Dynamic Models and Dynamic Markets for Electric Power Networks

Sean P. Meyn

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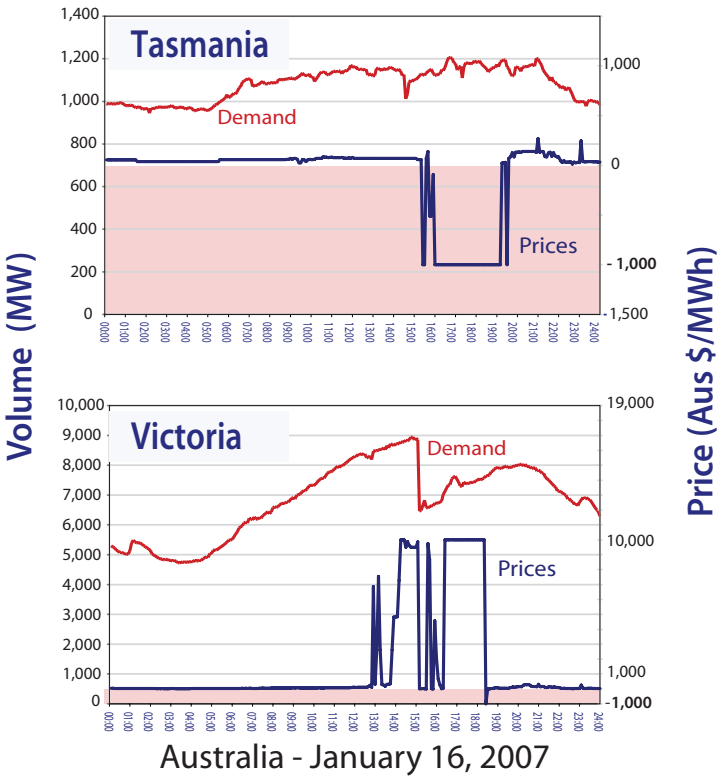
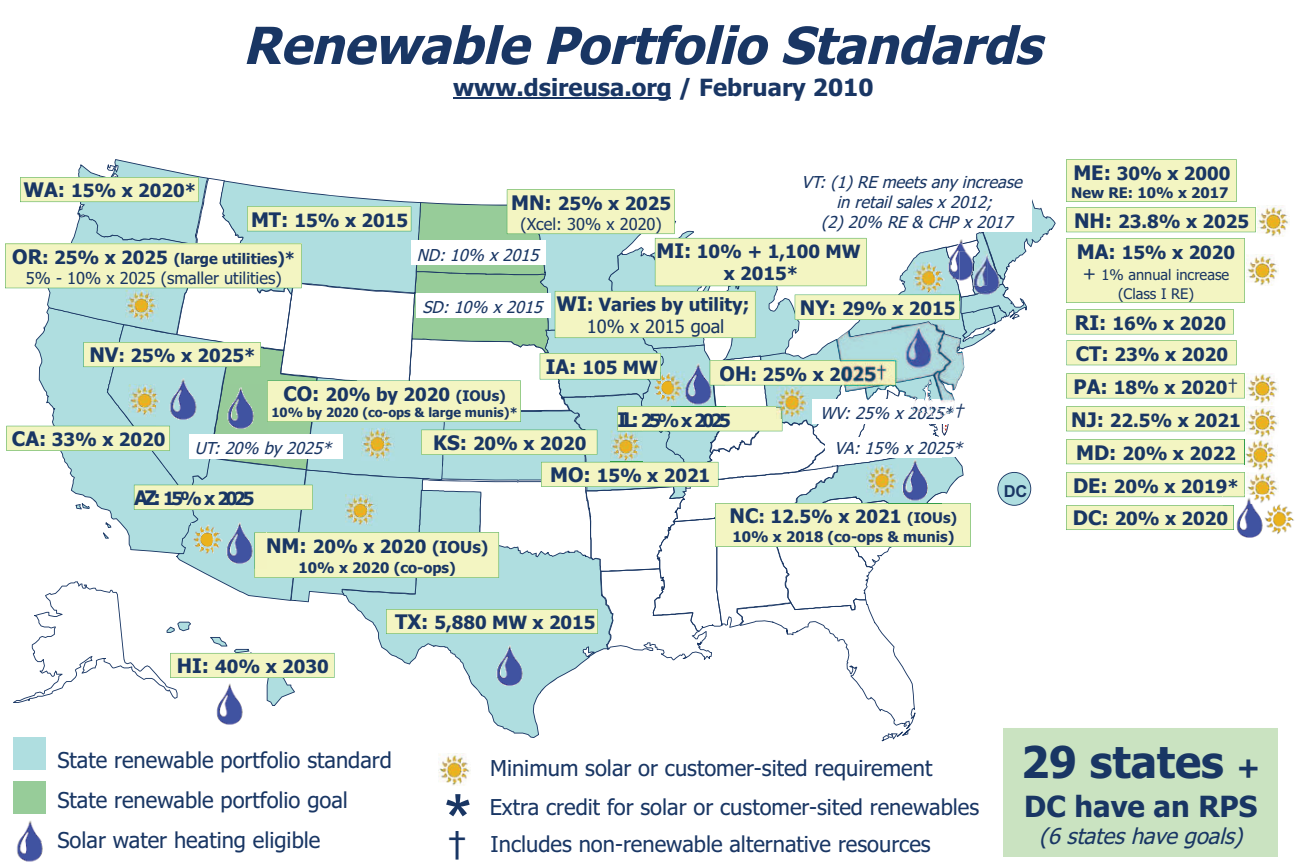
Outline

1. Issues and Approaches
2. Reliability by Design
3. Dynamic Markets and their Equilibria
4. Who Commands the Wind?
5. Research Frontiers
6. References

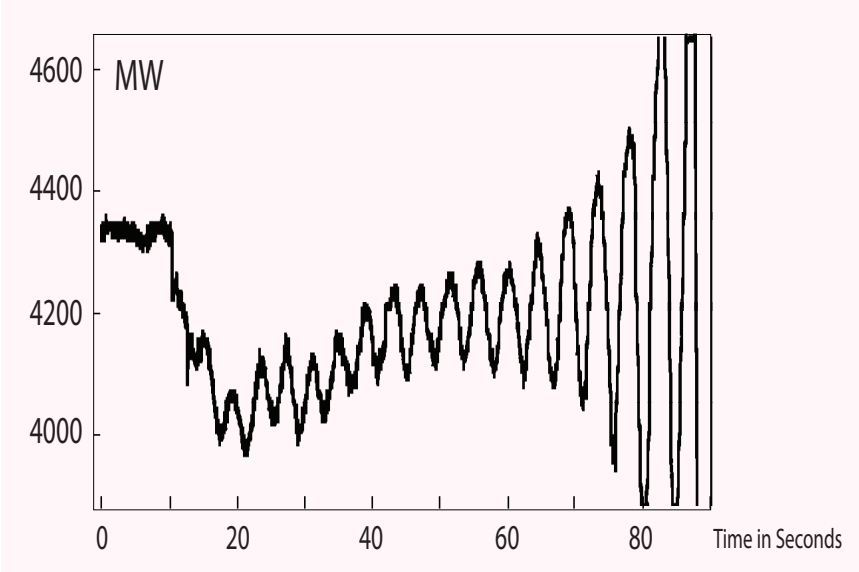


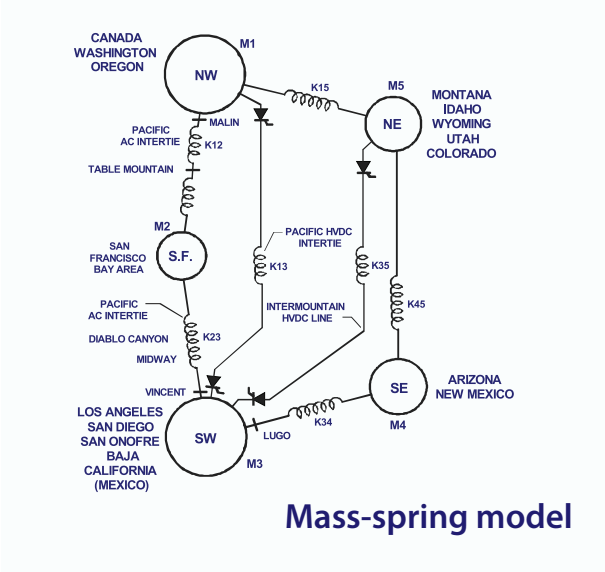
Issues and Approaches

Issues: Political *Mandates come, and go...*



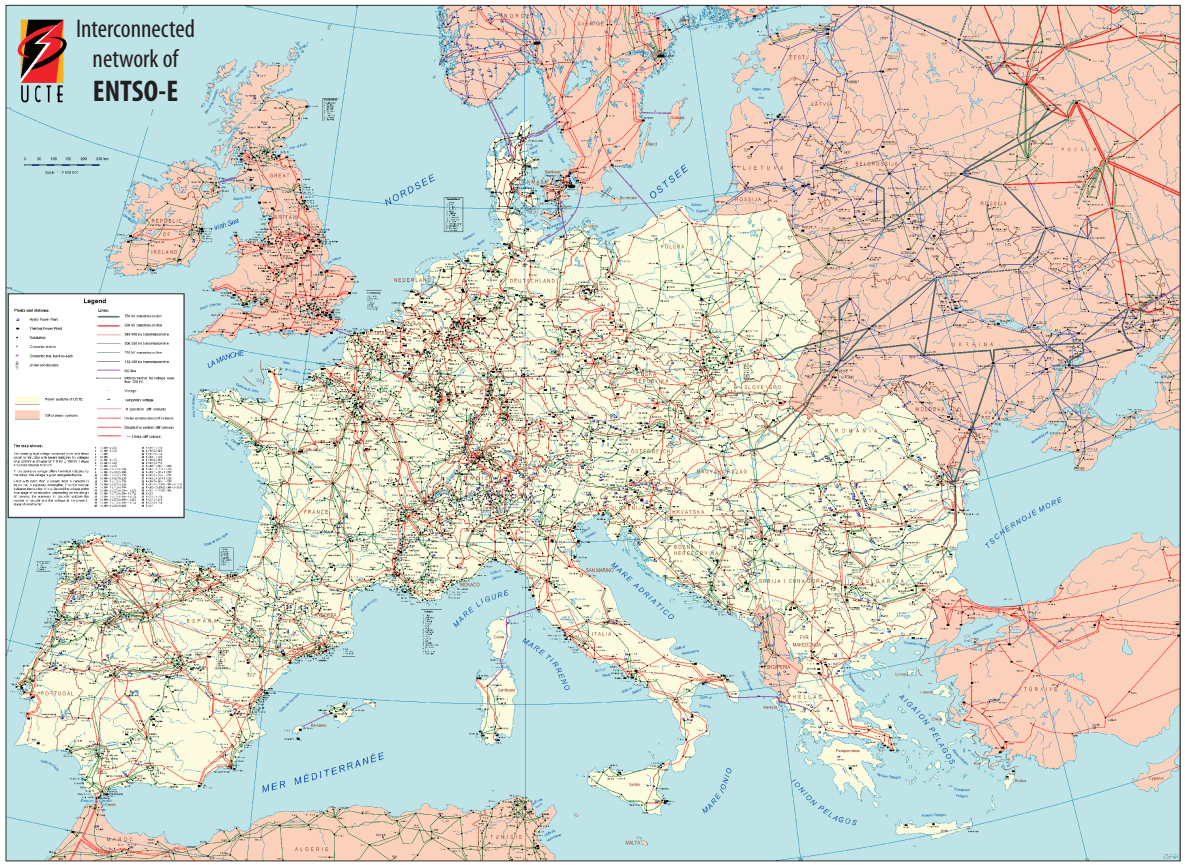
Issues: Dynamics *Coupled generators and consumers*





Instability in the North-West U.S.

Issues: Complexity



European Network of Transmission System Operators for Electricity

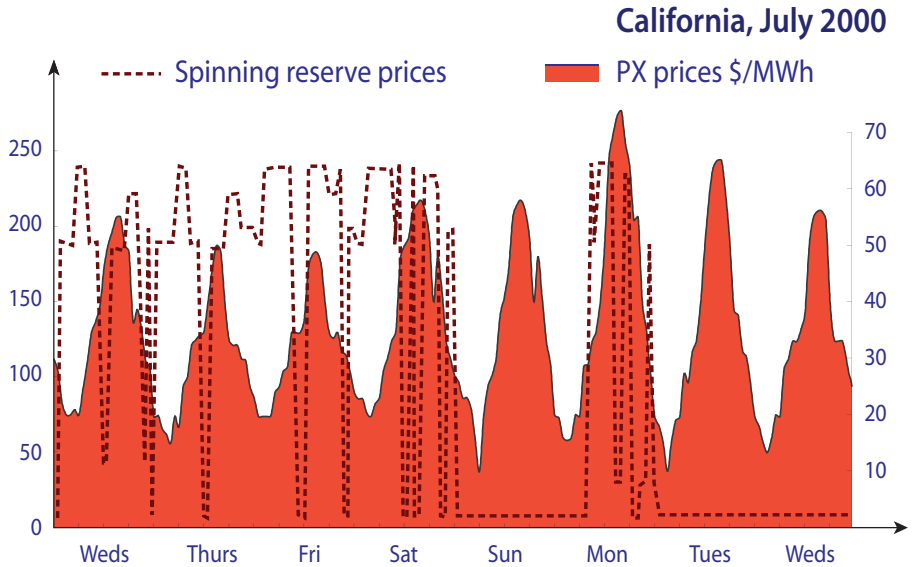
Issues: Market Power *Risk to consumers*

Enron traders openly discussed manipulating California's power market during profanity-laced telephone conversations in which they merrily gloated about ripping off "those poor grandmothers" during the state's energy crunch in 2000-01 ...

[AP by Kristen Hays, 06/03/04]

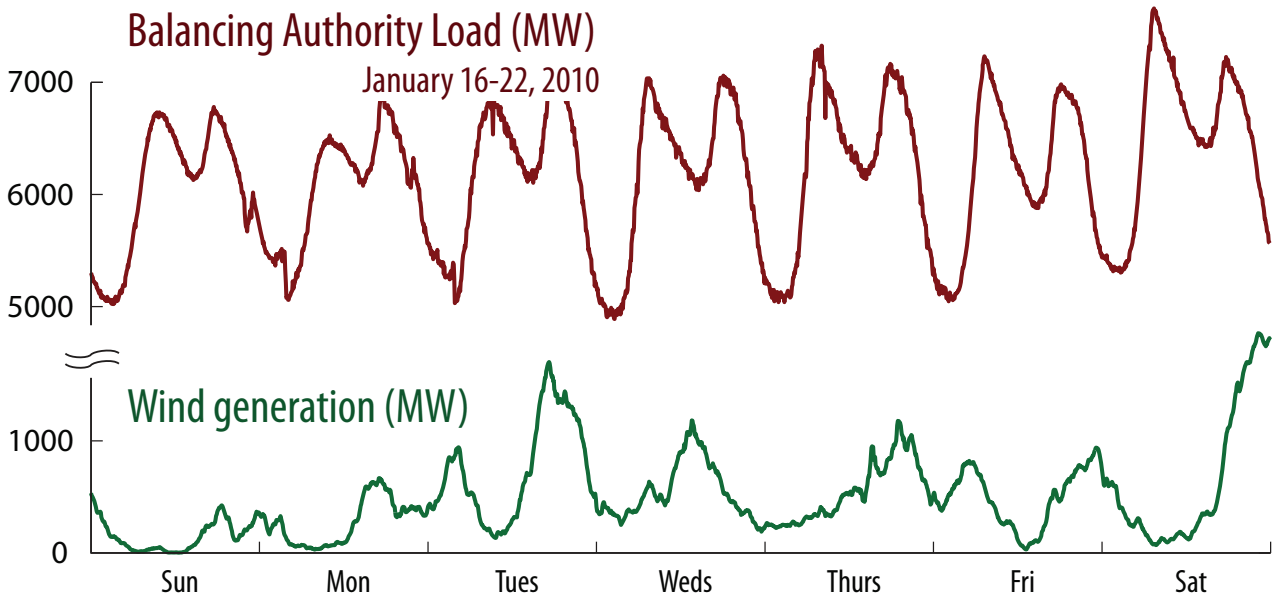
ENRON As Jeffrey Skilling, the first but sadly not the last guy to package up a big sample of nothing and sell it to a greedy public, Samuel West oozes self-belief; his boss, Ken Lay (Tim Piggott-Smith) and stooge, Andy Fastow (Tom Goodman-Hill) end up smeared with it. But Lucy Prebble's play is rather more than a simple tale ...

-Time Out, London 2010



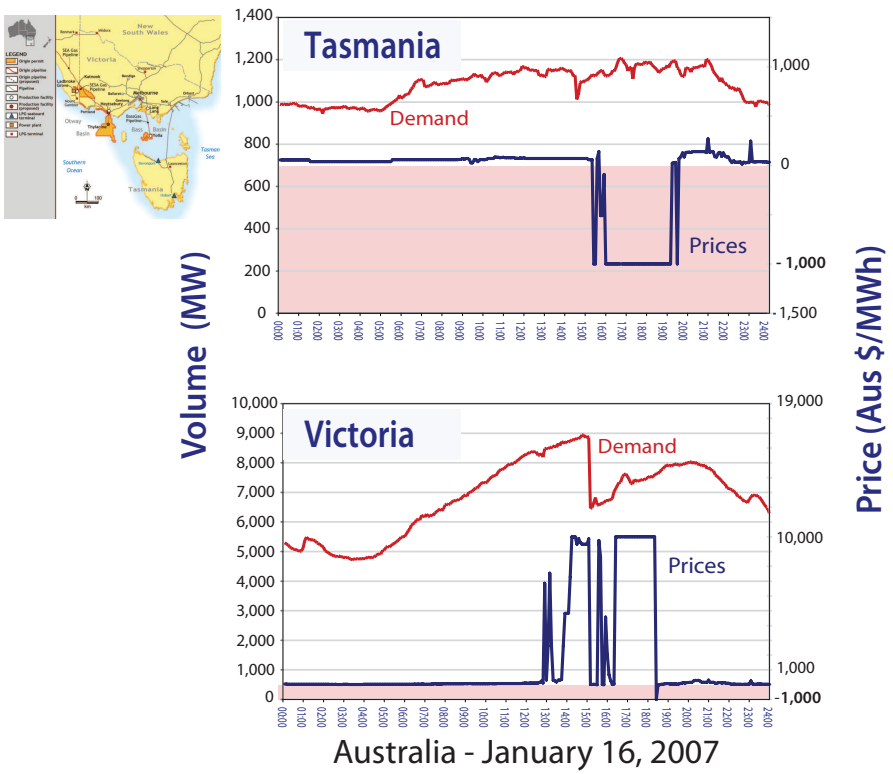
"Ripping off *those poor grandmothers*"

Issues: *Risks to suppliers*



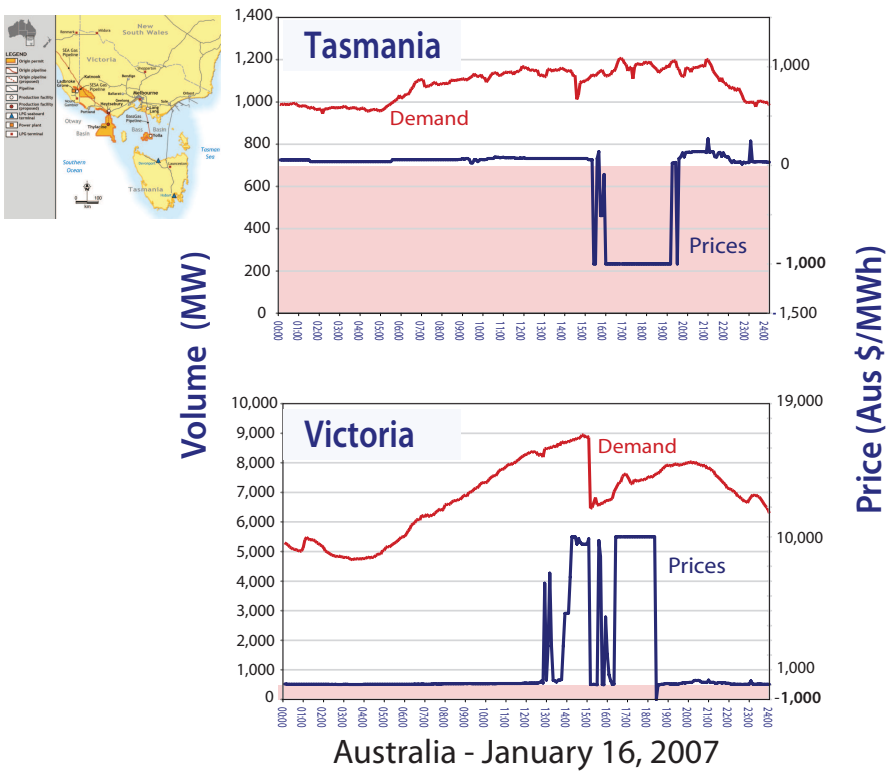
Who will buy my power if the wind is blowing?

Issues: Friction



Power flows according to Kirchoff's Laws, and subject to constraints ...

Issues: Friction



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... Mechanical and thermal constraints, ramp constraints, security constraints, ...

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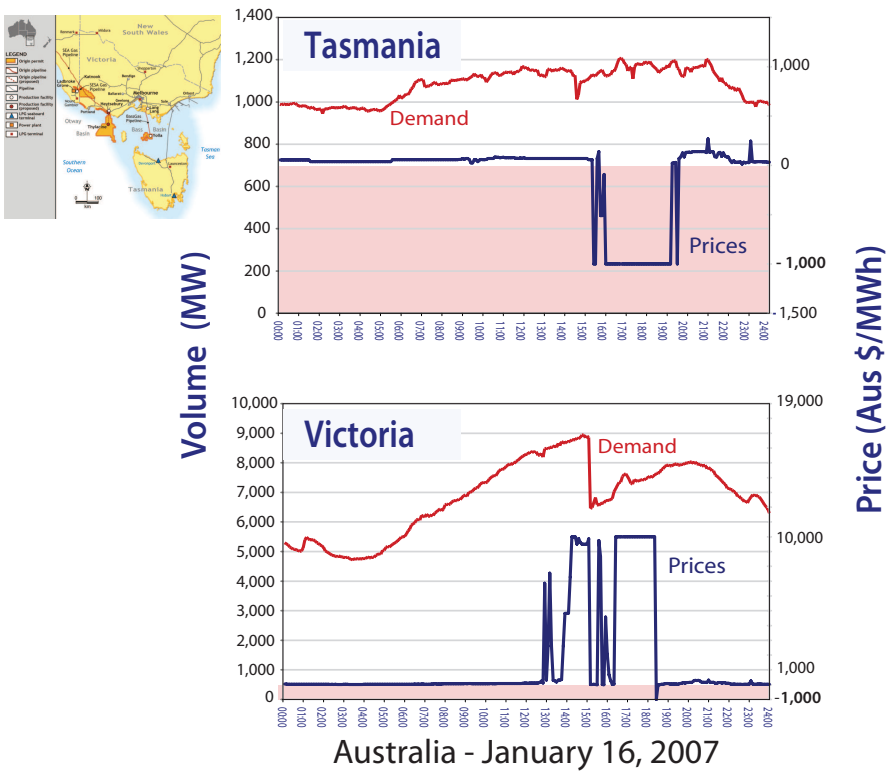

Power flows according to Kirchoff's Laws, and subject to constraints ...

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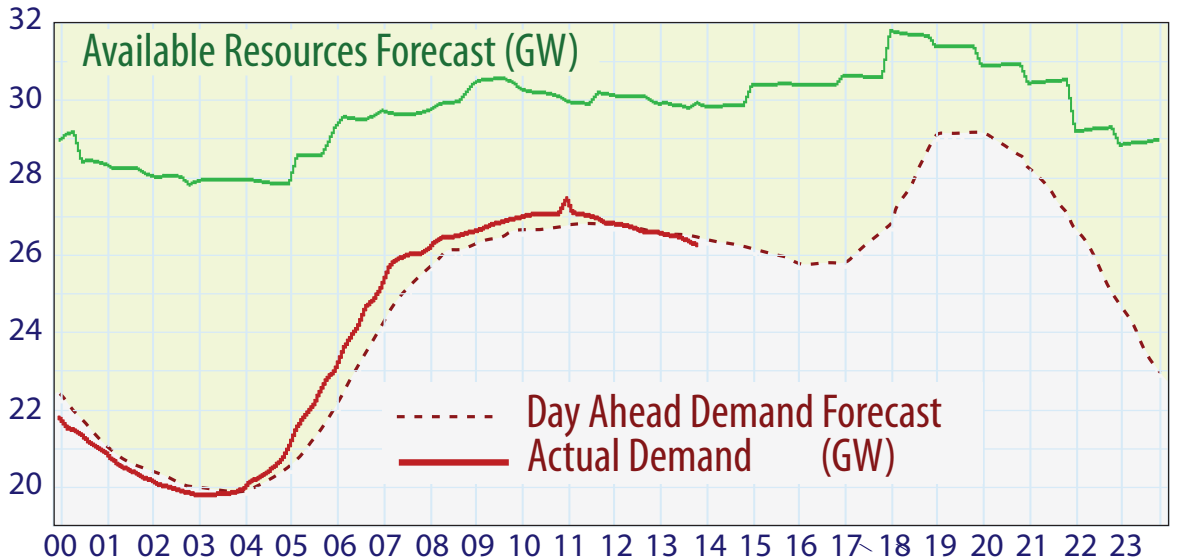
Market and physical system are tightly coupled

Issues: Uncertainty



California ISO
www.caiso.com

CAISO Mission:

- Operate the grid reliably and efficiently
- Provide fair and open transmission access
- Promote environmental stewardship
- Facilitate effective markets ...

Control solution: *Model based*

Simplest model that leads to useful policy

Listed in order of complexity:

1. Generation is modeled via overall ramp-rate:

$$\frac{G(t_1)-G(t_0)}{t_1-t_0} \le \zeta^+, \qquad 0 \le t_0 < t_1 < \infty$$

We may also impose lower bounds.

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   Power flow on transmission lines = linear function of nodal power values
   $F(t) = \Delta Z(t)$, $\Delta =$ *distribution factor matrix*

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Market Analysis: *Perfect competition*

A beautiful world...

In this lecture, Dynamic market equilibria
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Market Analysis: *Perfect competition*

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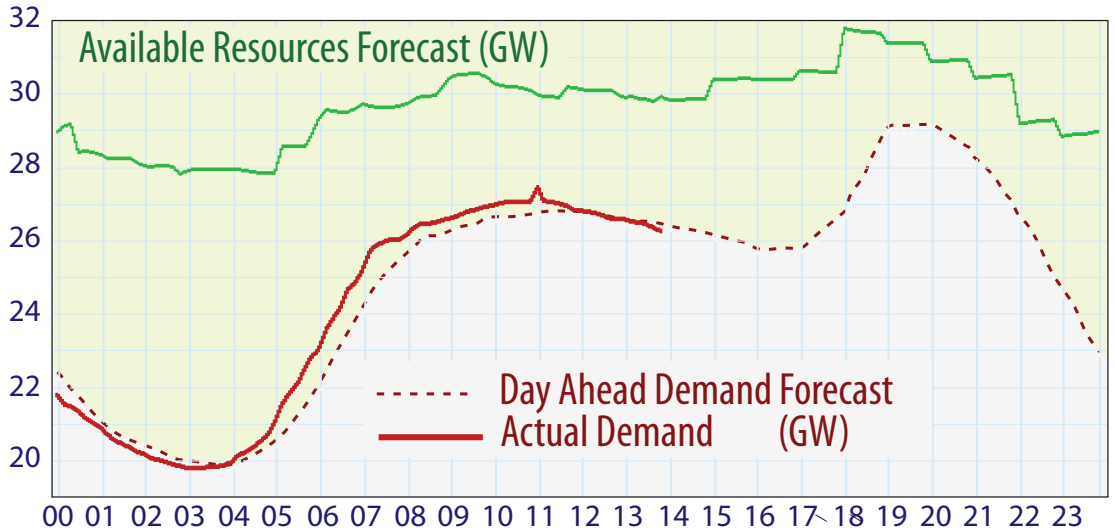
  No government mandates

- Consumers own available wind generation resources



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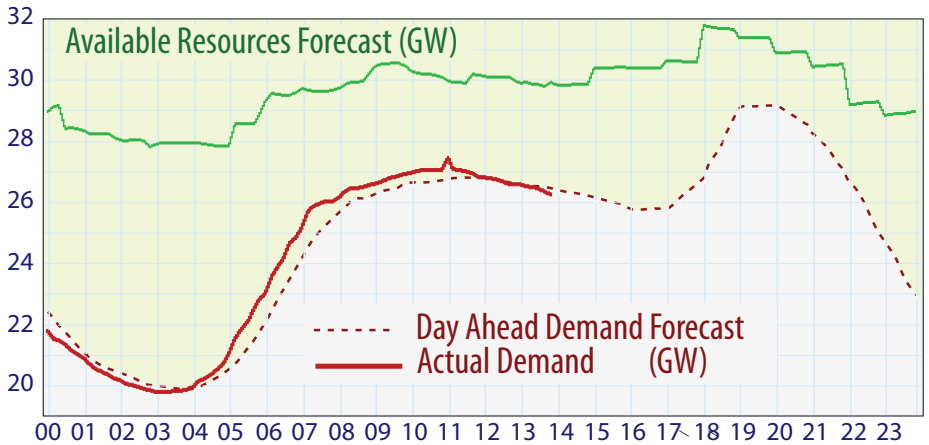
Reliability by Design

Reliability: *How to achieve the CAISO Mission?*



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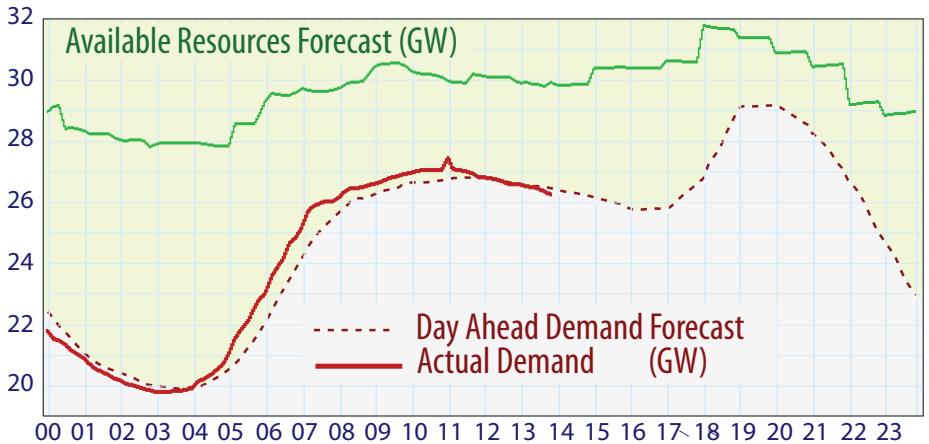
Begin with,

- **Operate the grid reliably and efficiently**

Reliability: *How to achieve the CAISO Mission?*



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These can wait:

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A diffusion model

Dynamic model for reserves

- $R(t) =$ Available power $-$ Demand $= G(t) - D(t)$

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- $R(t) =$ Available power $-$ Demand $= G(t) - D(t)$
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- $G(t)$: Deviation in on-line capacity from day-ahead market

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Volatility

Deviation in demand $\boldsymbol{D}$ is modeled as a driftless Brownian motion with instantaneous variance σ^2 (imposed for computation)

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Volatility

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Friction

Generation cannot increase instantaneously:

$$\frac{G(t') - G(t)}{t' - t} \leq \zeta\,, \quad \text{for all } t \geq 0, \text{ and } t' > t.$$

Social planner's objective

Discounted welfare: For given initial generation and demand,

$$K(g,d) = \mathsf{E}\Big[\int e^{-\gamma t}\big(\mathcal{W}_{\mathrm{S}}(t) + \mathcal{W}_{\mathrm{D}}(t) + \mathcal{W}_{\mathrm{E}}(t)\big)\,dt\Big].$$

Welfare functions:

- Supplier, $\mathcal{W}_{\mathrm{S}}(t)$: `payments - cost`

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Goal: *Maximize the discounted social welfare*

Social planner's objective

Piecewise linear welfare

Linear costs and values

$$\mathcal{W}_{\mathrm{S}}(t) := P(t)G(t) - cG(t)$$

$$\mathcal{W}_{\mathrm{D}}(t) := v \min(D(t), G(t)) - c^{\mathrm{bo}} \max(0, -R(t)) - P(t)G(t)$$

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$\Longrightarrow$ discounted welfare is a function of reserves

$$K(g,d) = (v-c)d + \mathsf{E}\Big[\int e^{-\gamma t} w(R(t))\, dt\Big]$$

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Threshold policy

Optimal threshold: $\bar{r}^* = \dfrac{1}{\theta_+}\log\left(\dfrac{c^{\mathrm{bo}}+v}{c}\right)$

where $\theta_+ > 0$ solves, $\frac{1}{2}\sigma^2\theta^2 - \zeta\theta - \gamma = 0$

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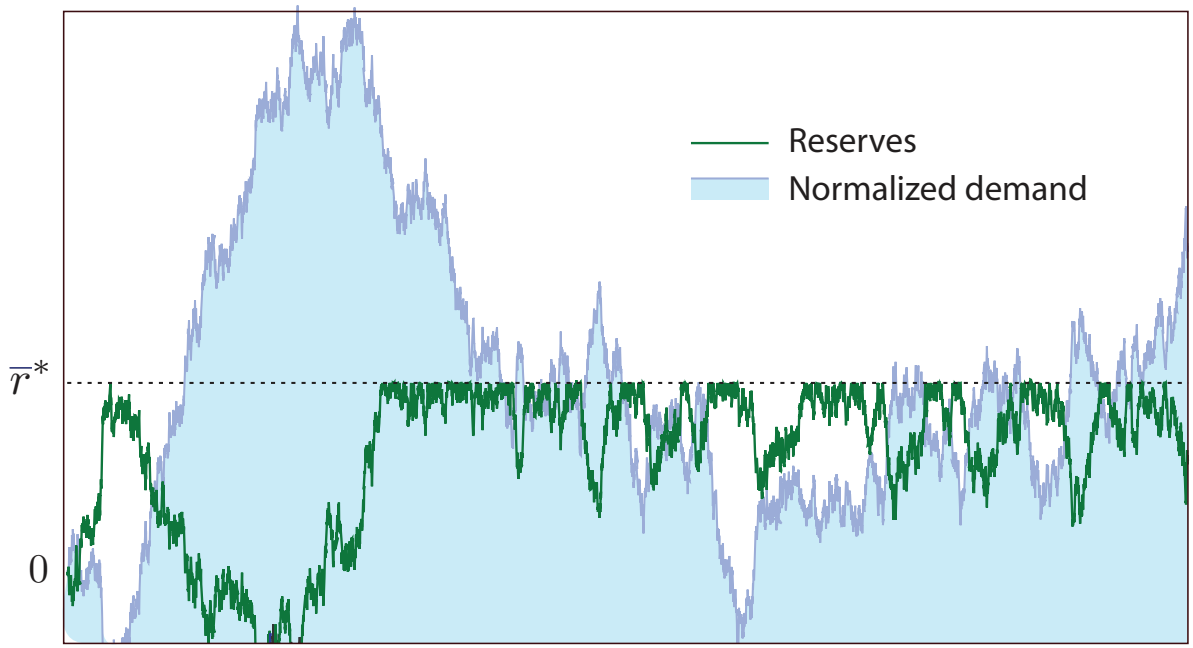
Average cost case: $\dfrac{1}{\theta_+} \to \frac{1}{2}\dfrac{\sigma^2}{\zeta}$, as $\gamma \downarrow 0$.

Social planner's objective

Reserve Dynamics

Optimal reserves = reflected Brownian motion:



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Primary and Ancillary Service

Multiple thresholds

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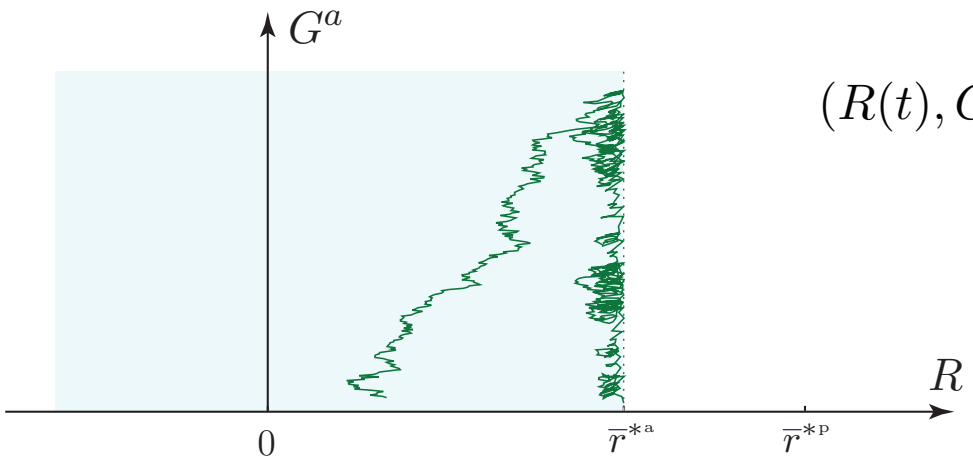

$(R(t), G^a(t))$ constrained by affine policy

Trajectory of the two-dimensional model under an affine policy

Primary and Ancillary Service

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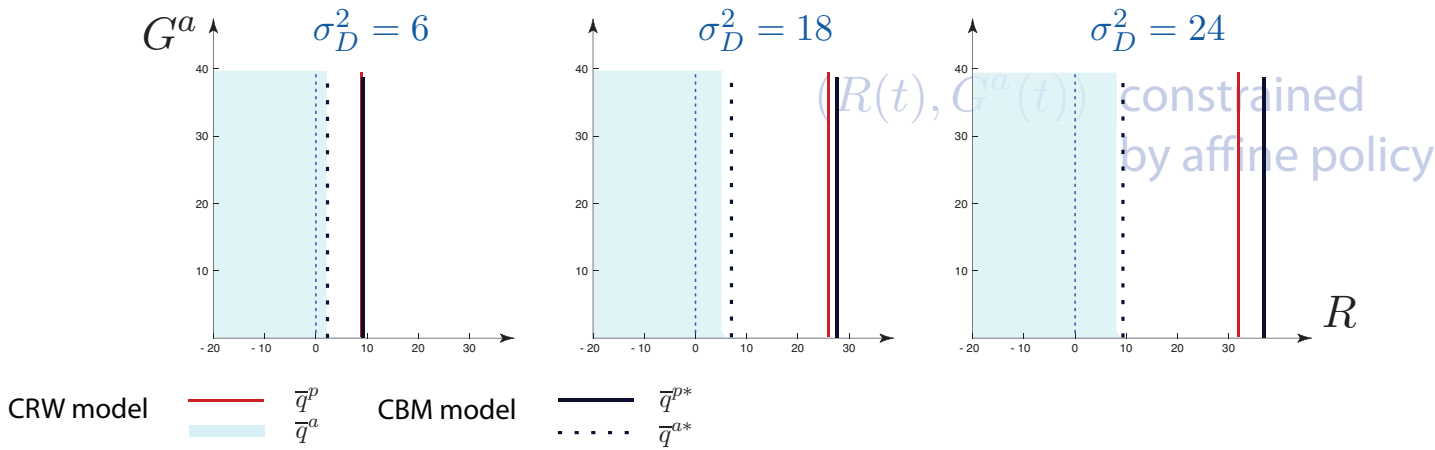
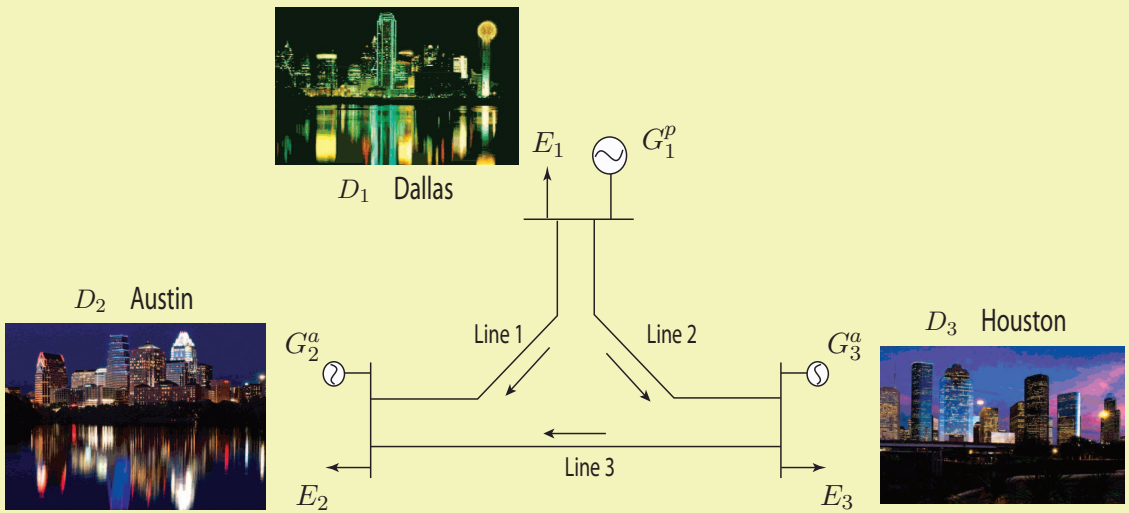


Solidarity between optimal policies for non-Brownian demand

Reliability by Design in Networks

Three-Node Example



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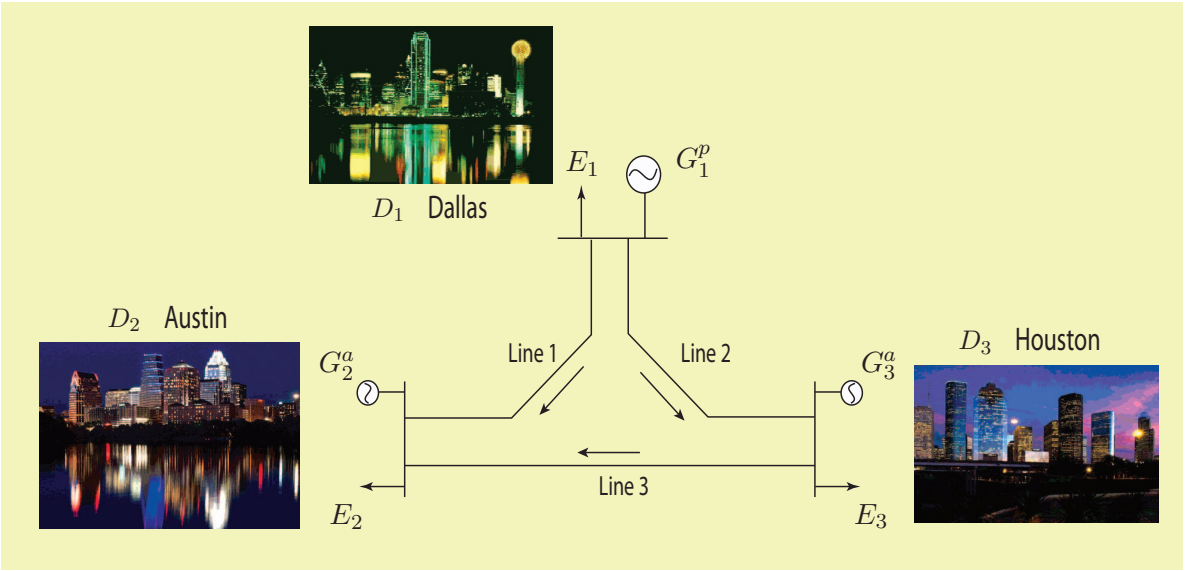

$$F(t) = \Delta Z(t)$$

- Power flow on transmission lines $F = (F_1, F_2, F_3)^T$
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$$\Delta = \frac{1}{3} \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 0 \end{bmatrix}$$

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Constraints: $F(t) \in \mathsf{F} := \{x \in \mathbb{R}^{\ell_t} : -f^+ \leq x \leq f^+\}$

Reliability by Design in Networks

Extension to general topology

- Primary and ancillary capacity processes at each node are subject to ramp constraints
- Deviation in demand is a multi-dimensional stochastic process – Brownian motion for computation.
- Cost in social planner's problem,

$$\sum_i \left(c_i^p g_i^p + c_i^a g_i^a + c_i^{bo} (e_i - d_i)_- \right)$$

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Optimization problem is now intractable!

Reliability by Design in Networks

Relaxations

Consider aggregate reserves and ancillary generalization:

$$R_A(t) = \sum R_i(t)\,, \quad G^a_A(t) = \sum G^a_i(t).$$

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For given

- Aggregate state $x_A = (r_A, g^a_A)^\intercal \in \mathsf{X}_A := \mathbb{R} \times \mathbb{R}_+$
- Demand vector $d \in \mathbb{R}^{\ell_n}$

Effective cost: Cost of cheapest state vector consistent with x_A:

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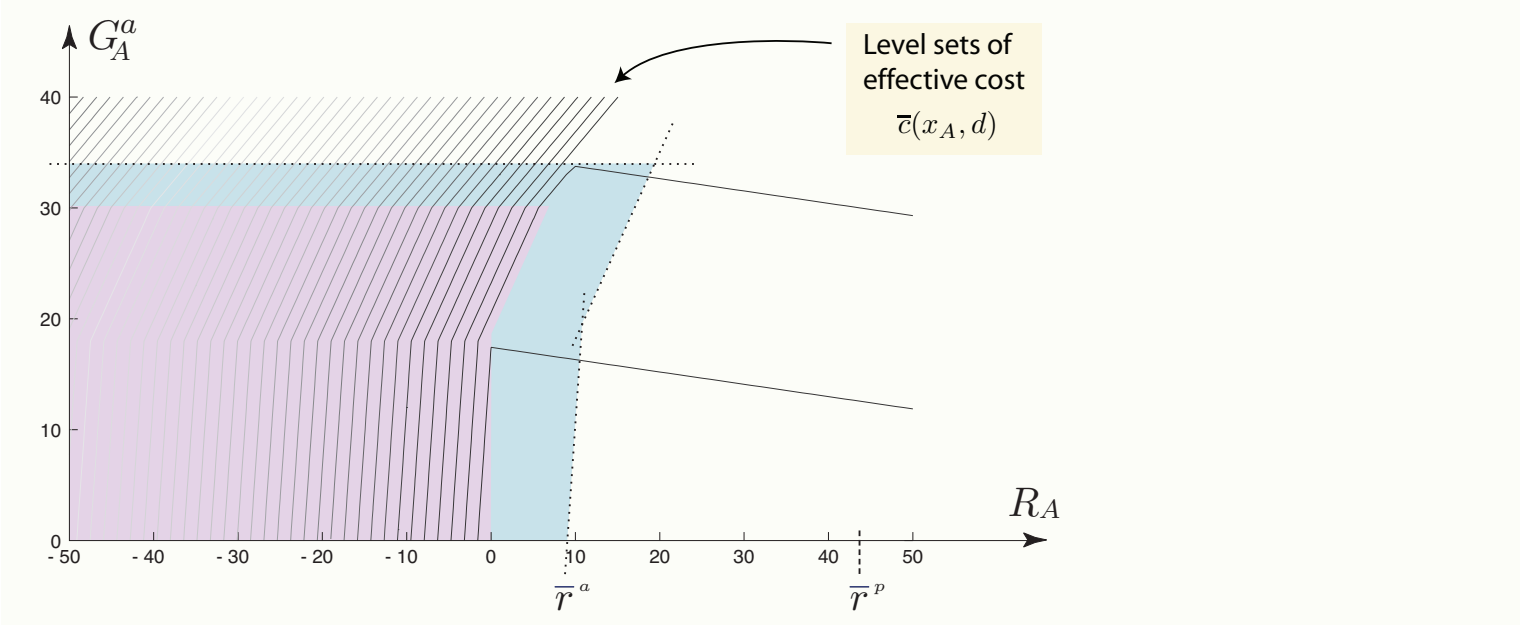
$$\textbf{min} \quad \sum_i (c^p_i g^p_i + c^a_i g^a_i + c^{bo}_i r_{i-})$$

$$\textbf{subject to} \quad \begin{array}{rclrcl} r_A & = & \sum_i (e_i - d_i) & g^a_A & = & \sum_i g^a_i \\ 0 & = & \sum_i (g^p_i + g^a_i - e_i) & 0 & = & e - d - q \end{array}$$

$$f = \Delta p \in \mathsf{F},\ e, q, g^p \in \mathbb{R}^{\ell_n},\ g^a \in \mathbb{R}^{\ell_n}_+$$

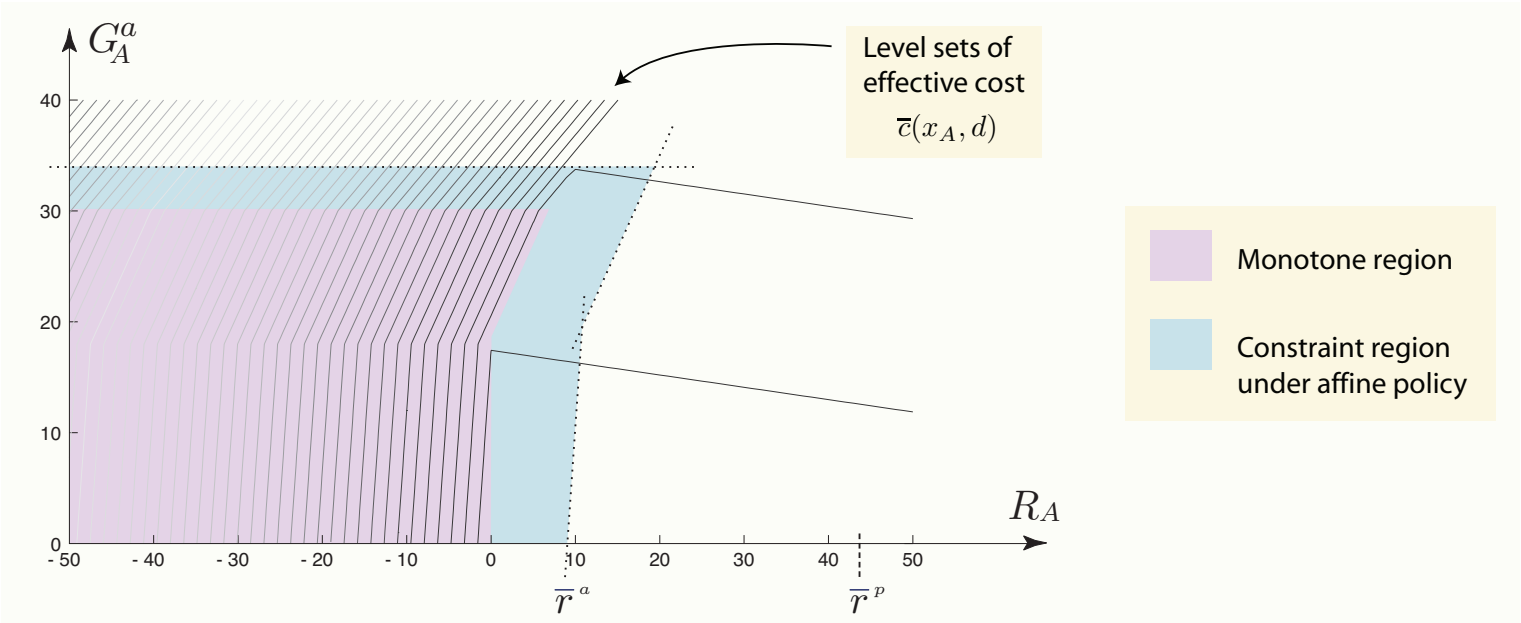
Reliability by Design in Networks

Relaxations and *Effective Cost*



Reliability by Design in Networks

Affine Policy for Relaxation



Affine policy is an affine enlargement of "monotone region" in which G_A^a ramps at maximum rate.

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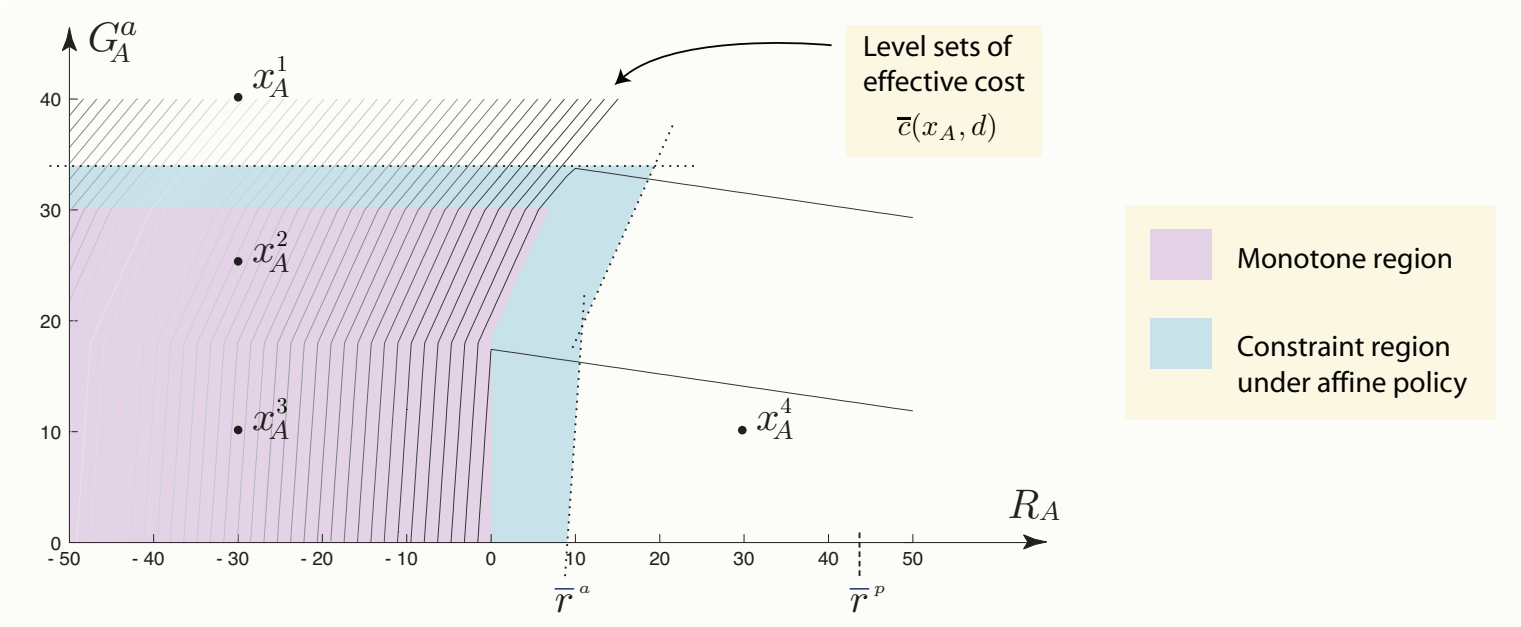
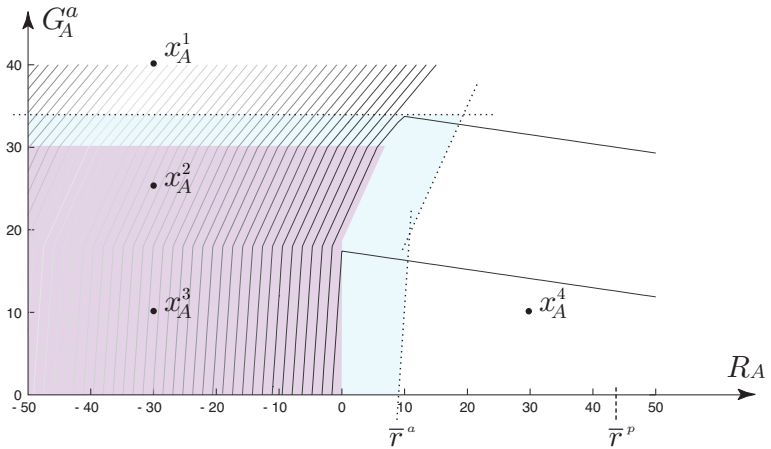


How does an affine policy respond in these four different scenarios?

Reliability by Design in Networks

Effective States in Relaxation

Effective state $\mathcal{X}^*(x_A)$:
Minimizer in the
computation of $\bar{c}(x_A, d)$.



- $\mathcal{X}^*(x_A^1)$: Dallas is experiencing black-out!

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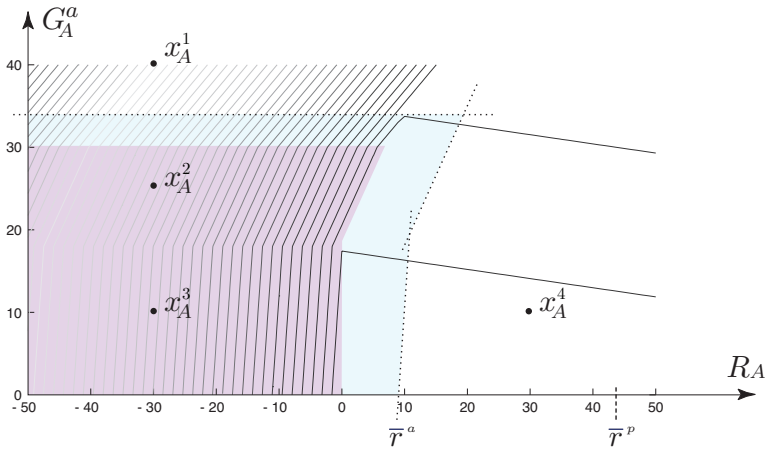

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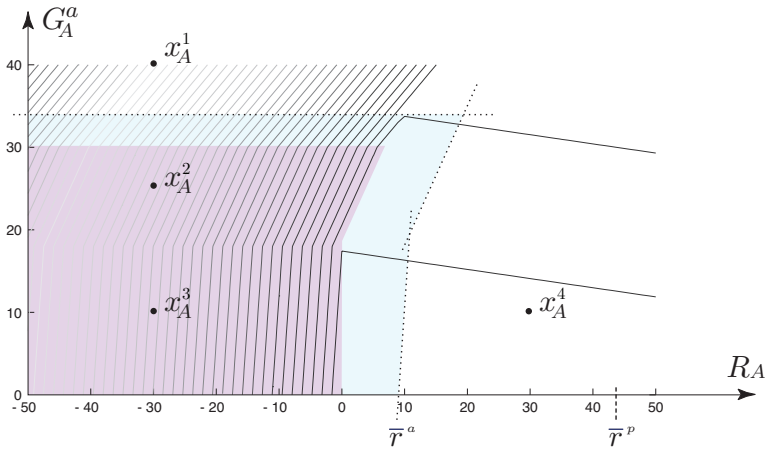

- $\mathcal{X}^*(x_A^1)$: Dallas is experiencing black-out!
  Yet ancillary service is ramping down at maximum rate!
  Each of the three transmission lines have reached their capacity limits: Nodes 2 and 3 cannot transfer reserves to node 1, and node 1 cannot extract more reserve to reduce the severity of black-out.

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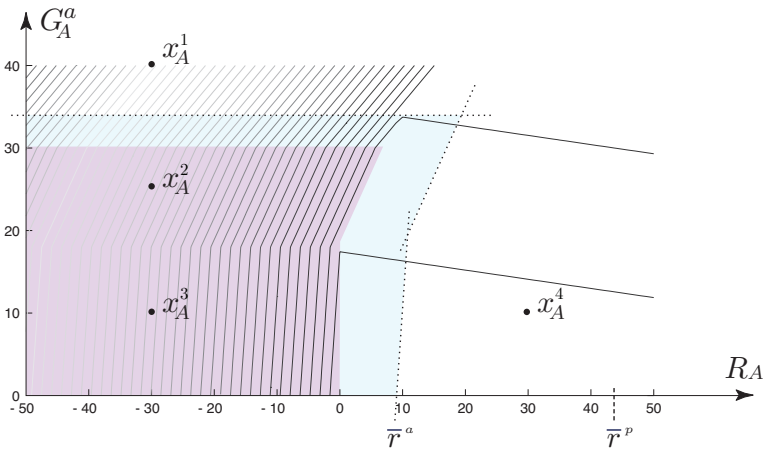

- $\mathcal{X}^*(x_A^2)$ and $\mathcal{X}^*(x_A^3)$: Black-out!

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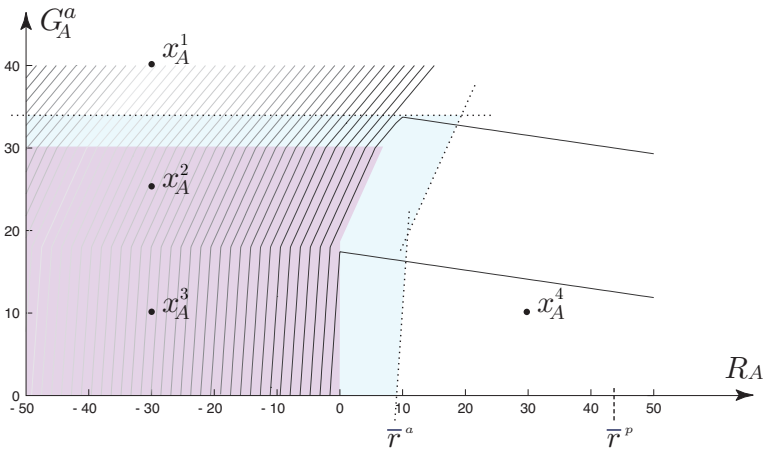

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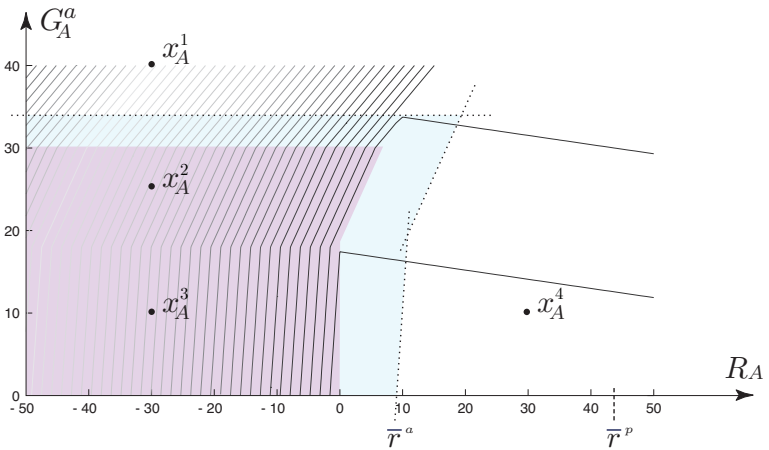

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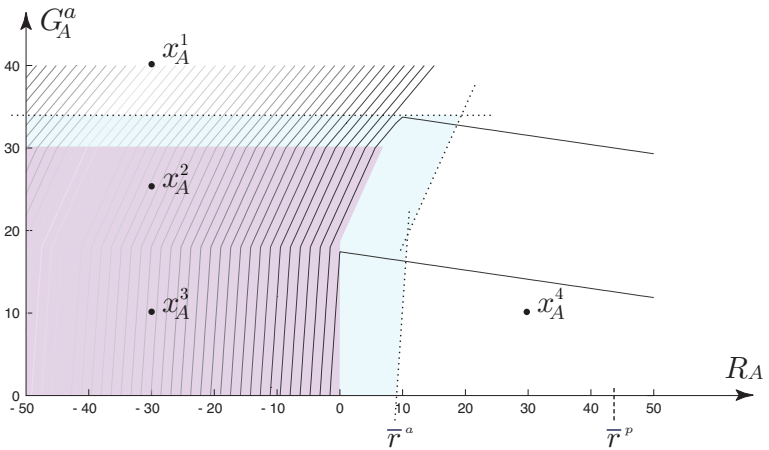

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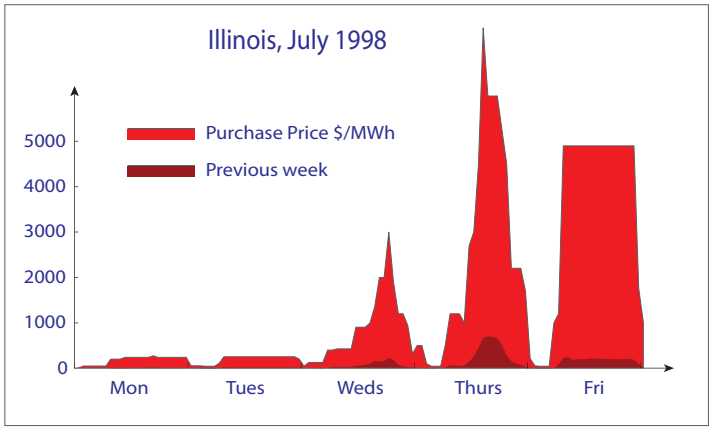
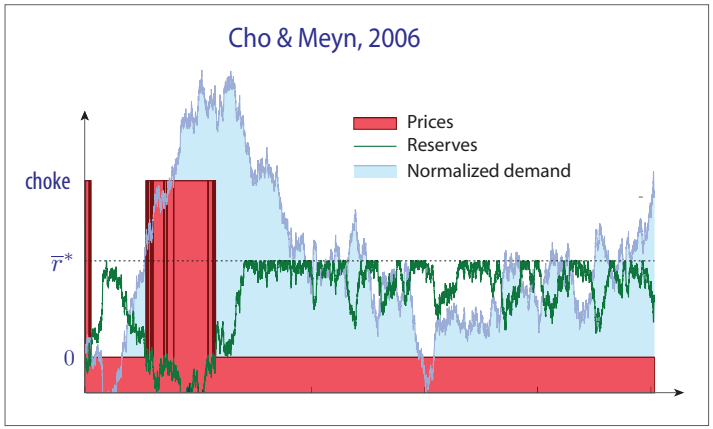
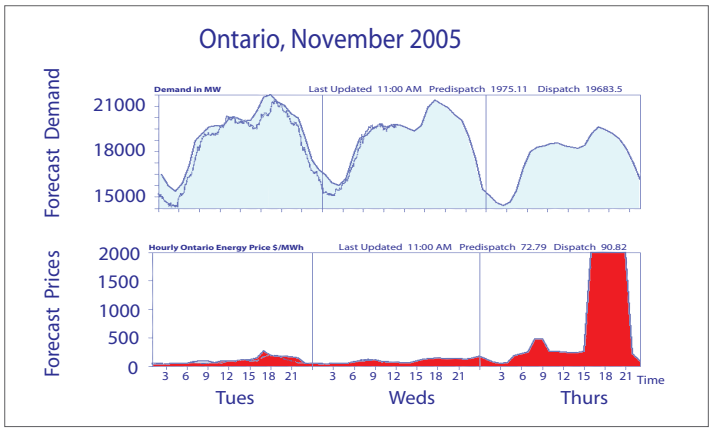
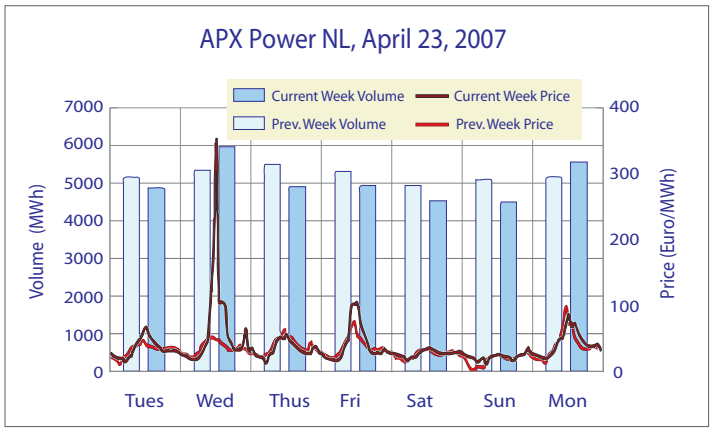
- $\mathcal{X}^*(x_A^4)$: High power reserves at each node, so that generation ramps down.









Dynamic Markets and their Equilibria

Market Structure

Day-ahead market

The day-ahead market (DAM) is cleared one day prior to the actual production and delivery of energy.

Economic world...

Forward markets for electricity which improve market efficiency and serve as a hedging mechanism

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...Physical world

Facilitate the scheduling of generating units

Market Structure

Real-time market

At close of the DAM: The ISO generates a schedule of generators to supply specific levels of power for each hour over the next 24 hour period.

RTM = Balancing Market

As supply and demand are not perfectly predictable, the RTM plays the role of fine-tuning this resource allocation process

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RTM is the focus here

Market Analysis
First Welfare Theorem

Efficient Equilibrium

$$\textbf{max} \quad K(g,d) = \mathsf{E}\Big[\int e^{-\gamma t}\big(\mathcal{W}_{\mathrm{S}}(t) + \mathcal{W}_{\mathrm{D}}(t)\big)\,dt\Big].$$

$$\textbf{subject to} \quad G_{\mathrm{S}}(t) = G_{\mathrm{D}}(t) \quad \text{for all } t$$

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"price-taking assumption"

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Price function $P(t)$ is irrelevant when $G_{\mathrm{S}}(t) = G_{\mathrm{D}}(t)$:

$$\mathcal{W}_{\mathrm{S}}(t) := P(t)G_{\mathrm{S}}(t) - cG(t)$$

$$\mathcal{W}_{\mathrm{D}}(t) := v\min(D(t), G_{\mathrm{D}}(t)) - c^{\mathrm{bo}}\max(0, -R(t)) - P(t)G_{\mathrm{D}}(t)$$

Market Analysis
First Welfare Theorem

First Welfare Theorem $\Longleftrightarrow$ Lagrangian Decomposition

$$\textbf{max} \quad K(g,d) = \mathsf{E}\Big[\int e^{-\gamma t}\Big\{\big(\mathcal{W}_{\mathrm{S}}(t) + \mathcal{W}_{\mathrm{D}}(t)\big) + \lambda(t)\big(G_{\mathrm{S}}(t) - G_{\mathrm{D}}(t)\big)\Big\}\, dt\Big]$$

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Market analysis assumptions

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Value of power

Consumer obtains v units of utility per unit of power consumed:
Utility to the consumer $= v \min(D(t), G(t))$.

What does an efficient equilibrium look like?

Answer: Marginal value

Equilibrium price

The *equilibrium price functional* is a piecewise constant function of the equilibrium reserve process,

$$p^e(r^e) = (v + c^{\mathrm{bo}})\mathbb{I}\{r^e < 0\}$$

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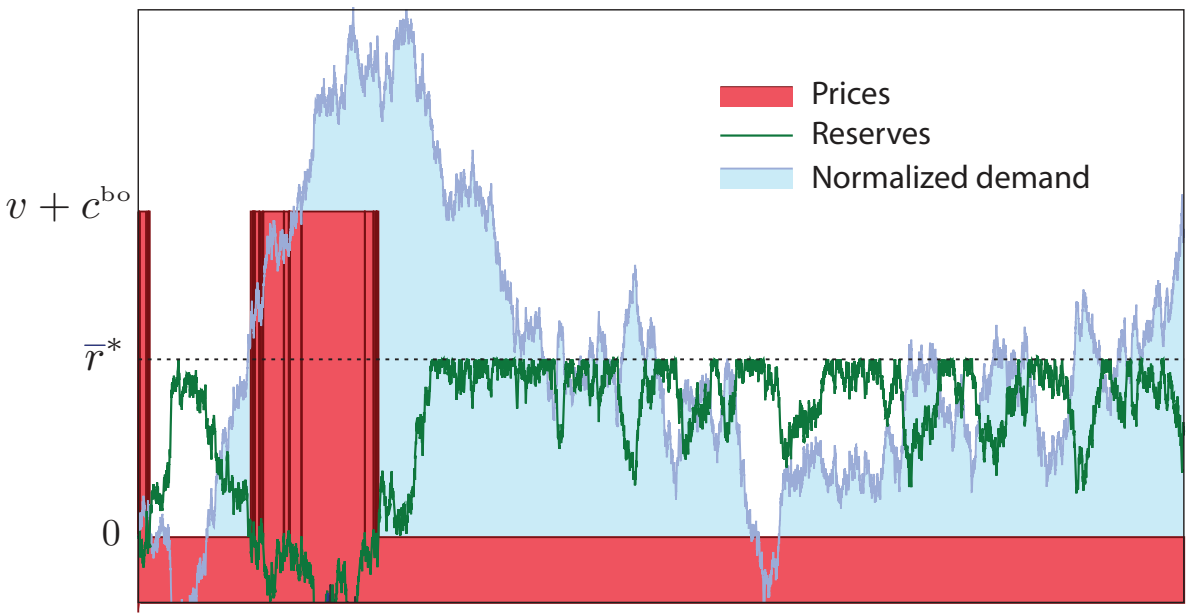
$P^*(t) = p^e(R^e(t))$ is the marginal value of power to the consumer at time t.

Market Equilibrium

Price Dynamics



$P^*(t) = p^e(R^e(t))$: The marginal value of power to the consumer

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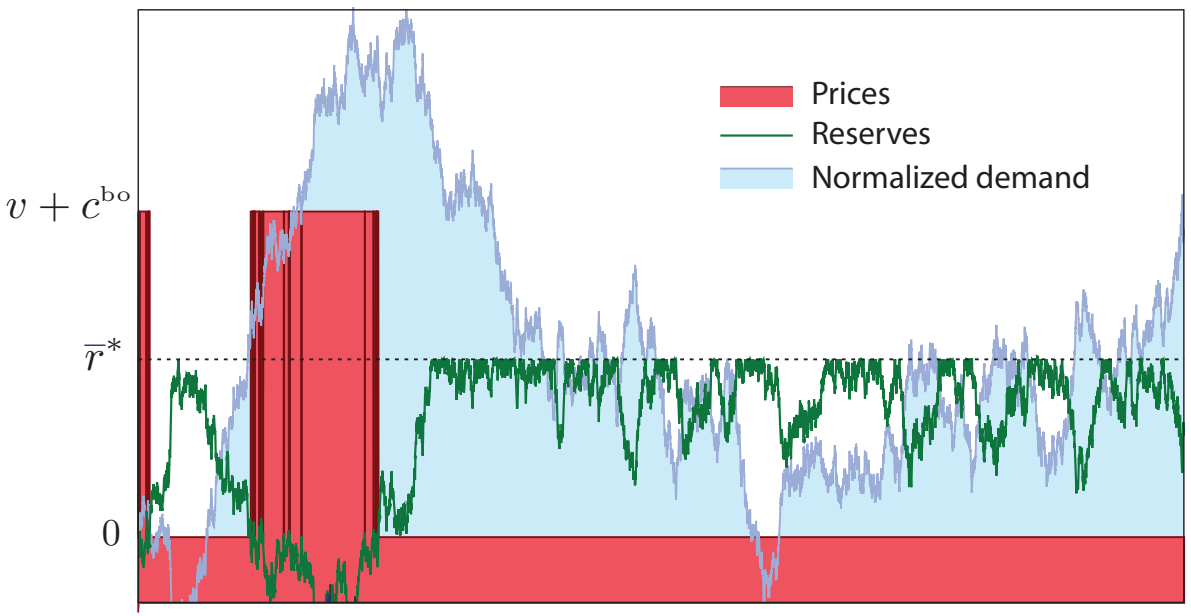
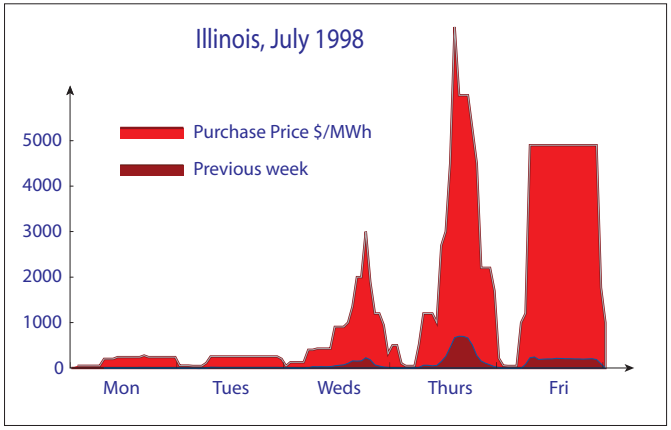
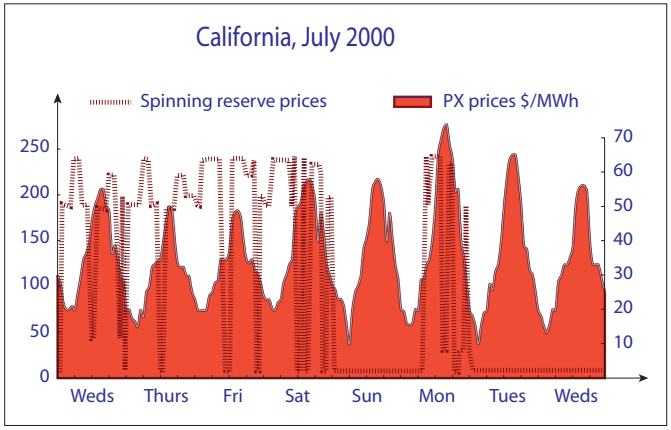


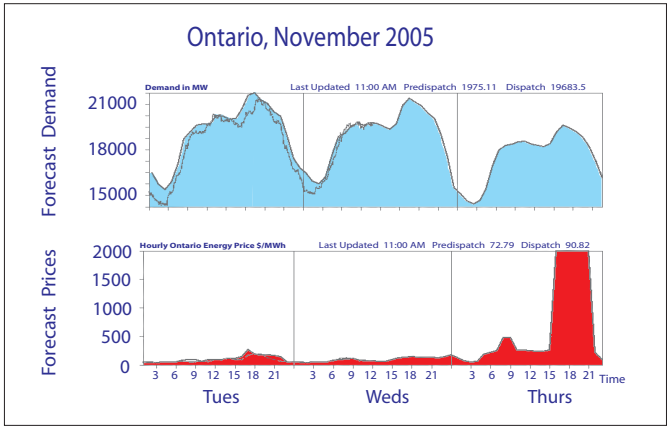
$P^*(t) = p^e(R^e(t))$: The marginal value of power to the consumer

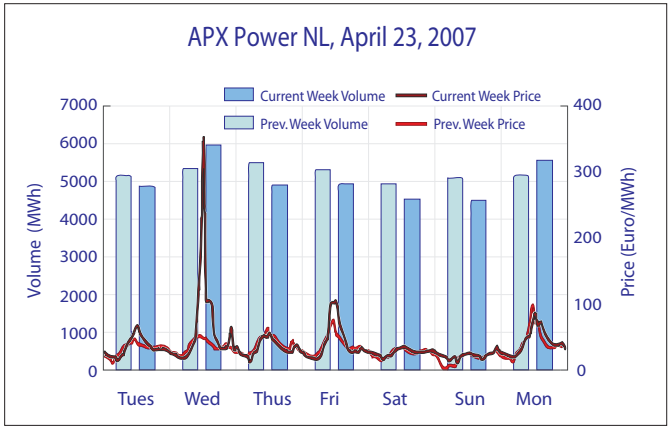
Smoother prices obtained when cost/utility are strictly convex

Familiar, right?



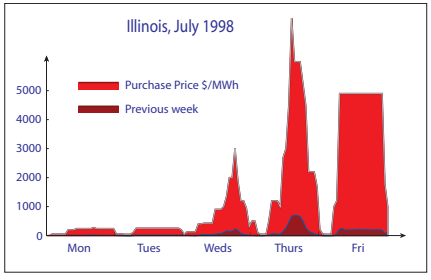


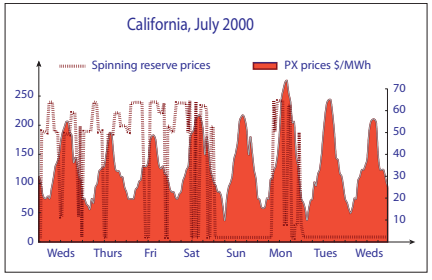


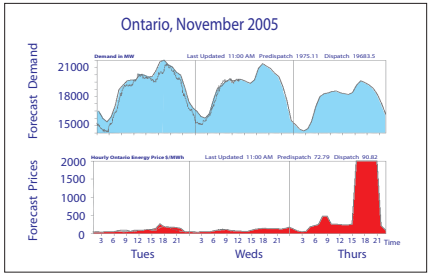


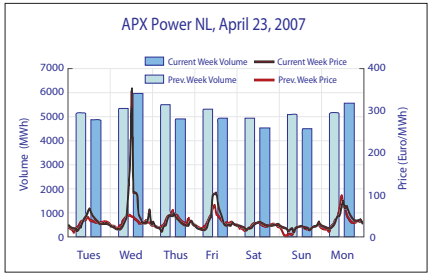
Sustainable business?

Marginal value of electricity may be $v + c^{\mathrm{bo}} =$ \$200,000/MWh!









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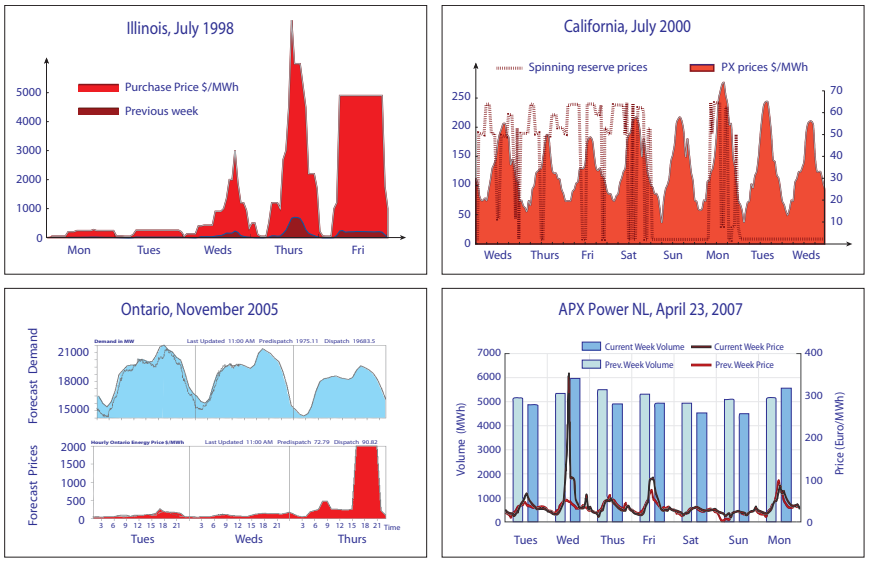

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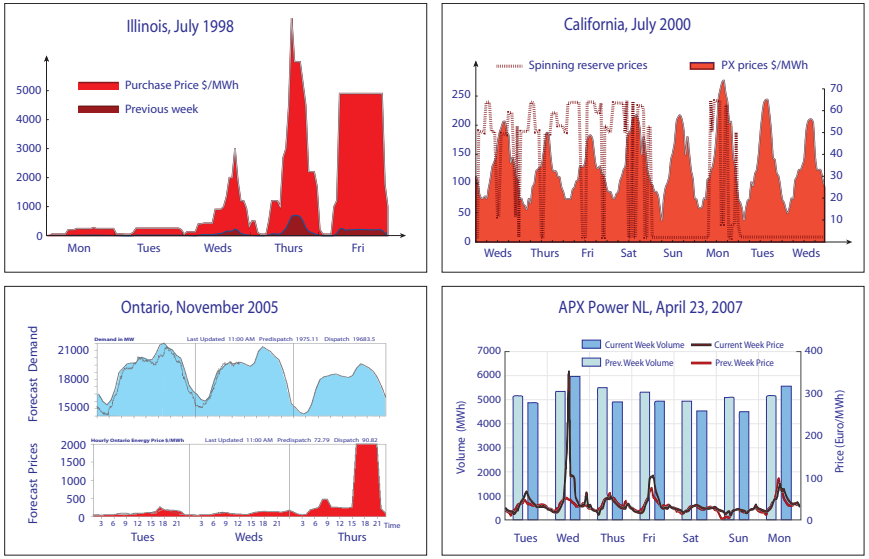

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Is this a sustainable business?

Who Commands the Wind?

Extending the dynamic model

Goal

Extend the DA/RT market model to differentiate power generated by wind and by conventional generation units

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Assumption

All the wind power available is injected into the system

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All the wind power available is injected into the system

Key issue

Conventional generators serve residual demand.

Volatility of the residual demand will result in higher reserves in the dynamic market equilibrium

Emerging issues

Not only mean energy

The details depend on both volatility of wind and its proportion of the overall generation

What about now?

If the penetration of wind resources is low, then the increase in load volatility will be negligible, and hence there will be negligible impact on the market outcomes

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What about in 2020?

Potential negative market outcomes are possible with a combination of high wind generation penetration and high volatility of wind

Who Commands the Wind?

Main findings

Volatility can have tremendous impact on the market outcome

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Asymmetric and surprising result

The supplier can achieve significant gains,
even when the consumer commands the wind

Consumers command the wind

Wind generating units are commanded by the demand side:

Consumers' and suppliers' welfare expressions

$$\begin{aligned}\mathcal{W}^{\mathrm{ttl}}_{\mathrm{D,W}}(t) = v \min(D^{\mathrm{ttl}}(t), G^{\mathrm{ttl}}(t) + G^{\mathrm{ttl}}_{\mathrm{W}}(t)) \\ - c^{\mathrm{bo}} \max(0, -R(t)) \\ - P(t)G(t) - p(t)g^{\mathrm{da}}(t)\end{aligned}$$

$$\mathcal{W}^{\mathrm{ttl}}_{\mathrm{S,W}}(t) = \big(P(t) - c^{\mathrm{rt}}\big)G(t) + \big(p(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t)$$

Consumers command the wind

Welfare expressions: DAM/RTM decomposition

$$\mathcal{W}^{\mathrm{ttl}}_{\mathrm{D,W}}(t) = \mathcal{W}^{\mathrm{rt}}_{\mathrm{D,W}}(t) + \left\{ v d^{\mathrm{da}}(t) - p(t)(d^{\mathrm{da}}(t) - g^{\mathrm{da}}_{\mathrm{W}}(t)) \right\} + \left\{ (P(t) - p(t)) r^{\mathrm{da}}_0 + v G_{\mathrm{W}}(t) \right\}$$

$$\mathcal{W}^{\mathrm{ttl}}_{\mathrm{S,W}}(t) = \mathcal{W}^{\mathrm{rt}}_{\mathrm{S,W}}(t) + \left(p(t) - c^{\mathrm{da}}\right) g^{\mathrm{da}}(t)$$

$\mathcal{W}^{\mathrm{rt}}_{\mathrm{D,W}}(t)$, $\mathcal{W}^{\mathrm{rt}}_{\mathrm{S,W}}(t)$: Welfare obtained in the RTM with residual demand $D^{\mathrm{net}}(t)$.

Suppliers command the wind

The other alternative is to consider wind as part of the supply side,

Consumers' welfare

$$\begin{aligned}\mathcal{W}^{\mathrm{ttl}}_{\mathrm{D,W}}(t) &= v\min(D^{\mathrm{ttl}}(t), G^{\mathrm{ttl}}(t) + G^{\mathrm{ttl}}_{\mathrm{W}}(t)) \\ &\quad - c^{\mathrm{bo}}\max(0, -R(t)) \\ &\quad - P(t)(G(t) + G_{\mathrm{W}}(t)) - p(t)(g^{\mathrm{da}}(t) + g^{\mathrm{da}}_{\mathrm{W}}(t)) \\ &= \mathcal{W}^{\mathrm{rt}}_{\mathrm{D,W}}(t) \\ &\quad + \left\{(v - p(t))d^{\mathrm{da}} + (P(t) - p(t))r^{\mathrm{da}}_0\right\} \\ &\quad + (v - P(t))G_{\mathrm{W}}(t)\end{aligned}$$

Suppliers command the wind

Suppliers' welfare

$$\begin{aligned}\mathcal{W}^{\mathrm{ttl}}_{\mathrm{S,W}}(t) &= \big(P(t) - c^{\mathrm{rt}}\big)G(t) + P(t)G_{\mathrm{W}}(t) \\ &\quad + \big(p(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t) + p(t)g^{\mathrm{da}}_{\mathrm{W}}(t) \\ &= \mathcal{W}^{\mathrm{rt}}_{\mathrm{S,W}}(t) \\ &\quad + \big(p(t) - c^{\mathrm{da}}\big)g^{\mathrm{da}}(t) + P(t)G_{\mathrm{W}}(t) + p(t)g^{\mathrm{da}}_{\mathrm{W}}(t)\end{aligned}$$

The welfare gain $p(t)g^{\mathrm{da}}_{\mathrm{W}}(t)$ is now taken by the suppliers

Numerical Results: Setting the stage I

Parameters used in all experiments:

Parameters [$/MWh]

Cost of blackout and value of consumption:

$$c^{\mathrm{bo}} = 200,000 \quad v = 50$$

Cost and prices of generation: $c^{\mathrm{rt}} = 30$, and

$$c^{\mathrm{da}} = 0.75 * c^{\mathrm{rt}}, \quad p^{\mathrm{da}} = 0.85 * c^{\mathrm{rt}}$$

Discount factor: $\gamma = 1/12$ (12 hour time horizon).

Numerical Results: Setting the stage II

Parameters (physical)

Ramp limit: $\zeta = 200$ MW/min

Mean demand: $\bar{D} = 50,000$ MW

Standard deviation: $\sigma = 500$ MW.

Numerical Results: Setting the stage III

Penetration and volatility

Coefficient of variation to capture the relative volatility of wind:

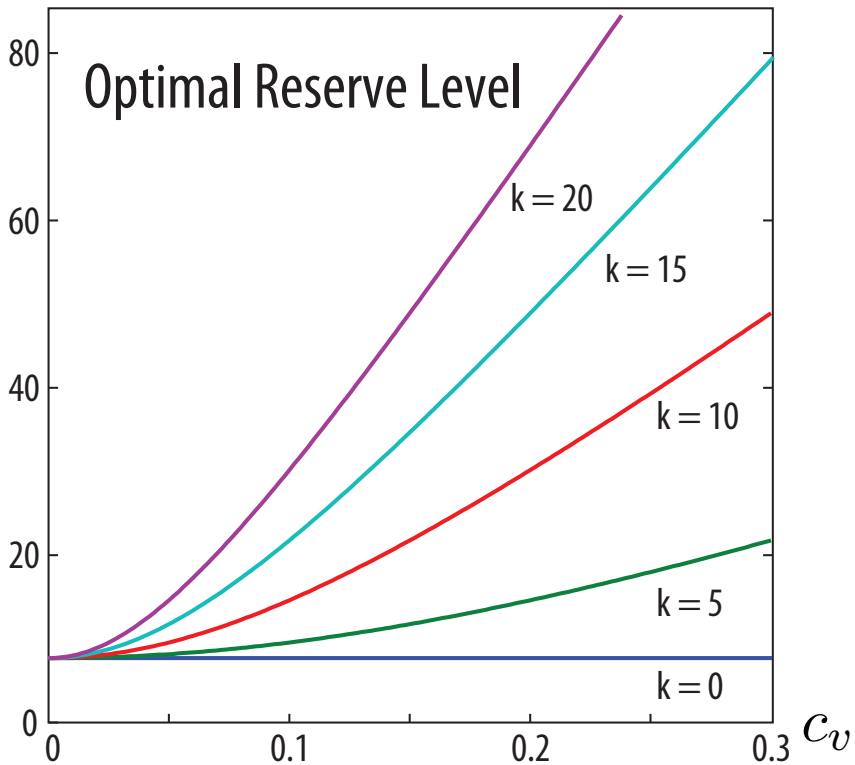
$$c_v = \frac{\sigma_w}{\mathsf{E}[G_{\mathrm{W}}^{\mathrm{ttl}}(t)]}$$

Percentage of wind penetration:

$$k = 100 * \mathsf{E}[G_{\mathrm{W}}^{\mathrm{ttl}}(t)]/\bar{D}$$

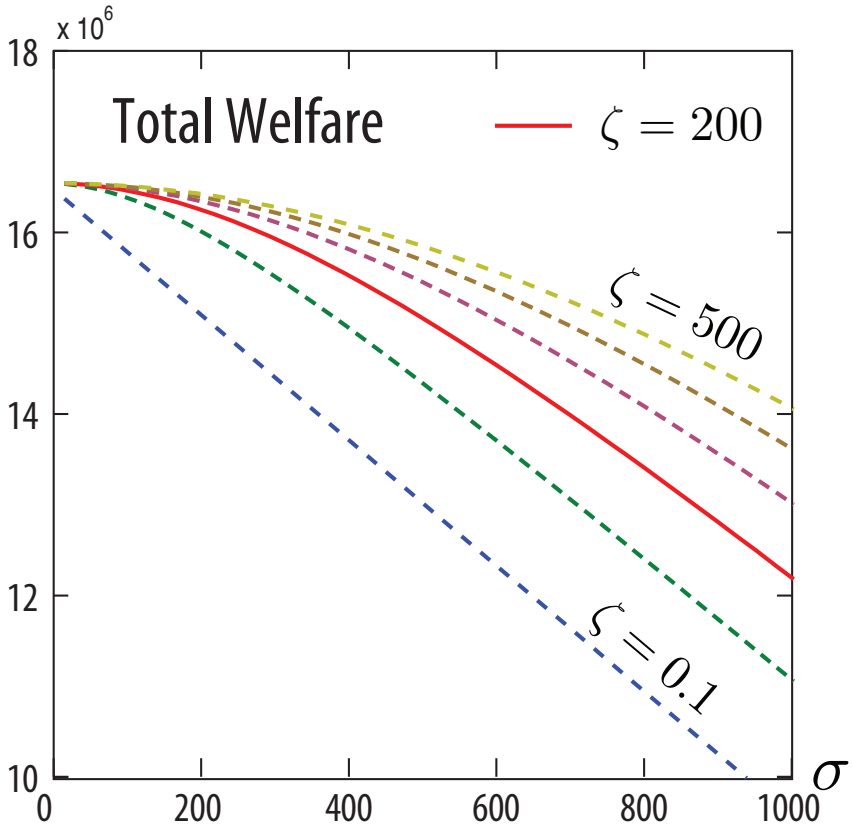
Numerical Results: Volatility Impacts

Optimal reserves rise with volatility



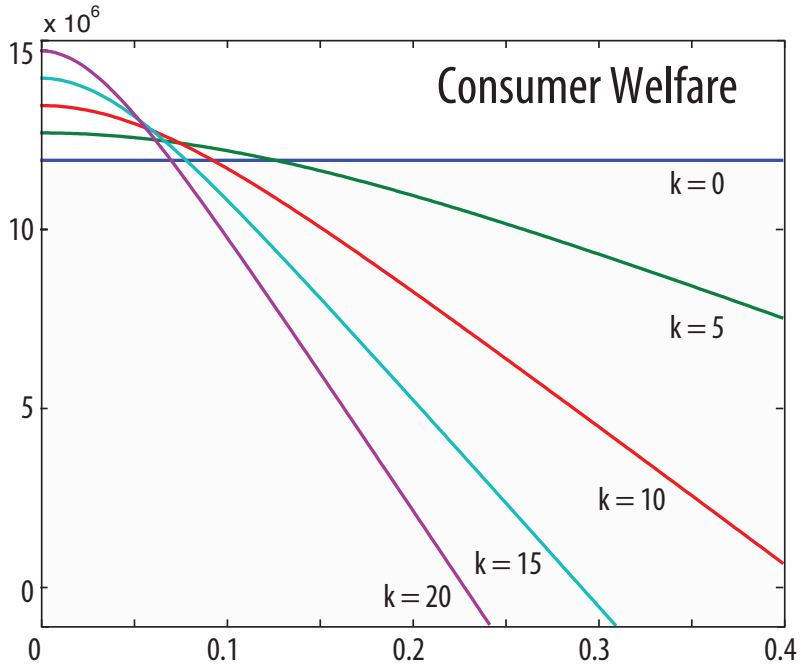
Numerical Results: Volatility Impacts

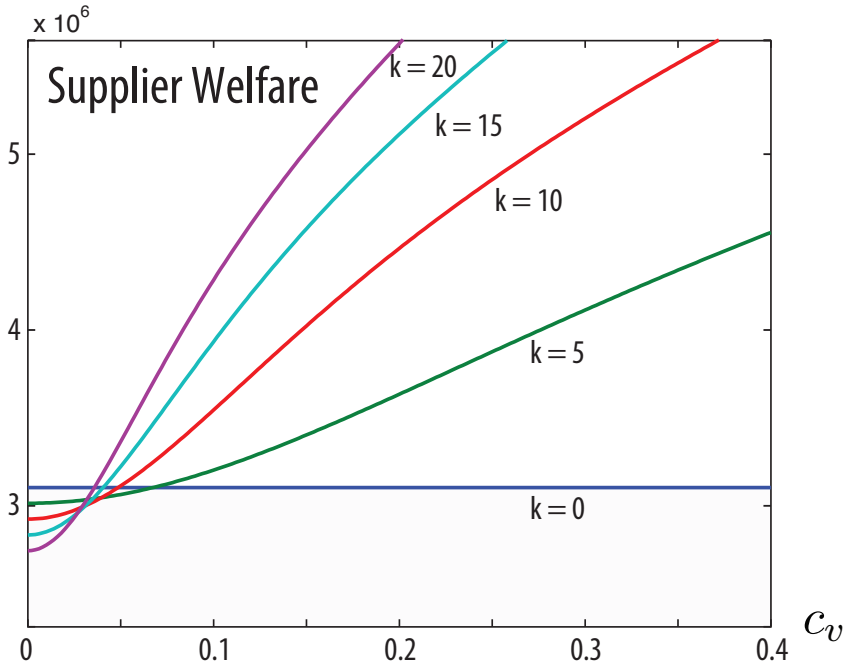
Social planner's welfare falls with volatility



Numerical Results: Consumers command the wind

Consumer welfare falls and supplier welfare rises with volatility





Research Frontiers

Research Frontiers

Strategic behavior

Q1

Building bridges

Research Frontiers

Strategic behavior

Q1

Building bridges between this research and what was described on Monday by Hobbs & Ralph — How to incorporate dynamics and uncertainty in a strategic setting?

> How could two smart people come to such different conclusions? I had to get to the bottom of this. –MacKay 2009

Research Frontiers

Strategic behavior

Q1

Building bridges between this research and what was described on Monday by Hobbs & Ralph — How to incorporate dynamics and uncertainty in a strategic setting?

> How could two smart people come to such different conclusions? I had to get to the bottom of this. –MacKay 2009

Analysis of strategic behavior will be possible if agent behavior is suitably constrained

Research Frontiers

Strategic behavior

Q2

Learning

Research Frontiers

Strategic behavior

Q2

Learning Once a market framework is in place, agents will learn over time to optimize their reward

Research Frontiers

Strategic behavior

Q2

Learning Once a market framework is in place, agents will learn over time to optimize their reward *Q-learning and TD-learning are potential approaches*

Research Frontiers

Controlling supply and demand

Q3

The impact of demand management and storage on the market

Research Frontiers

Controlling supply and demand

Q3

The impact of demand management and storage on the market

Q4

The impact of *supply* management and storage on the market.
e.g., modern wind turbines allow pitch angle control.

Research Frontiers

Controlling supply and demand

Q3

The impact of demand management and storage on the market

Q4

The impact of *supply* management and storage on the market. e.g., modern wind turbines allow pitch angle control.

Beyond arithmetic: What are the dynamical properties of the power grid in LA county with dynamic pricing, hybrid cars, and automated water pumping?

Research Frontiers

Facing volatility

Q5

Not just "missing money" —

Research Frontiers

Facing volatility

Q5

Not just "missing money" — Mechanisms to reduce consumers *and* suppliers exposure to volatility.

Research Frontiers

Facing volatility

Q5

Not just "missing money" — Mechanisms to reduce consumers *and* suppliers exposure to volatility.

Especially supply-side volatility with introduction of renewable energy, such as wind

Research Frontiers

Long-term incentives

Q6

How to create policies to protect participants on both sides of the market, while creating incentives for R&D on renewable energy?

Research Frontiers

Long-term incentives

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The economic notion of *efficiency* disregards issues such as emissions, or public safety.

Research Frontiers

Long-term incentives

Q6

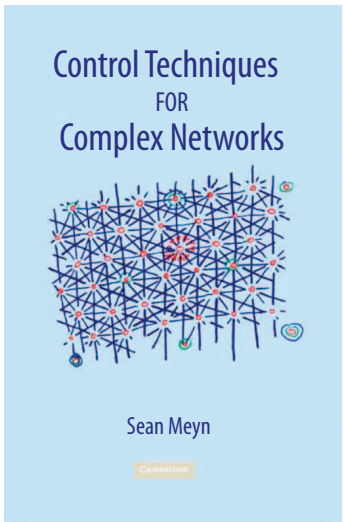
How to create policies to protect participants on both sides of the market, while creating incentives for R&D on renewable energy?

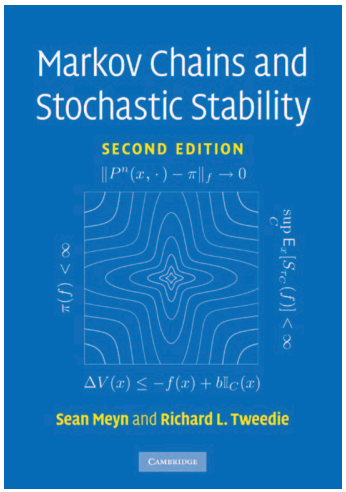
The economic notion of *efficiency* disregards issues such as emissions, or public safety.

We hope that the ideas described here will form a building block for constructing and analyzing far more intricate models, taking into account a broader range of issues.

Thanks!

Celebrating with Dutch Babies after finishing *The value of volatile resources...*


Control Techniques
FOR
Complex Networks
Sean Meyn


Markov Chains and
Stochastic Stability
SECOND EDITION
Sean Meyn and Richard L. Tweedie

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