Optimal Cooperation in Large Wireless Networks

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Spatial Network Models for Wireless Communications
Cambridge, April 9, 2010
Percolation?
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\[ p \approx 0.5416 \]
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Simple network model
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- **uniform topology:**
  
  $n$ nodes independently and uniformly distributed in a square area $A$
Simple network model

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  - $n$ nodes independently and uniformly distributed in a square area $A$
- **uniform traffic:**
  - order $n$ source-destination pairs chosen at random in the network
Question

What is the maximum throughput scaling in such a network?
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- **per-node throughput**: $R(n) =$ data rate per S-D pair
What is the maximum throughput scaling in such a network?

- per-node throughput: $R(n) = \text{data rate per S-D pair}$
- aggregate throughput: $T(n) = n R(n)$
Communication model

\[ y_i = \sum_{k \in \tau} h_{ik} x_k + z_i \]

- \( \{x_k, k \in \tau\} = \) transmitted signals - power \( P \) each
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- \( h_{ik} = \) fading coefficient between node \( k \) and \( i \), modelled as

\[ h_{ik} = \sqrt{G} \frac{\exp(2\pi j r_{ik}/\lambda)}{r_{ik}} \]

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- free space propagation model: no scatterers, path loss exponent \( \alpha = 2 \)
Two sets of results in the literature (non-exhaustive list)

- **dense networks**: increasing number of nodes $n$, fixed power per node $P$, fixed network area $A$
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  - Franceschetti-Migliore-Minero ’09
Claim

- there is not one universal scaling law for wireless networks
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so coupling the system parameters is not a good idea
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  number of nodes \( n \)

  power per node \( P \)
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- instead, consider the following parameters to be independent:

  number of nodes $n$
  power per node $P$
  network area $A$
Our plan

Characterize the maximum throughput scaling of the network when

1. \( n \) grows large and there is neither power nor space limitation (i.e. \( P \) and \( A \) are “as large as we want”)
Our plan

Characterize the maximum throughput scaling of the network when

1. $n$ grows large and there is neither power nor space limitation (i.e. $P$ and $A$ are “as large as we want”)

2. $n$ grows large, but there is power limitation (i.e. $P$ is the limiting factor)
Our plan

Characterize the maximum throughput scaling of the network when

1. $n$ grows large and there is neither power nor space limitation (i.e. $P$ and $A$ are “as large as we want”) 

2. $n$ grows large, but there is power limitation (i.e. $P$ is the limiting factor) 

3. $n$ grows large, but there is space limitation (i.e. $A$ is the limiting factor)
1. No power or space limitation
1. No power or space limitation

**Question:**

Is the number of nodes itself a factor limiting the per-node throughput?
Time-division

one communication at a time in the network
Time-division

one communication at a time in the network
Time-division

one communication at a time in the network

per-node throughput $R(n) = \Theta(1/n)$
Multi-hop (Gupta-Kumar ’00)

- spatial reuse: order $n$ local simultaneous communications are feasible
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- relaying burden: order $\sqrt{n}$ hops are needed to reach destinations
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per-node throughput $R(n) = \Theta(1/\sqrt{n})$
Parenthesis: distributed MIMO systems

between two clusters of $M$ nodes, it is possible to transfer $M$ information bits simultaneously (provided no power or space limitation!)
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- between two clusters of $M$ nodes, it is possible to transfer $M$ information bits simultaneously (provided no power or space limitation!)
- but this requires first a dissemination phase at the transmit cluster
between two clusters of $M$ nodes, it is possible to transfer $M$ information bits simultaneously (provided no power or space limitation!)

but this requires first a dissemination phase at the transmit cluster

... as well as an aggregation phase at the receive cluster
Hierarchical cooperation scheme (Özgür-L-Tse ’07)

- first phase: local exchange of information inside clusters of nodes
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- first phase: local exchange of information inside clusters of nodes
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- first phase: local exchange of information inside clusters of nodes
- second phase: long range MIMO transmissions across the network
- third phase: local exchange of information inside clusters of nodes
- recursion: perform the same operation inside clusters now
- after \( h \) levels of recursion: per-node throughput \( R(n) = \Theta \left( n^{-\frac{1}{h+1}} \right) \)
in a network with a large number of nodes, it is possible to sustain a per-node throughput arbitrarily close to $\Theta(1)$ (provided that there is neither power nor space limitation)
2. Power limitation
2. Power limitation

**Question:**

does hierarchical cooperation still outperform multi-hop in this case?
Power requirements

- multi-hop transmissions require \( SNR = \frac{P}{d^2} \geq 0 \text{ dB}, \)
  where \( d = \text{average distance between nearby nodes} \)
Power requirements

- multi-hop transmissions require $SNR = P/d^2 \geq 0 \text{ dB}$, where $d =$ average distance between nearby nodes
- what about long range MIMO transmissions?
Power requirements

- multi-hop transmissions require $SNR = \frac{P}{d^2} \geq 0$ dB, where $d =$ average distance between nearby nodes
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  - communication distance $\sim$ network diameter $= D \sim \sqrt{n} \ d$
Power requirements

- Multi-hop transmissions require $SNR = \frac{P}{d^2} \geq 0$ dB, where $d =$ average distance between nearby nodes.
- What about long range MIMO transmissions?
  - Communication distance $\sim$ network diameter $= D \sim \sqrt{n} \cdot d$
  - MIMO gain: in a MIMO transmission involving $n$ nodes, the power received at each node is amplified by a factor $n$. 
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  - communication distance $\sim$ network diameter $= D \sim \sqrt{n} d$
  - MIMO gain: in a MIMO transmission involving $n$ nodes, the power received at each node is amplified by a factor $n$
  - $\Rightarrow$ same condition: $SNR = P/d^2 \geq 0$ dB
- so hierarchical cooperation still outperforms multi-hop in this case, and a per-node throughput $R(n)$ arbitrarily close to $\Theta(1)$ is achievable
And below? (i.e. when $SNR \ll 0 \text{ dB}$)

- how to compensate for the lack of available power?
And below? (i.e. when $\text{SNR} \ll 0 \text{ dB}$)

- how to compensate for the lack of available power?
- a simple solution: bursty transmissions!
  - i.e. wait for a duration of $1/\text{SNR}$ time-slots before any transmission
And below? (i.e. when $SNR \ll 0 \text{ dB}$)

- how to compensate for the lack of available power?
- a simple solution: **bursty transmissions!**
  i.e. wait for a duration of $1/SNR$ time-slots before any transmission
- $\Rightarrow$ per-node throughput $R(n)$ arbitrarily close to $\Theta(SNR)$
Conclusion #2

- when there is power limitation but no space limitation, hierarchical cooperation achieves the optimal throughput scaling
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- a per-node throughput $R(n)$ arbitrarily close to $\Theta(1)$ can be maintained, provided that $SNR = P/d^2 \geq 0 \text{ dB}$
when there is power limitation but no space limitation, hierarchical cooperation achieves the optimal throughput scaling

a per-node throughput $R(n)$ arbitrarily close to $\Theta(1)$ can be maintained, provided that $SNR = P/d^2 \geq 0$ dB

rephrasing in terms of the aggregate throughput: an aggregate throughput $T(n)$ arbitrarily close to $\Theta(n)$ is achievable, provided that $SNR = P/d^2 \geq 0$ dB.
Conclusion #2

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- rephrasing in terms of the aggregate throughput: an aggregate throughput $T(n)$ arbitrarily close to $\Theta(n)$ is achievable, provided that $SNR = P/d^2 \geq 0$ dB.

Remark:
The situation where the power path loss exponent $\alpha > 2$ is a different story! (see Özgür-Johari-Tse-L, Trans. on IT 2010).
3. Space limitation
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Question:

does hierarchical cooperation still outperform multi-hop in this case?
Space limitation

Franceschetti-Migliore-Minero, Trans. on IT 2009:

In an extended network, $T(n) = O(\sqrt{n})$. 
Franceschetti-Migliore-Minero, Trans. on IT 2009:

In an extended network, \( T(n) = O(\sqrt{n}) \).

In a more general context, this result translates into:

\[
T(n) = \begin{cases} 
O(\sqrt{n}) & \text{if } \sqrt{A}/\lambda \leq \sqrt{n} \\
O(\sqrt{A}/\lambda) & \text{if } \sqrt{n} \leq \sqrt{A}/\lambda \leq n \\
O(n) & \text{if } \sqrt{A}/\lambda \geq n
\end{cases}
\]

where \( \lambda \) is the carrier wavelength.
Temporary conclusion

- when area is a scarce resource, i.e. when $\sqrt{A}/\lambda \leq \sqrt{n}$, multi-hop achieves the optimal aggregate throughput scaling $T(n) = \Theta(\sqrt{n})$
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- when area is a scarce resource, i.e. when $\sqrt{A/\lambda} \leq \sqrt{n}$, multi-hop achieves the optimal aggregate throughput scaling $T(n) = \Theta(\sqrt{n})$

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Temporary conclusion

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- on the other hand, when $\sqrt{A}/\lambda \geq n$, hierarchical cooperation achieves the optimal aggregate throughput scaling $T(n) = \Theta(n)$

- can we do better than multi-hop in the intermediary regime?
Theorem

When $\sqrt{n} \leq \sqrt{A}/\lambda \leq n$ and $SNR = P/d^2 \geq 0$ dB, an aggregate throughput scaling arbitrarily close to

$$T(n) = \Theta(\sqrt{A}/\lambda)$$

is achievable via hierarchical cooperation (Özgür-L-Tse, ITA 2010).
Theorem

When $\sqrt{n} \leq \sqrt{A/\lambda} \leq n$ and $\text{SNR} = P/d^2 \geq 0$ dB, an aggregate throughput scaling arbitrarily close to

$$T(n) = \Theta(\sqrt{A/\lambda})$$

is achievable via hierarchical cooperation (Özgür-L-Tse, ITA 2010).

Remark:
A similar result has been obtained independently by Lee-Chung, ISIT 2010.
Proof idea: where does the spatial limitation kick in?
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- the number of bits that can be transmitted simultaneously between two clusters of $M$ nodes and area $A$ is $O(\sqrt{A}/\lambda)$
Proof idea: where does the spatial limitation kick in?

- The number of bits that can be transmitted simultaneously between two clusters of $M$ nodes and area $A$ is $O(\sqrt{A}/\lambda)$.
- In the case where clusters are far apart from each other, the situation might even get worse!
Proof idea: where does the spatial limitation kick in?

- the number of bits that can be transmitted simultaneously between two clusters of $M$ nodes and area $A$ is $O(\sqrt{A}/\lambda)$
- in the case where clusters are far apart from each other, the situation might even get worse!
- what we show: if the distance $D$ between the two clusters is of the order of $\sqrt{A}$ or more, then $\Omega(A/\lambda D)$ bits can be transmitted simultaneously
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- the number of bits that can be transmitted simultaneously between two clusters of $M$ nodes and area $A$ is $O(\sqrt{A}/\lambda)$
- in the case where clusters are far apart from each other, the situation might even get worse!
- what we show: if the distance $D$ between the two clusters is of the order of $\sqrt{A}$ or more, then $\Omega(A/\lambda D)$ bits can be transmitted simultaneously
- at the highest level of the hierarchical scheme, it turns out that $D \sim \sqrt{A}$, so $\Omega(\sqrt{A}/\lambda)$ bits can be transmitted simultaneously
Proof idea (cont’d)

- how to compensate for the lack of spatial degrees of freedom?
Proof idea (cont’d)

- how to compensate for the lack of spatial degrees of freedom?
- a simple solution again: reduce the number of nodes communicating simultaneously, so as to meet the spatial limitation
Proof idea (cont’d)

- how to compensate for the lack of spatial degrees of freedom?
- a simple solution again: **reduce the number of nodes communicating simultaneously**, so as to meet the spatial limitation
- !!! the area occupied by the nodes should be kept **fixed** !!!
Proof idea (cont’d)

- how to compensate for the lack of spatial degrees of freedom?
- a simple solution again: reduce the number of nodes communicating simultaneously, so as to meet the spatial limitation
- !!! the area occupied by the nodes should be kept fixed !!!
- this way, a throughput of order arbitrarily close to $\Theta(\sqrt{A}/\lambda)$ is achievable via hierarchical cooperation
Conclusion #3

If \( \sqrt{A/\lambda} \leq \sqrt{n} \), then multi-hop is optimal
if \( \sqrt{A/\lambda} \geq \sqrt{n} \), then hierarchical cooperation is optimal
(provided that \( SNR = P/d^2 \geq 0 \) dB in both cases)
Wireless network example: EPFL learning center
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- \( n \sim 1'000 \) students at peak hours
Wireless network example: EPFL learning center

- $n \sim 1'000$ students at peak hours
- $A = 200 \times 100 = 20'000$ square meters (discarding the holes!)
Wireless network example: EPFL learning center

- \( n \sim 1'000 \) students at peak hours
- \( A = 200 \times 100 = 20'000 \) square meters (discarding the holes!)
- carrier frequency = 3 GHz \( \Rightarrow \) carrier wavelength \( \lambda = 0.1 \) m
Wireless network example: EPFL learning center

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- $A = 200 \times 100 = 20'000$ square meters (discarding the holes!)
- Carrier frequency $= 3$ GHz $\Rightarrow$ carrier wavelength $\lambda = 0.1$ m
- so $\sqrt{A}/\lambda \sim 1'400 \geq n$: no spatial limitation
Wireless network example: EPFL learning center

- $n \sim 1’000$ students at peak hours
- $A = 200 \times 100 = 20’000$ square meters (discarding the holes!)
- carrier frequency $= 3 \text{ GHz} \Rightarrow$ carrier wavelength $\lambda = 0.1 \text{ m}$
- so $\sqrt{A}/\lambda \sim 1’400 \geq n$: no spatial limitation
- and no power limitation either ($d \sim 4 \text{ m}, \text{ SNR} \gg 0 \text{ dB}$)
Open problem

What happens when both power and area are limiting factors?