
10
ε, λ
Ω/Ω_P

Multi-passage problem

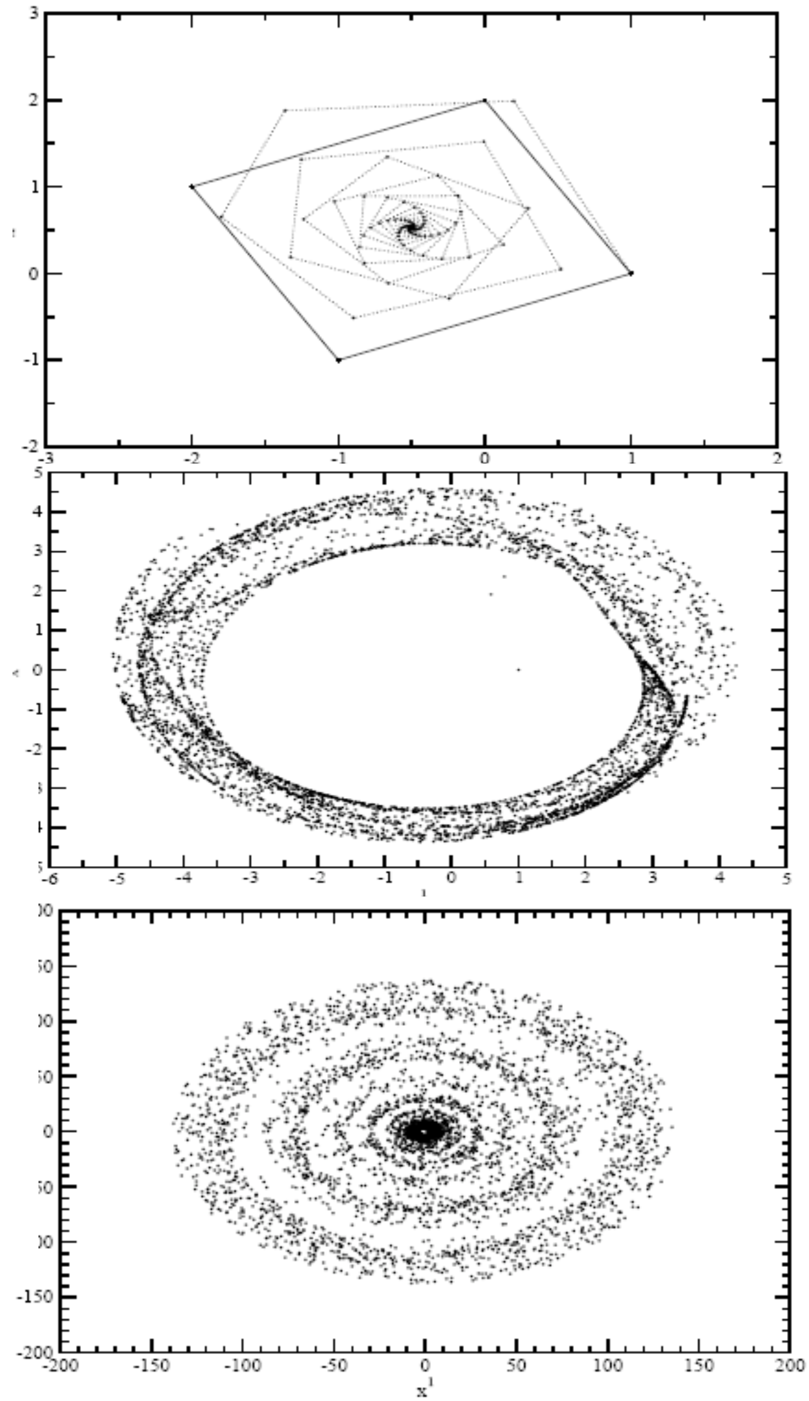
The planet can gain energy either when dissipation is significant or when the mode amplitudes and phases are evolving in stochastic regime (Kosovichev Novikov 1992, Kochanek 1992, Mardling, 1994).

This happens when $a > a_{st,}$ where

$$a_{st} = \left(\frac{\alpha_{crit} E_{pl}}{6\pi\tilde{\omega}_{00}(\Delta E_{ns} + \Delta E_{ns*})}\right)^{2/5} \left(\frac{M}{m_{pl}}\right)^{3/5} R_{pl}$$

$$\approx 30.8(\tilde{\omega}_{00}\epsilon_{ns})^{-2/5} \left(\frac{M_J}{m_{pl}}\right)^{3/5} \left(\frac{R_{pl}}{R_J}\right) Au,$$

$$\epsilon_{ps} = 10^9(\Delta E_{ps} + \Delta E_{ns*})/E_{pl}.$$



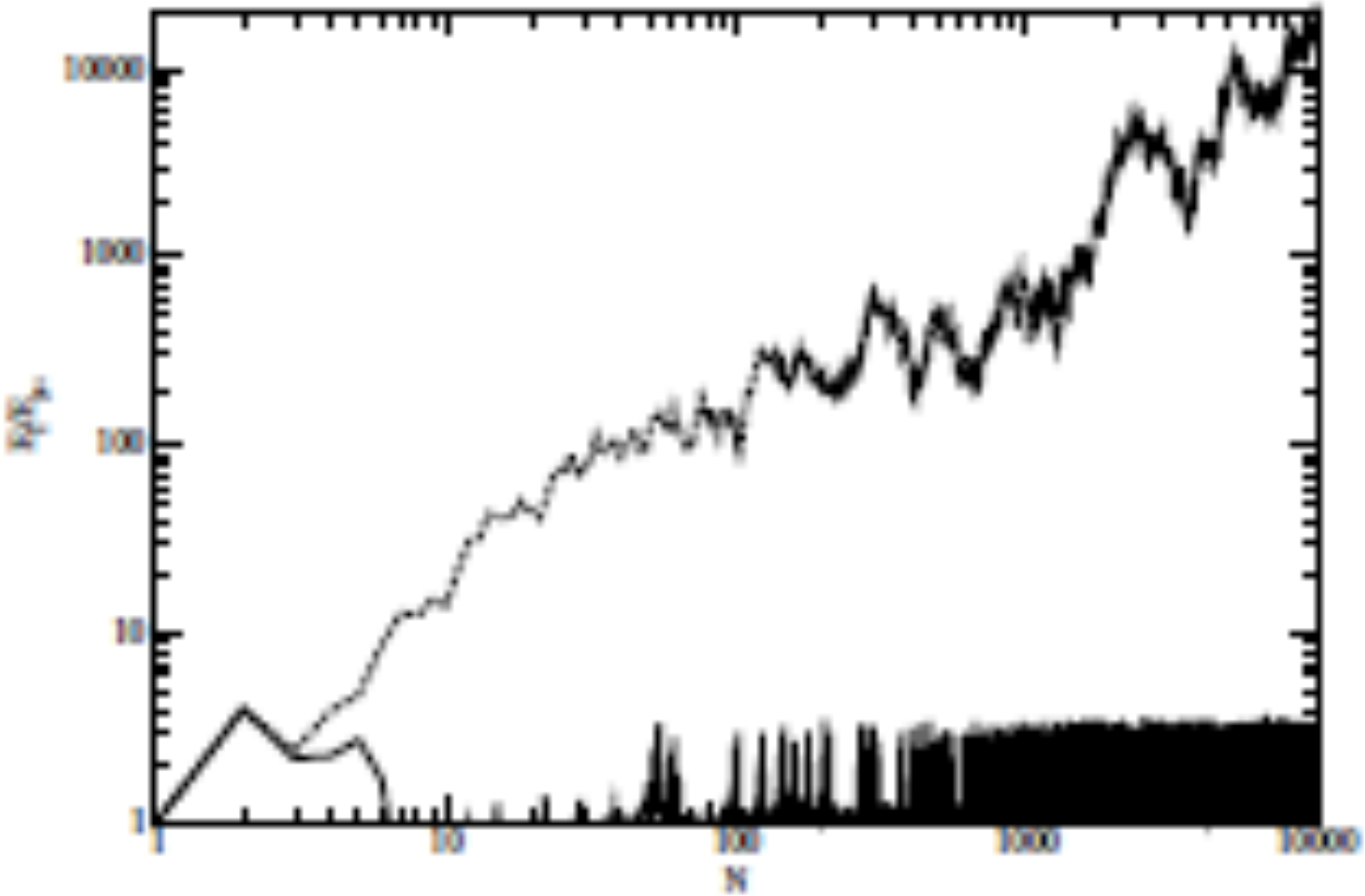


Figure 2. E_1 as a function of the number of iterations N. The case $\phi_1(0) = 2.32$ corresponding to the maximal value of $\alpha_{crit} = 1.04$ is shown. The solid curve corresponds to $\alpha = 0.9$ and the dashed curve corresponds to $\alpha = 1.1$.

Circularisation time scales

- In principal, the energy/angular momentum transfer due to dynamic tides should be included in numerical calculations of interactions of several planets with each other and with the central star (eg. Nasagawa, Ida, Bessho 2008). However, a simple approach may help to understand
- the importance of this effect.

- Assume that the dynamic tides operate either because of dissipation or due to stochastic instability. In the limit of highly eccentric orbit this leads to evolution of its semi-major axis while orbital angular momentum (and η) is conserved. η can be
- related to orbital period after the orbital circularisation

$$\eta = 9.1\eta_0 \left(\frac{R_\odot}{R_*}\right)^{3/2} P_3, \quad \eta_0 = \sqrt{\frac{M_*}{M}\frac{R^3}{R_*^3}}.$$

where P_3 is the orbital period after circularisation, in units of 3 days.

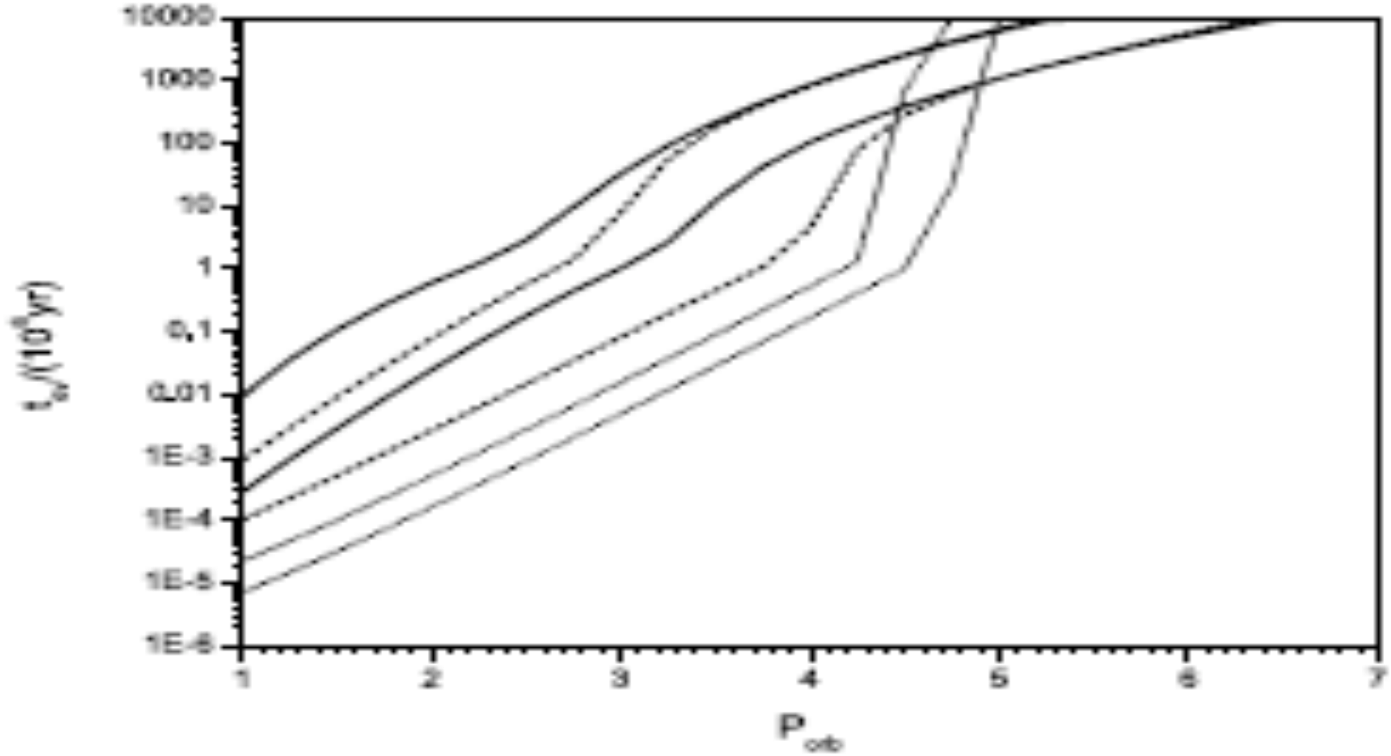


Figure 25. The circularization time-scale t_{cv} (in units of 10^6 yr) for $1M_J$ as a function of the planet orbital period after the stage of tidal circularization, P_{orb} (in units of days). The solid curves correspond to t_{cv} determined solely by the inertial modes. The dashed curves are calculated with help of the sum of contributions corresponding to the inertial modes, the fundamental modes and the modes excited in the star. The energy transfers determined by the inertial and fundamental modes are given by expressions corresponding to rotation of the planet at the equilibrium angular velocities Ω_{ps} and Ω^{f}_{ps}, respectively. The dotted curves correspond to the case of a non-rotating planet. Accordingly, the contribution of inertial modes is not taken into account. The lower (upper) curves of the same type correspond to $a_{in} = 1$ ($a = 10$ au) [$a_{in} = 10$ ($a = 100$ au)].

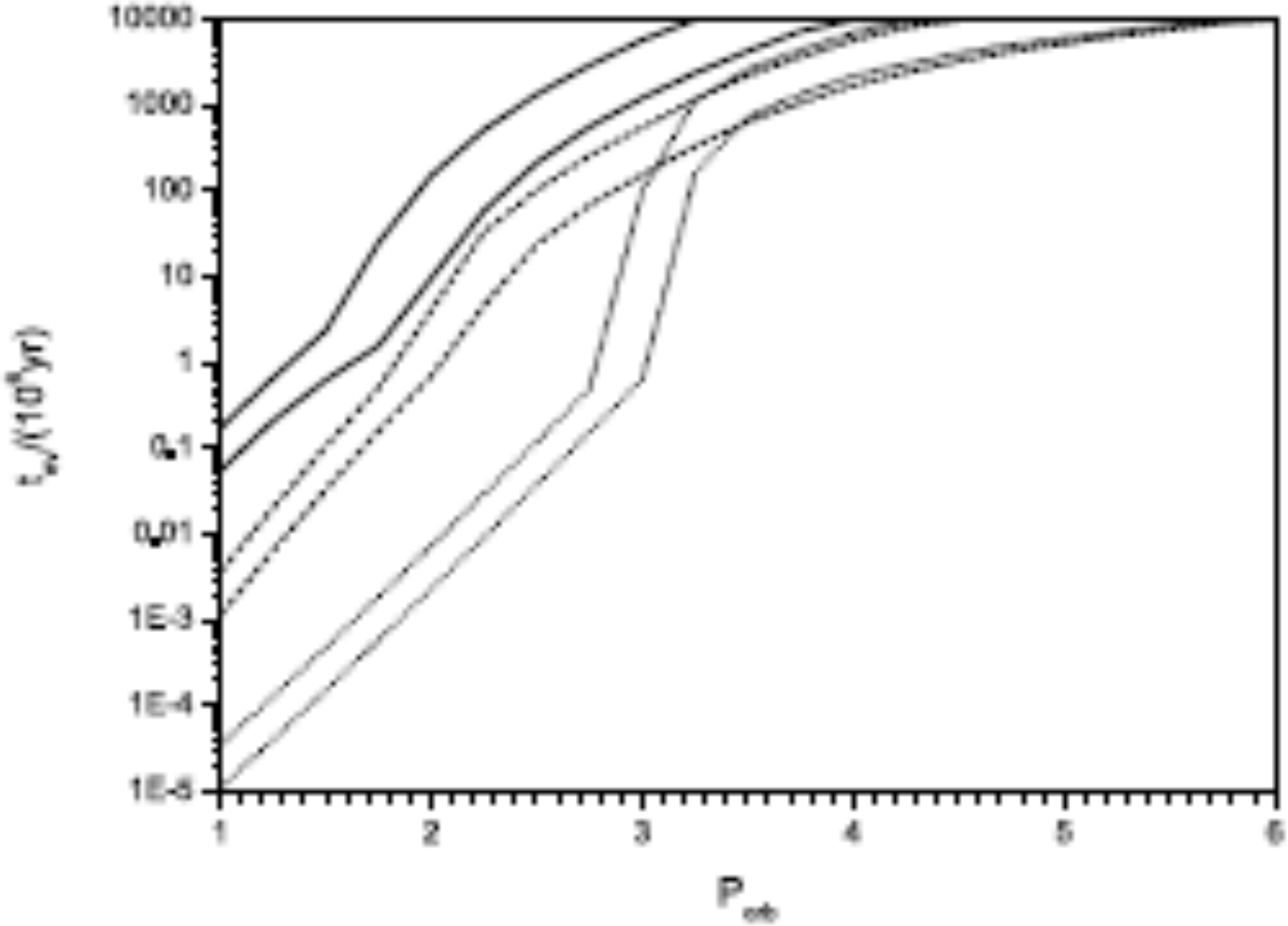


Figure 26. Same as Fig. 25 but for $M_* = 5M_J$.

conclusions

- 1) Simple expressions for energy and angular momentum transfer due to tidal excitation of fundamental and inertial modes were obtained. They can be used
- in numerical calculations of evolution of
- planetary systems.
- 2) Inertial waves appear to be important at sufficiently large values of orbital periods after circularisation – 4-5 days.

The formalism developed has much wider range of applicability, eg. it can be applied to compact objects in globular cluster (Ivanov & Papaloizou 2007b)

Self consistent calculations, which take into account dissipation of mode energy, evolution of planet's radius, its rotation as well as quasi-static tides and tides exerted on the star are needed.

More can be found in

- Ivanov & Papaloizou, MNRAS, 2004
- Papaloizou & Ivanov, MNRAS, 2005

  Ivanov, Papaloizou, MNRAS 2007

  Ivanov, Papaloizou, A&A, 2007

  Ivanov, Papaloizou, MNRAS, (submitted)

  Papaloizou, Ivanov, MNRAS, (submitted)