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Grasping Plasma Turbulence Fundamentals: Where do we stand?

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Kinetic-scale turbulence in laboratory and space plasmas
Cambridge University, 20 July 2010

The plasma turbulence challenge

The ultimate answer to Life,
the Universe and Everything is...

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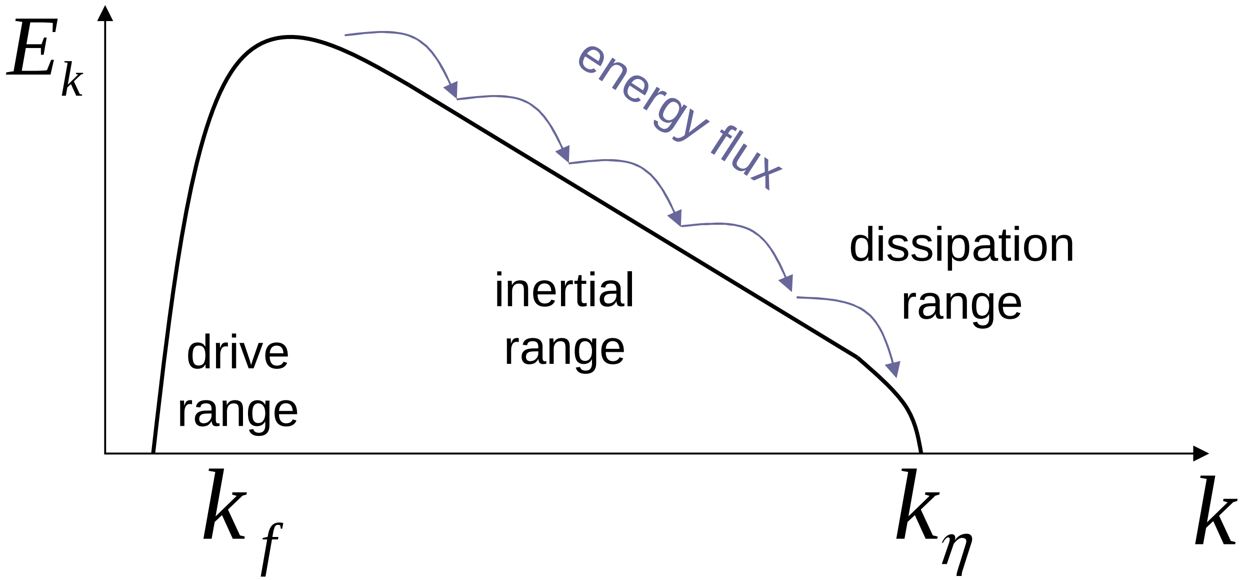
How about our neighbors?
The fluid turbulence challenge

The Navier-Stokes equation in action

$$(\partial_t + \vec{v}\cdot\nabla)\,\vec{v} = -\nabla p + \nu\nabla^2\vec{v} \qquad \nabla\cdot\vec{v} = 0$$



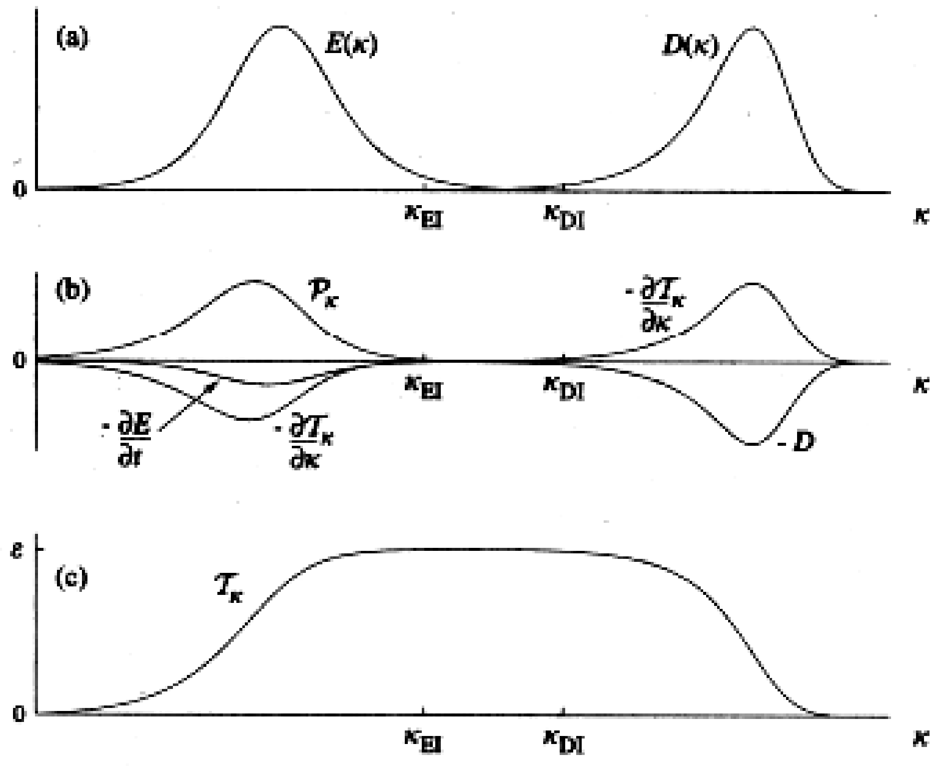
Turbulence as a **local cascade in wave number space…**

Spectral energy balance

For homogeneous turbulence:

$$\frac{\partial}{\partial t}E(k,t) = \boxed{P_k(k,t)} \boxed{-\frac{\partial}{\partial k}T_k(k,t)} \boxed{-2\nu k^2 E(k,t)}$$

Production **Spectral transfer** **Dissipation**



Kolmogorov’s theory from 1941

K41 is based merely on intuition and dimensional analysis – it is *not* derived rigorously from the Navier-Stokes equation

Key assumptions:

- Scale invariance – like, e.g., in critical phenomena
- Central quantity: energy flux ε

$$E = \frac{1}{2V}\int v^2 \, d^3x = \int_0^\infty E(k)\, dk$$

$$\boxed{E(k) = C\, \varepsilon^{2/3}\, k^{-5/3}}$$

Quantity	Dimension
Wave number	1/length
Energy per unit mass	$\text{length}^2/\text{time}^2$
Energy spectrum $\mathcal{E}(k)$	$\text{length}^3/\text{time}^2$
Energy flux ε	$\text{energy/time} \sim \text{length}^2/\text{time}^3$

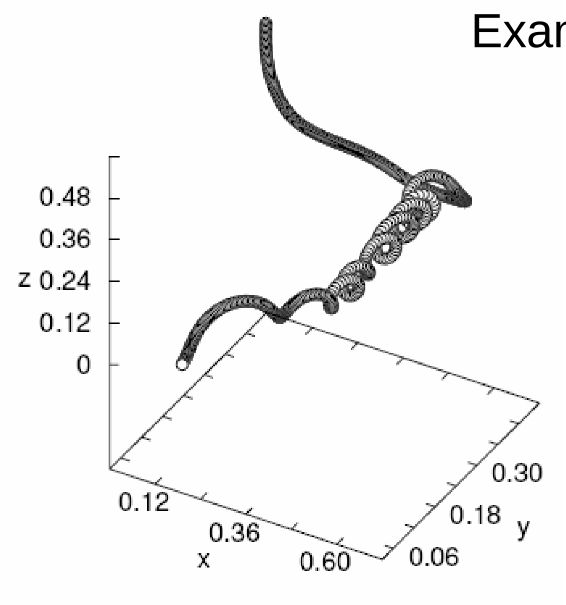
This is the most famous turbulence result: the “-5/3” law.

However, K41 is fundamentally flawed: Scale invariance is broken!

Key open issues: Inertial range

- Is the inertial range physics universal (for Re $\rightarrow \infty$)?
- If so, can one derive a rigorous IR theory from the NSE?
- How should one, in general, handle the interplay between randomness and coherence?

Biferale et al. 2005



Example: Trapping of tracers in vortex filaments

Note:

The observed deviations from self-similarity can be reproduced qualitatively by relatively simple vortex models.

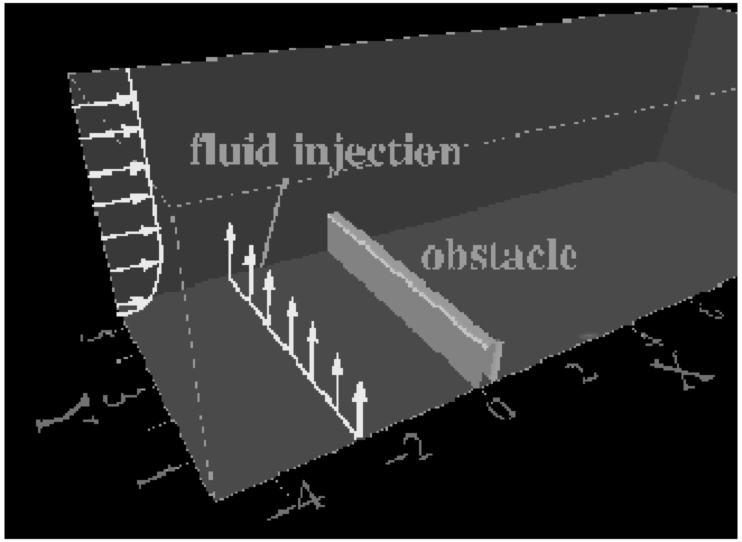
See, e.g., Wilczek, Jenko & Friedrichs, PRE 2008

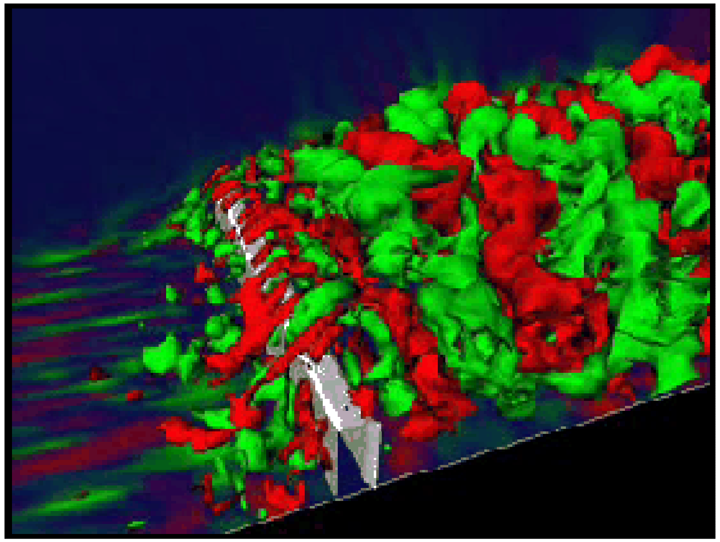
Key open issues: Drive range

- Often, one is interested mainly in the *large* scales. Here, one encounters an interesting interplay between linear (drive) and nonlinear (damping) physics. – Is it possible to remove the small scales?

- Candidates: LES, Dynamical systems approach etc.

Orellano and Wengle 2001





How about us?
Fundamental open issues in plasma turbulence research

The gyrokinetic equations in action

$$f = f(\mathbf{X}, v_{\|}, \mu; t)$$

Advection/Conservation equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{X}} \cdot \frac{\partial f}{\partial \mathbf{X}} + \dot{v}_{\|} \frac{\partial f}{\partial v_{\|}} = 0$$

X = gyrocenter position
$v_{\|}$ = parallel velocity
μ = magnetic moment

$$\dot{\mathbf{X}} = v_{\|}\mathbf{b} + \frac{B}{B_{\|}^{*}}\left(\frac{v_{\|}}{B}\bar{\mathbf{B}}_{1\perp} + \mathbf{v}_{\perp}\right)$$

$$\mathbf{v}_{\perp} \equiv \frac{c}{B^2}\bar{\mathbf{E}}_1 \times \mathbf{B} + \frac{\mu}{m\Omega}\mathbf{b} \times \nabla(B + \bar{B}_{1\|}) + \frac{v_{\|}^2}{\Omega}(\nabla \times \mathbf{b})_{\perp}$$

$$\dot{v}_{\|} = \frac{\dot{\mathbf{X}}}{m v_{\|}} \cdot \left(e\bar{\mathbf{E}}_1 - \mu \nabla(B + \bar{B}_{1\|})\right)$$

Appropriate field equations

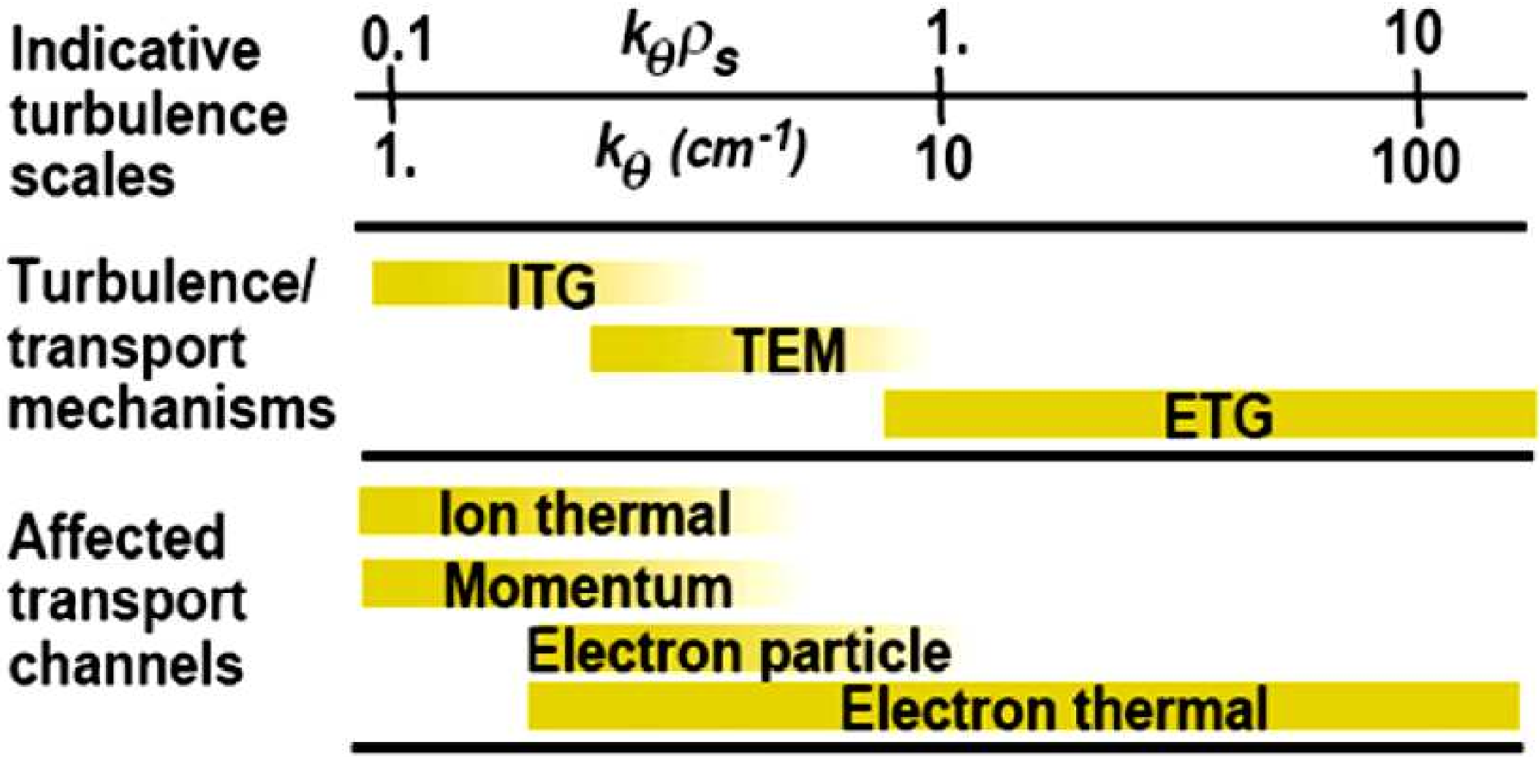
$$\frac{n_1}{n_0} = \frac{\bar{n}_1}{n_0} - \left(1 - \|I_0^2\|\right)\frac{e\phi_1}{T} + \|x I_0 I_1\|\frac{B_{1\|}}{B}$$

$$\nabla_{\perp}^2 A_{1\|} = -\frac{4\pi}{c}\sum \bar{J}_{1\|}$$

$$\frac{B_{1\|}}{B} = -\sum \epsilon_{\beta}\left(\frac{\bar{p}_{1\perp}}{n_0 T} + \|x I_1 I_0\|\frac{e\phi_1}{T} + \|x^2 I_1^2\|\frac{B_{1\|}}{B}\right)$$

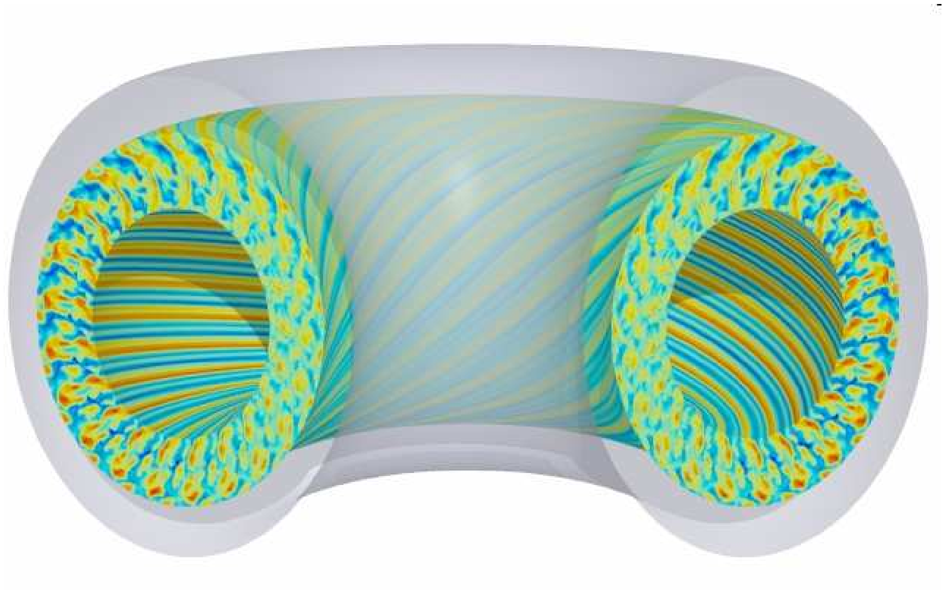
The kinetic version of reduced MHD…

Microinstabilities driving plasma turbulence



Doyle et al.

Some fundamental open issues



- How do the various microinstabilities saturate?
- How is the (free) energy distributed and dissipated?
- How useful is the concept of an inertial range?
- What is the role of sub-ion-gyroradius scales?

What do we really know about nonlinear saturation?

The drift wave / zonal flow paradigm

Many years of studying ITG turbulence
(mostly using adiabatic electrons)
led us to think that
the physics of nonlinear saturation is
synonymous with zonal flow shearing.

Is this view really correct?

Historical parallels

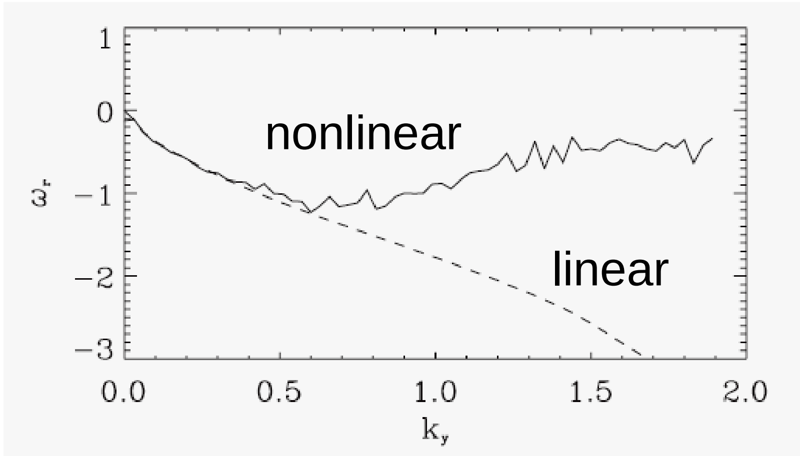
Gaul is entirely occupied by the Romans.

Well, not entirely...

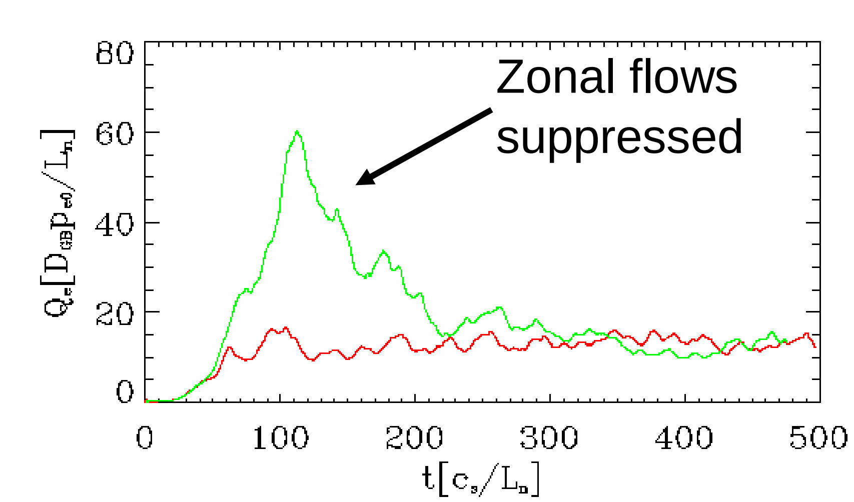
Trapped electron mode turbulence

Pure TEM turbulence simulations [Dannert & Jenko, PoP 2005]:

- In the drive range, nonlinear and linear frequencies are identical



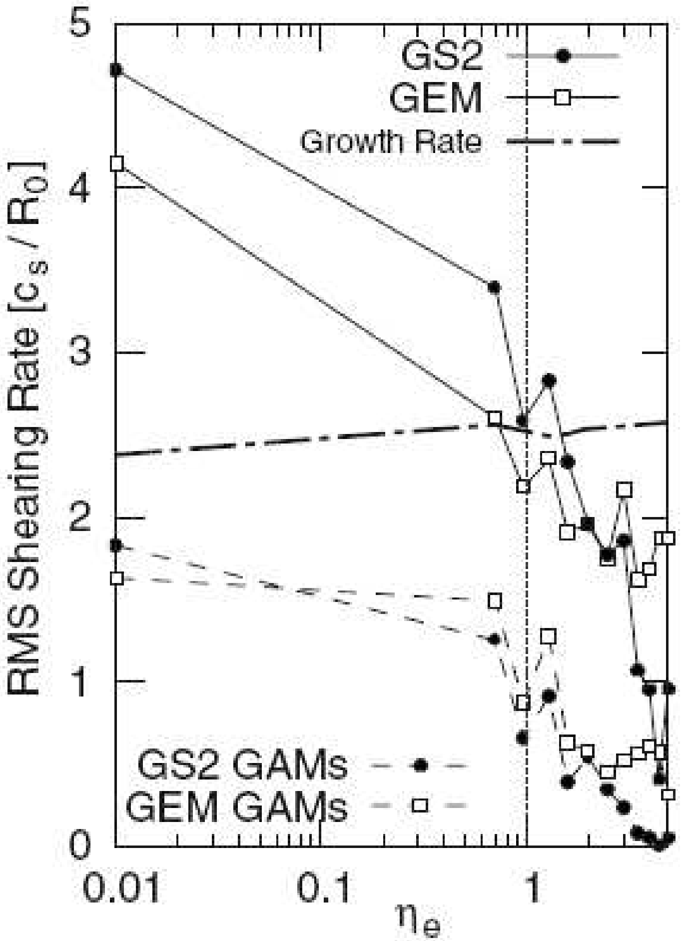
- In the drive range, there is no significant shift of cross phases w.r.t. linear ones



– No dependence of transport level on presence or absence of zonal flows

ZF / Non-ZF regimes

Ernst et al., PoP 2009



ExB shearing rates exceed the growth rate *only* for $\eta_e < 1$

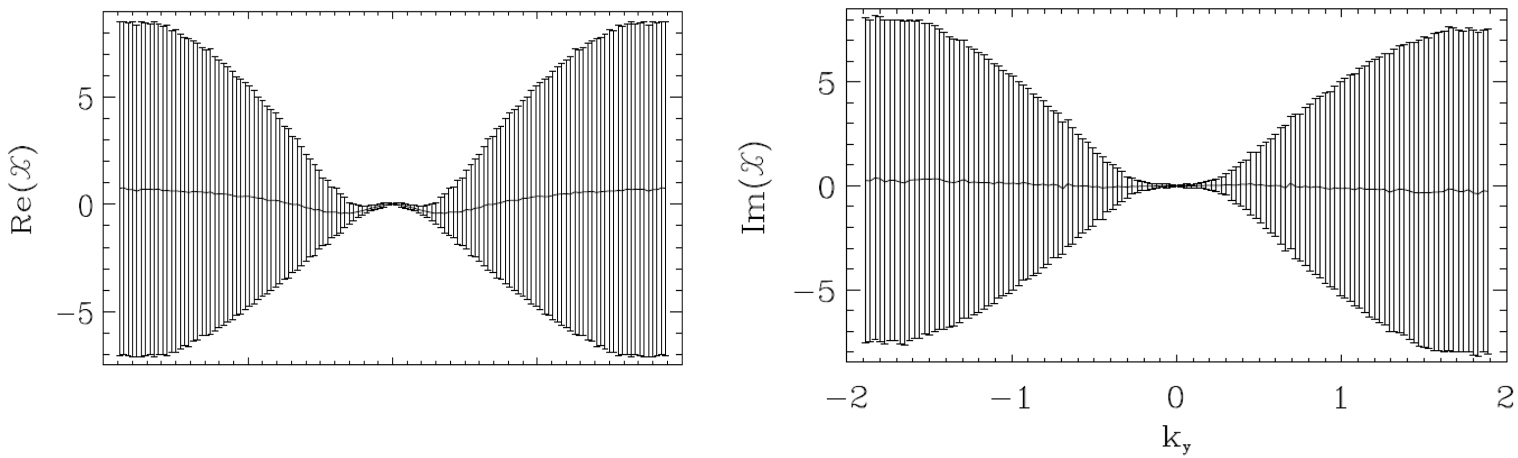
For mainly temperature gradient driven TEM turbulence, ZFs (and GAMs) are relatively weak

Thus, in a wide region of parameter space, the standard drift-wave / ZF paradigm does not hold

Theory-motivated statistical analysis

- Both weak and strong turbulence theories suggest that the ExB nonlinearity can be represented by a coherent part $\mathcal{N}l[g] \sim g$ and a random noise part

- $\mathcal{N}l[g]$ and g are fluctuating quantities; minimizing the model error $\langle|\mathcal{N}[g] - \mathcal{X}g|^2\rangle$, we obtain $\mathcal{X} = \langle g^*\mathcal{N}[g]\rangle / \langle|g|^2\rangle$

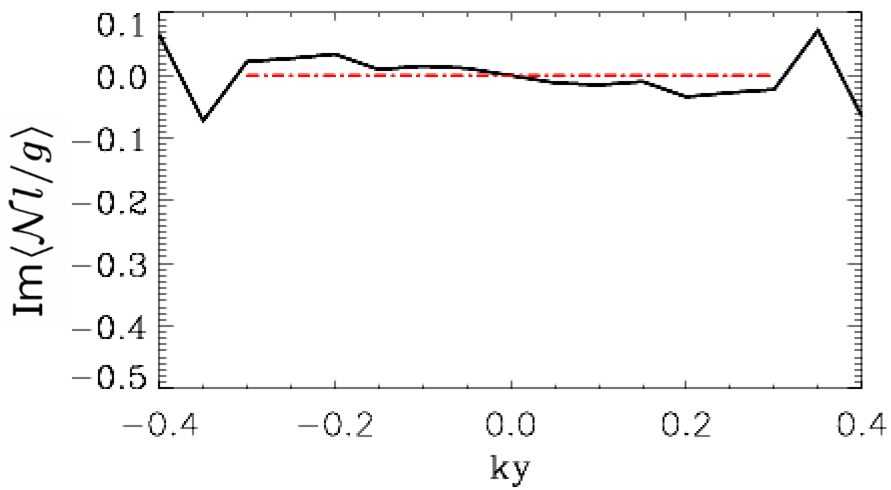


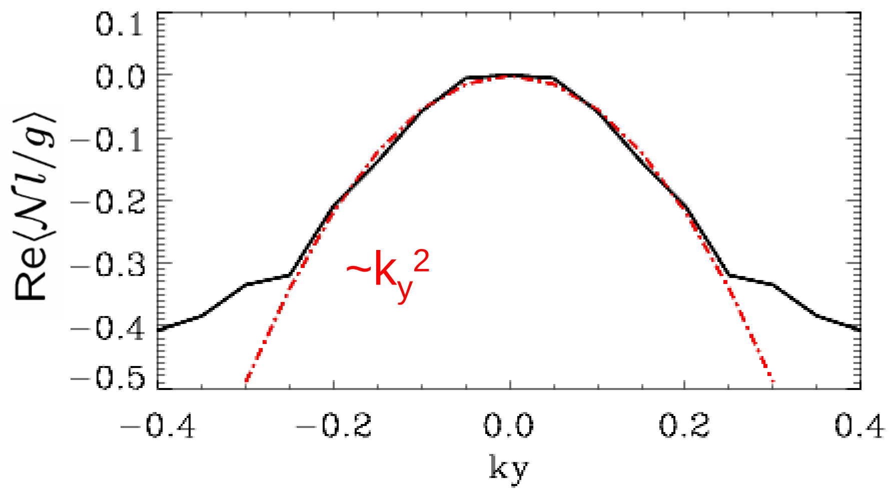
Saturation of TEMs: "eddy damping"

Merz & Jenko, PRL 2008

Low-k_y drive range: large transport contributions, but small random noise; here, one finds:

$$\mathcal{N}l[g] \simeq D(-k_\perp^2)g = D\nabla_\perp^2 g$$



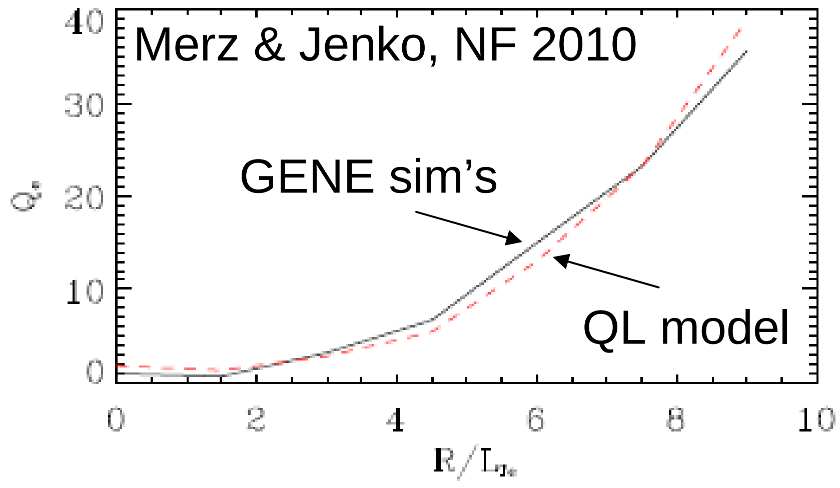


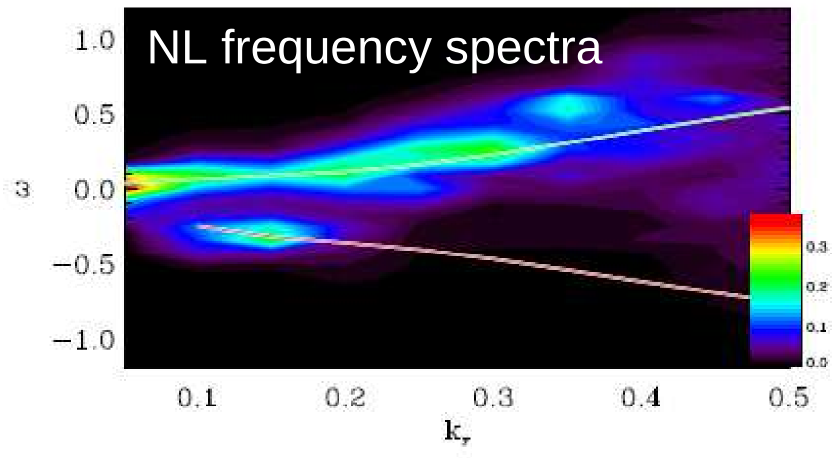
This is in line with various theories, including Resonance Broadening Theory (Dupree), MSR formalism (Krommes), Dressed Test Mode Approach (Itoh).

Implications for transport modeling

Results motivate use of the following quasilinear model:

$$Q_e \propto \max_{k_y} \left[\frac{\gamma_l(k_y)}{k_y^2 \left(1 + \hat{s}^2 \| z^2 \|\right)} \right] \left(\frac{R}{L_{T_e}} + \frac{R}{L_n} \right)$$





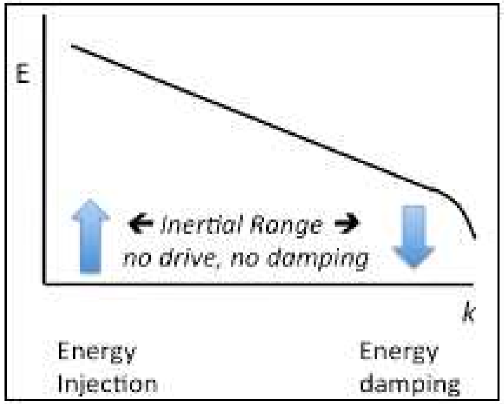
In situations where ITG modes and TEMs compete, they can coexist, and there can be non-trivial nonlinear interactions

Dissipation & cascades in plasma turbulence

D.R. Hatch, P.W. Terry, W.M. Nevins, F. Merz & FJ

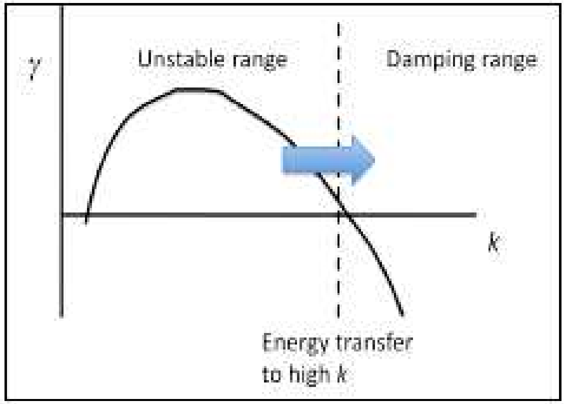
Turbulence in fluids and plasmas – Three basic scenarios

1.Hydrodynamic cascade



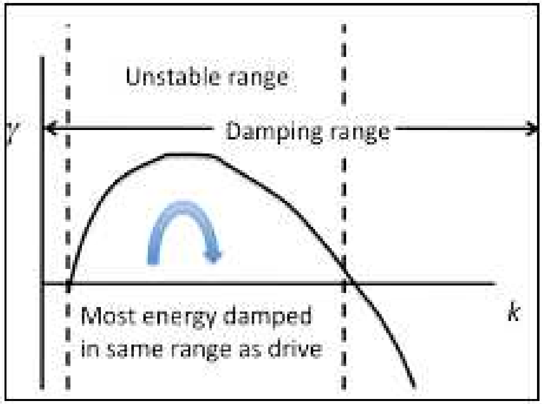
Inertial range
→ no dissipation
→scale invariant dynamics
→power law spectrum

2. Conventional μ-turbulence



Energy transfer to high *k*
like hydro – no inertial range
adjacent unstable,
damping ranges

3. Saturation by damped eigenmode



Energy can go to high *k*
but most of it is lost at
low *k* in driving range

Saturation via damped eigenmodes

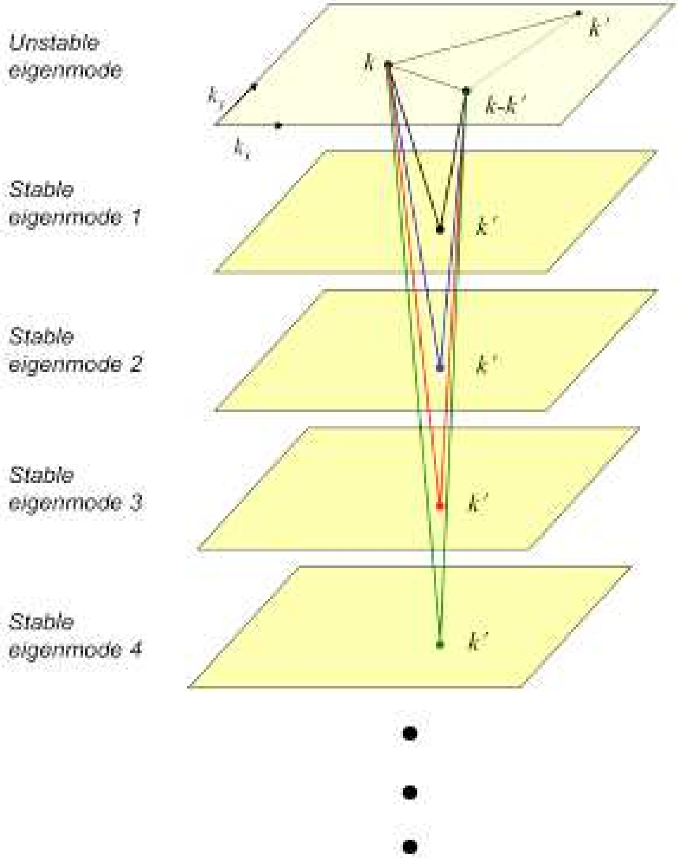
Plasma dispersion relation has multiple roots

- One root unstable → drives turbulence (TEM, ITG, ETG...)
- Other roots can be damped for all *k*
- Fluid models: one root per equation
- Gyrokinetics: infinite in principle;
  discretization yields large but finite number

3-wave interactions drive damped eigenmodes

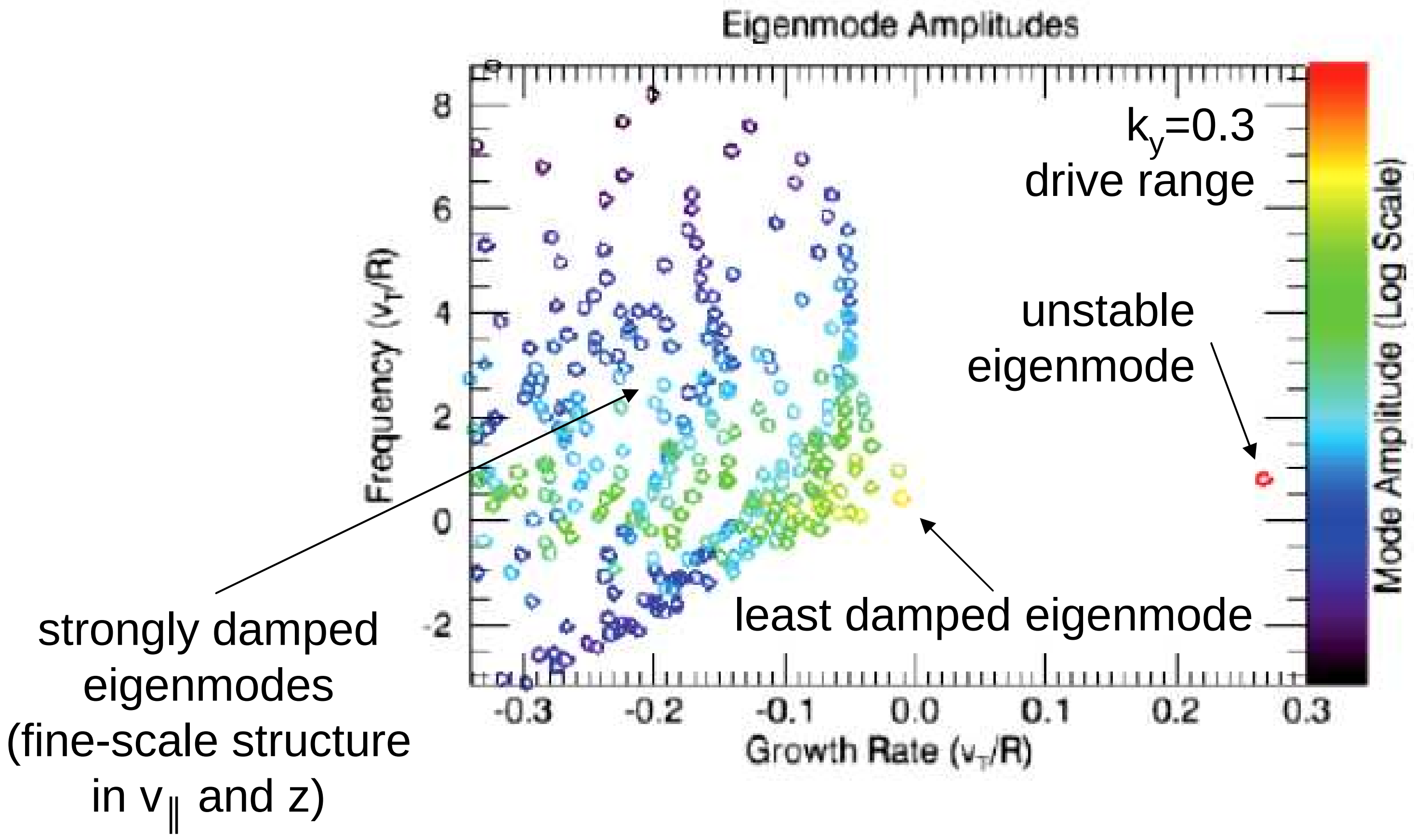
- Pumped by unstable mode through parametric instability
  Only condition: $Amp_{damp} << Amp_{ustable}$ initially
- Each eigenmode driven by combo of all nonlinearities
  => Large *multiplicity* of coupling channels
  => Many eigenmodes are excited

Consistent phenomenology across many models



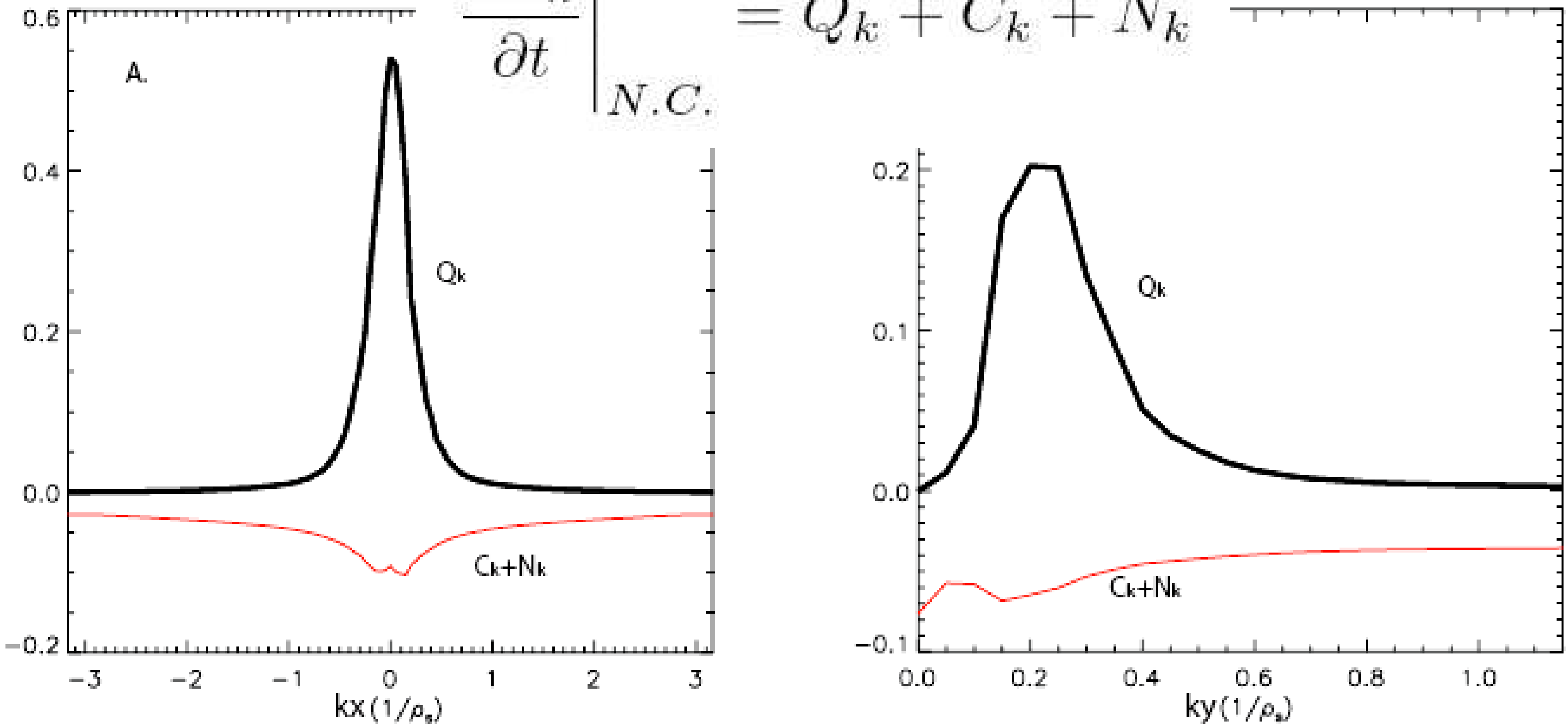
Excitation of damped eigenmodes

Using GENE as a linear eigenvalue solver to analyze nonlinear ITG runs via projection methods, one finds…



Energetics in wavenumber space

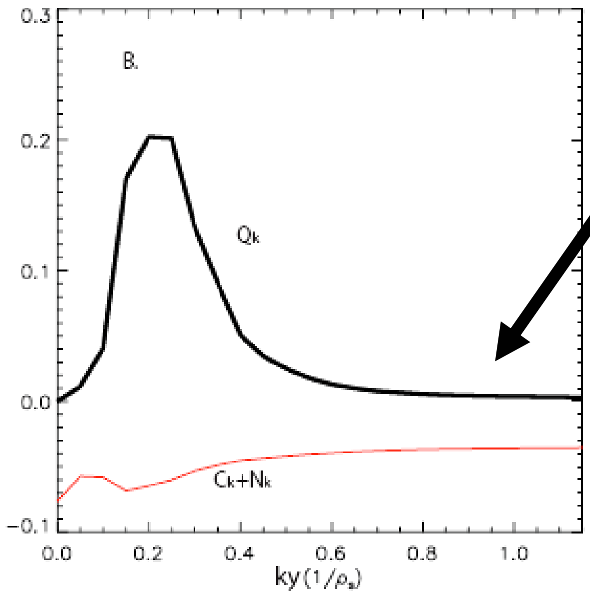
$$\left.\frac{\partial E_k}{\partial t}\right|_{N.C.} = Q_k + C_k + N_k$$



Damped eigenmodes are responsible for significant dissipation in the drive range.

Some energy escapes to high k

From finite amplitude dissipation rate diagnostic, high k dissipation is constant in *k*



Calculate spectrum of residual of energy that is transferred to high *k*

Use attenuation condition:

d/dk (transfer rate) = Energy dissipation rate

Do simple calculation for flow field

Dissipation rate = const. $E(k) = \alpha\, E(k)$

$$E(k) = \int dx\, v^2 e^{ikx}$$

Transfer rate = $T(k) = v_k^3 k$

Use closure of Terry and Tangri, PoP ‘09

Resulting spectrum decays exponentially @lo k, asymptotes to power law @hi k

Spectrum from k space attenuation of $T(k)$ by dissipation $\alpha E(k)$:

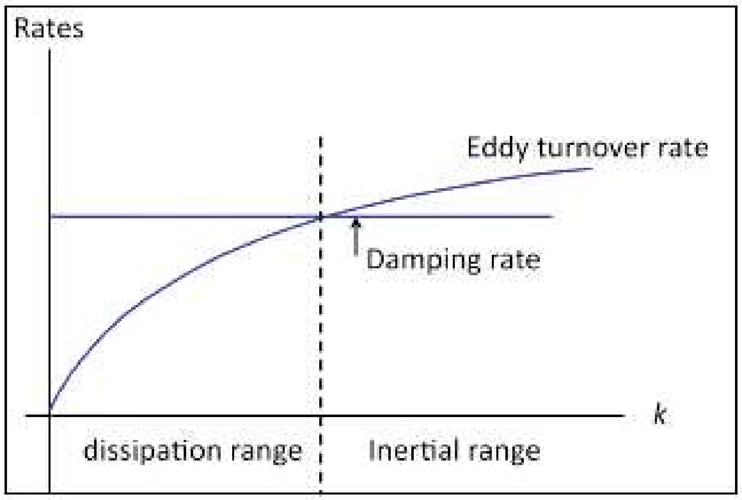
$$\frac{dT(k)}{dk} = \frac{d(v_k^3 k)}{dk} = aE(k)$$

Corrsin closure procedure: $v_k^3 k = v_k^2 \cdot v_k k = E(k)k \cdot \varepsilon^{1/3} k^{-1/3} k$

Solving attenuation ODE:

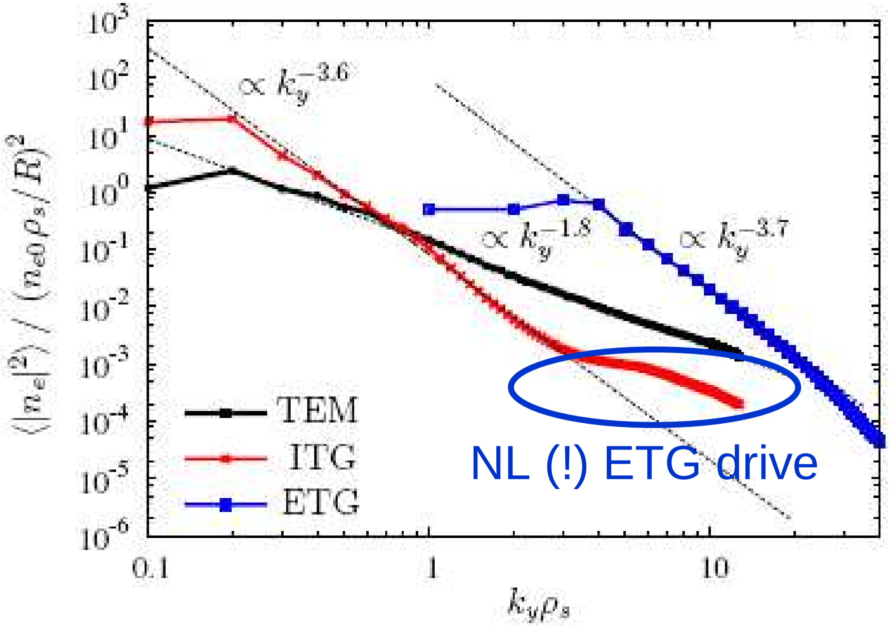
$$E(k) = \beta \varepsilon^{2/3} k^{-5/3} \exp\left[\frac{3}{2} \alpha \varepsilon^{-1/3} k^{-2/3}\right]$$

Spectrum becomes power law in range where eddy turnover rate exceeds constant dissipation rate

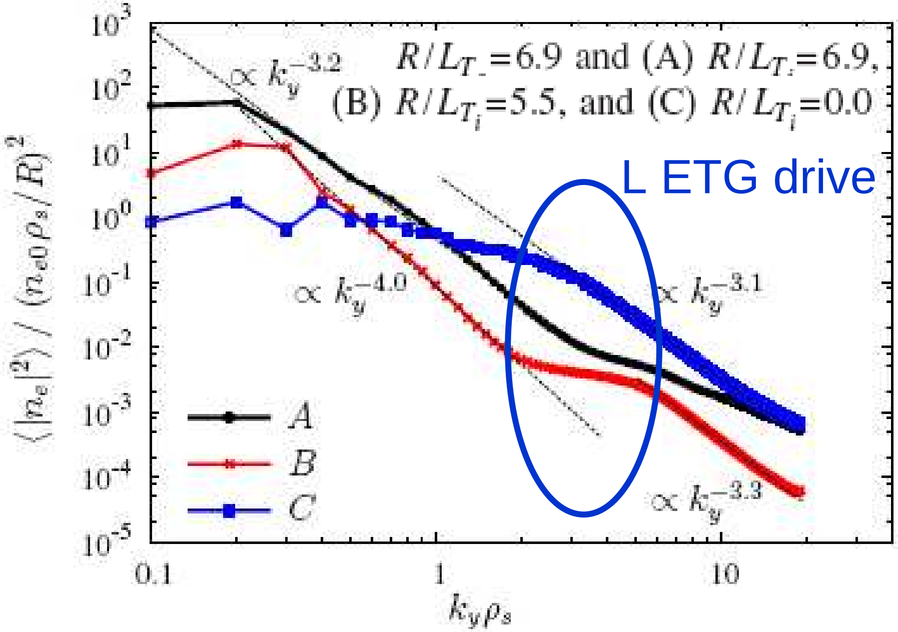


Multiscale wavenumber spectra

Poloidal wavenumber spectra of density fluctuations for pure TEM / ITG / ETG turbulence



Poloidal wavenumber spectra of density fluctuations for mixed TEM / ITG – ETG turbulence

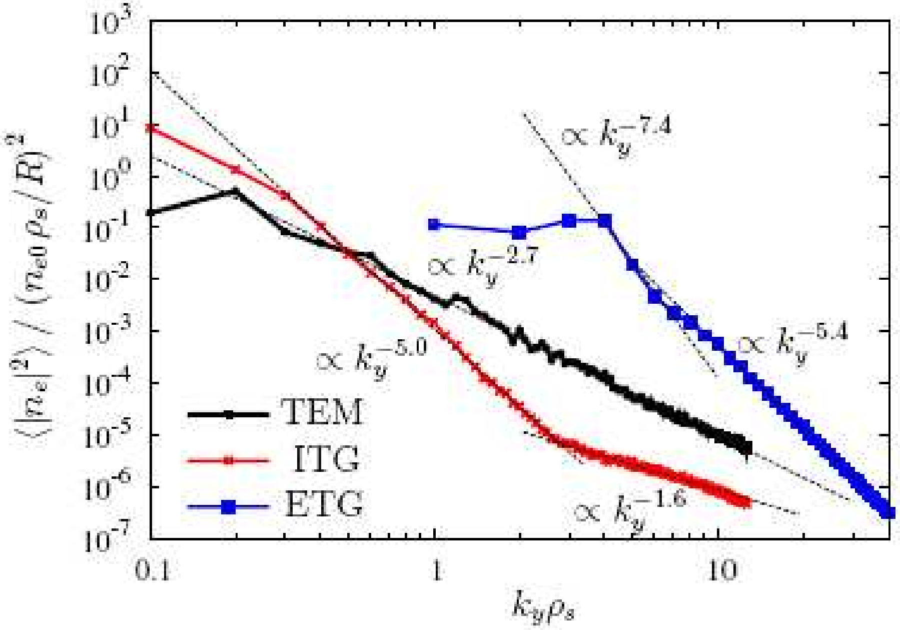


Universality?

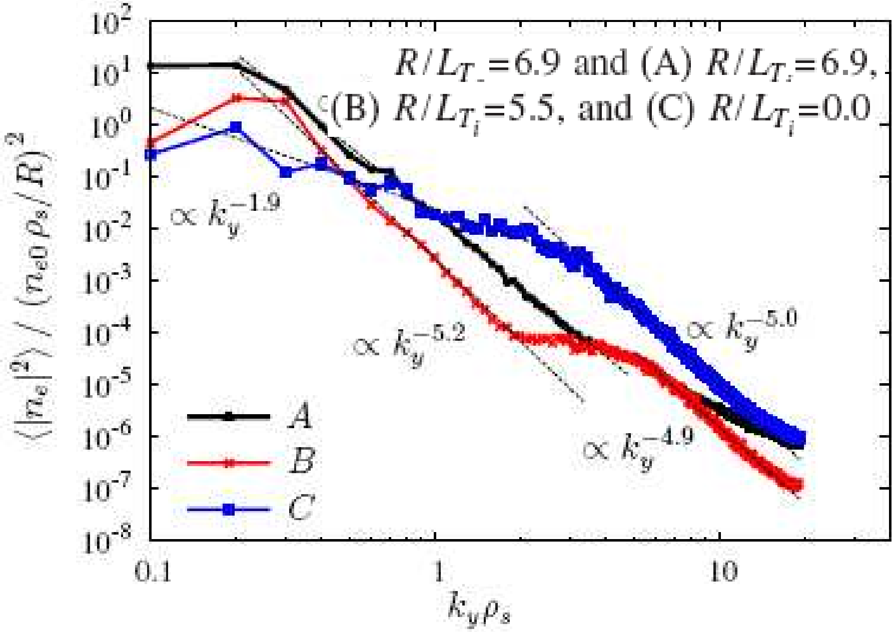
T. Görler & FJ, PoP **15**, 102508 (2008)

k_y spectra at k_x=0 are steeper

Poloidal wavenumber spectra of density fluctuations for pure TEM / ITG / ETG turbulence



Poloidal wavenumber spectra of density fluctuations for mixed TEM / ITG – ETG turbulence



k_x spectra differ

T. Görler & FJ, PoP **15**, 102508 (2008)

What is the role of sub-ion-gyroradius scales?

High-k turbulence simulations

ETG turbulence can induce significant electron heat transport:

$\chi_e^{ETG} \gg \frac{\rho_e^2 v_{te}}{L_{T_e}}$ is possible (Jenko et al., PoP 2000; Dorland et al., PRL 2000)

For comparison: $\chi_i^{ITG} \approx 0.7 \frac{\rho_s^2 c_s}{L_{T_i}}$ (Cyclone base case)

Confirmed, e.g., by (Idomura *et al*., NF 2005) and (Nevins *et al.*, PoP 2006).

Latter paper: A prefactor of ≤10 is sufficient to explain certain experiments.

TABLE IV. DIII-D electron transport analysis.

	$\chi_e/\chi_{e,GB}$	T (keV)	L_T (m)
Fig. 1 and 2, t=1.82s, r/a=0.35	0.84	3.5	0.17
Figs. 4–6, r/a=0.35	0.16	3.5	0.13
Fig. 1 and 2, t=1.82s, r/a=0.6	10.0	1.5	0.17
Figs. 4–6, r/a=0.6	8.6	1.3	0.17

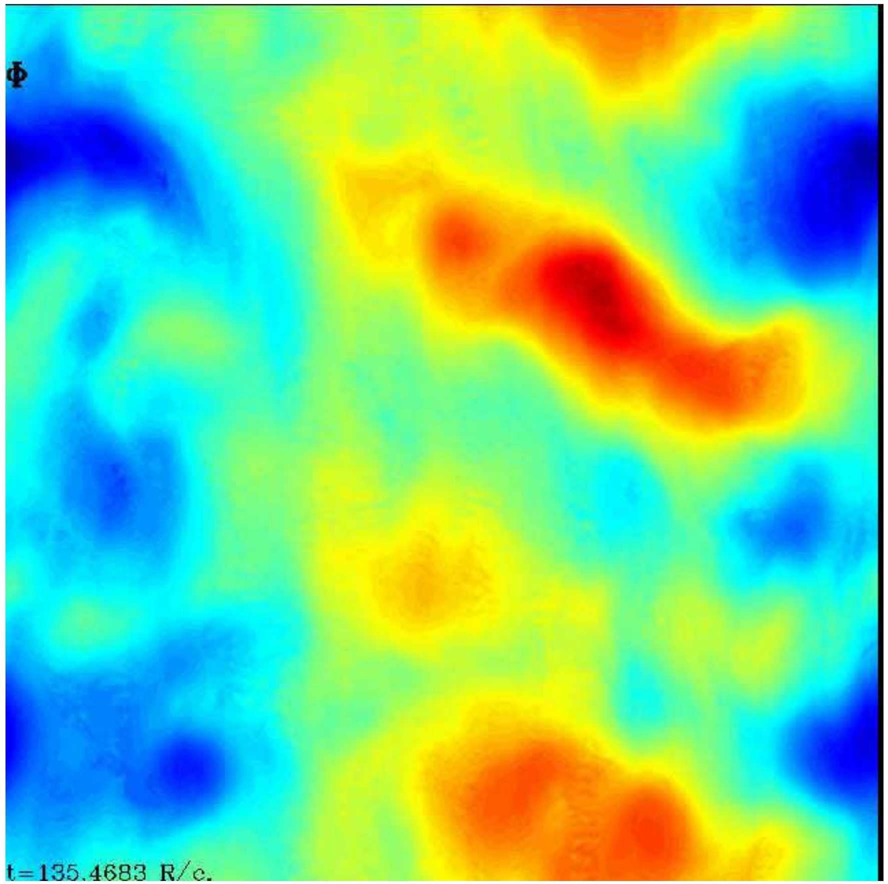
Note: DIII-D transport analysis[36] shows $\chi_e \sim \chi_{e,GB}$ within the internal transport barrier at r/a=0.35, while $\chi_e < 10\chi_{e,GB}$ in the L-mode edge plasma (r/a=0.6).

TABLE V. NSTX transport analysis.

	$\chi_e/\chi_{e,GB}$
shot #1080213@t=0.3 s, r/a=0.3	4.4
shot #1080213@t=0.3 s, r/a=0.4	6.4
shot #1080213@t=0.3 s, r/a=0.5	7.5
shot #112581@t=0.55 s, r/a=0.7	6.0
shot #106194@t=2.43 s, R=1.2 m	7.4
shot #109070@t=0.45 s, R=13.5 m	10.4

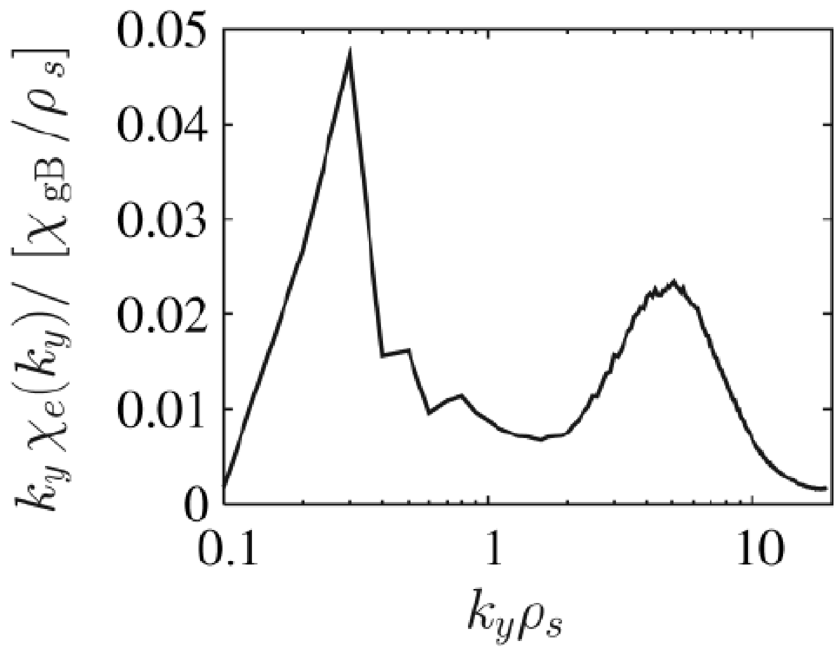
Coexistence of ITG and ETG modes

box size: ~64 ρ_i resolution: ~2ρ_e



Ion-scale transport much larger than in experiments.

[Görler & Jenko, PRL 2008]



ITG/TEM/ETG turbulence: Large fraction of electron heat transport is carried by electron scales.

Special case: The H-mode edge

Physics of H-mode barriers

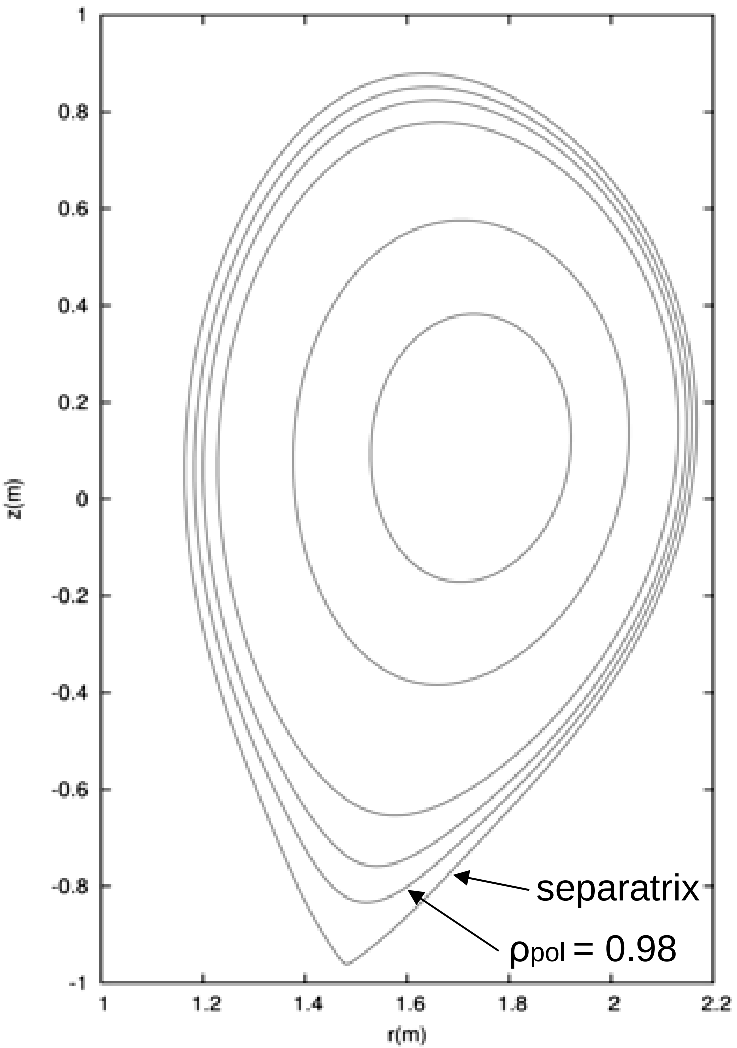
- Strong ExB shear flows thought to suppress long-wavelength turbulence
- Ion heat transport close to neoclassical, but other transport channels remain anomalous
- What sets the residual electron heat transport?

Some candidates for setting the residual electron heat transport

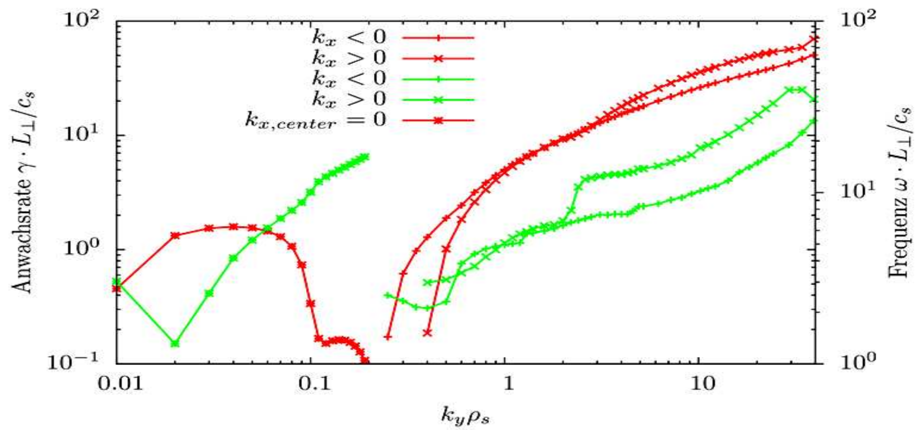
- Paleoclassical transport (theoretical foundations are disputed)
- Residual long-wavelength turbulence (not ITG)
- High-wavenumber turbulence (e.g., ETG)

This possibility will be investigated by means of gyrokinetic simulations…

ASDEX Upgrade #20431 (ρ_{pol} = 0.98)



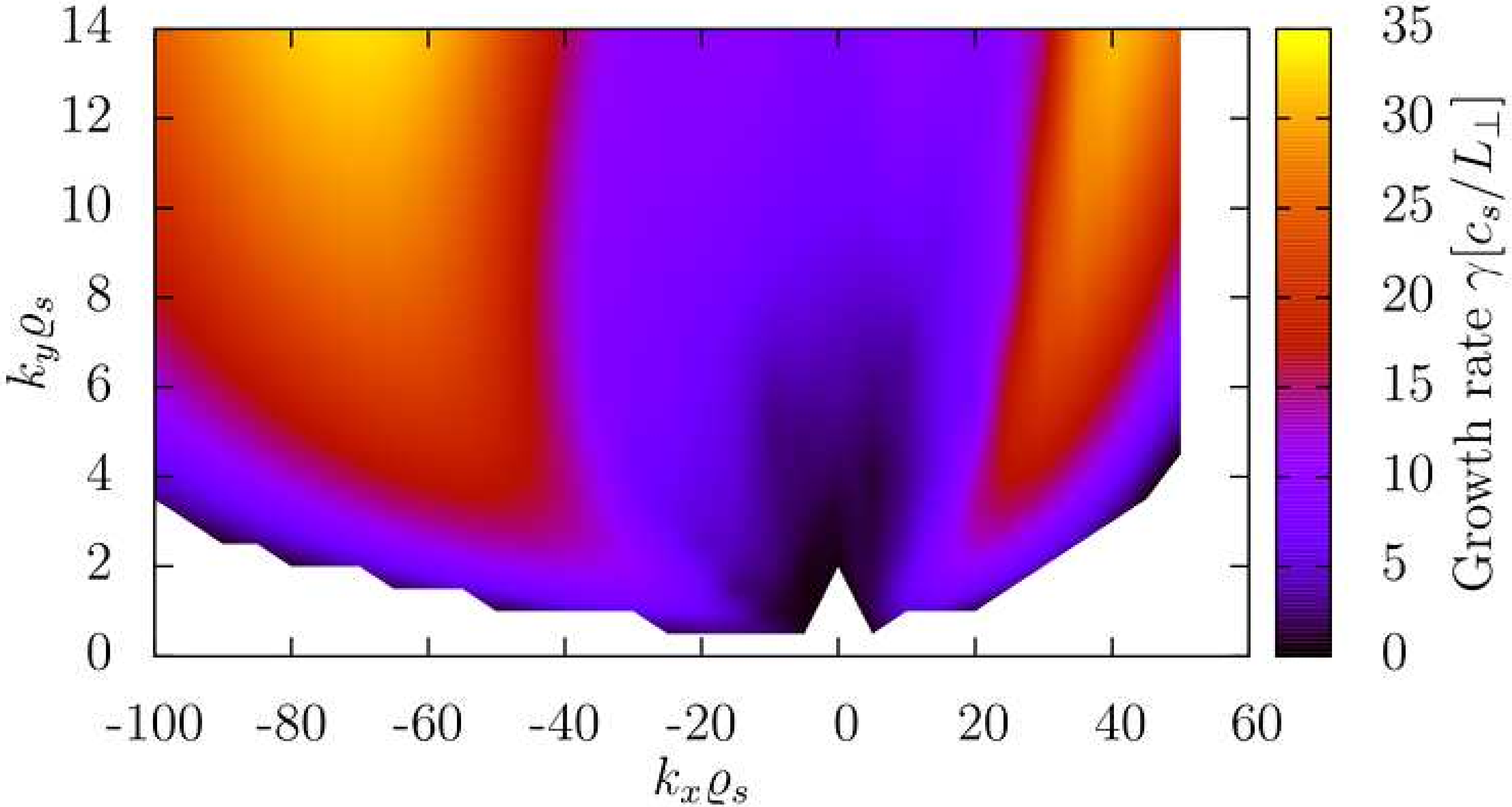
Told et al., PoP 2008



<u>Edge transport barrier region:</u>

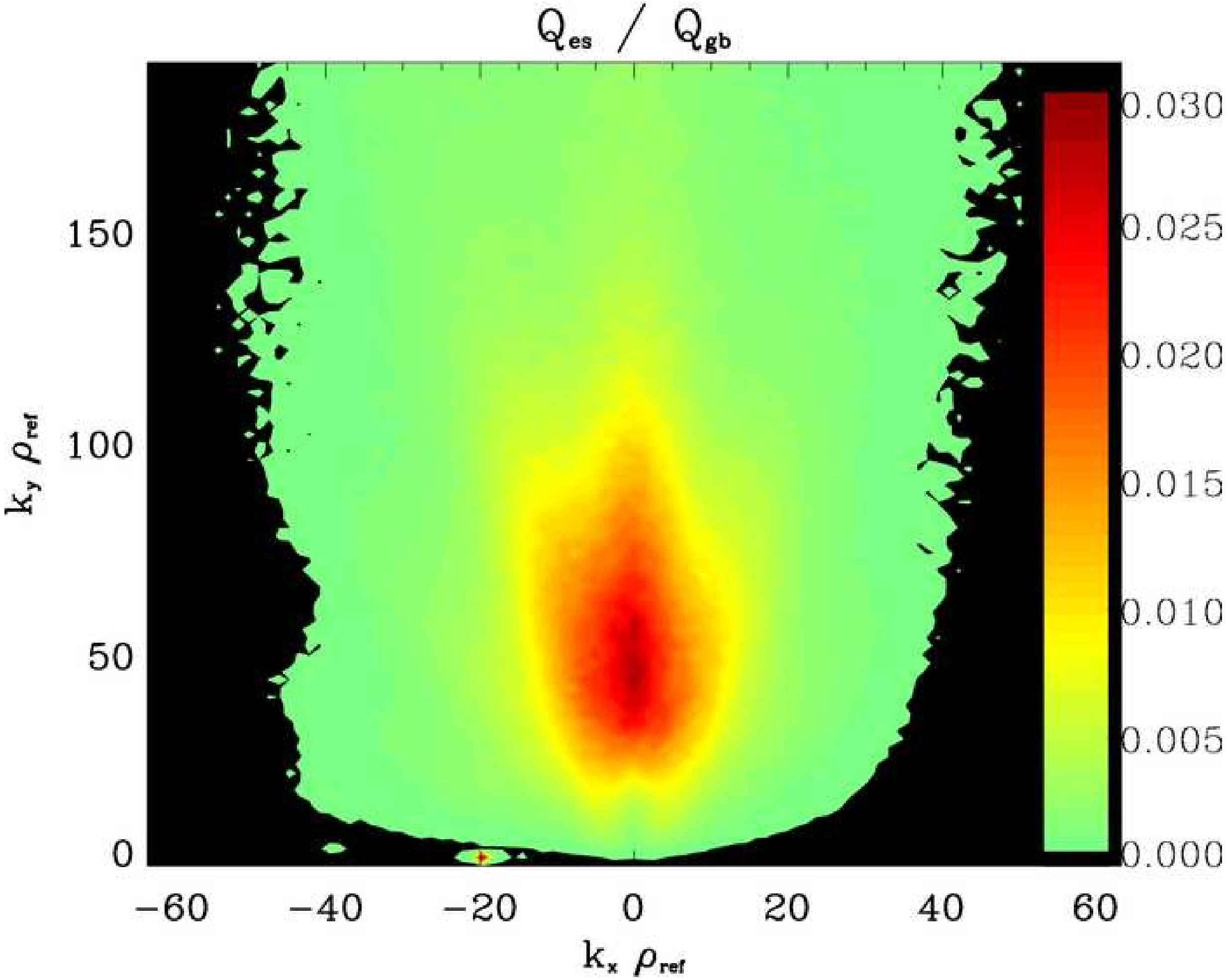
- $k_y\rho_s < 0.1$ → ITG mode
- $k_y\rho_s \sim 0.15$ → microtearing mode
- $k_y\rho_s > 0.2$ → ETG mode

Linear stability of edge ETG modes



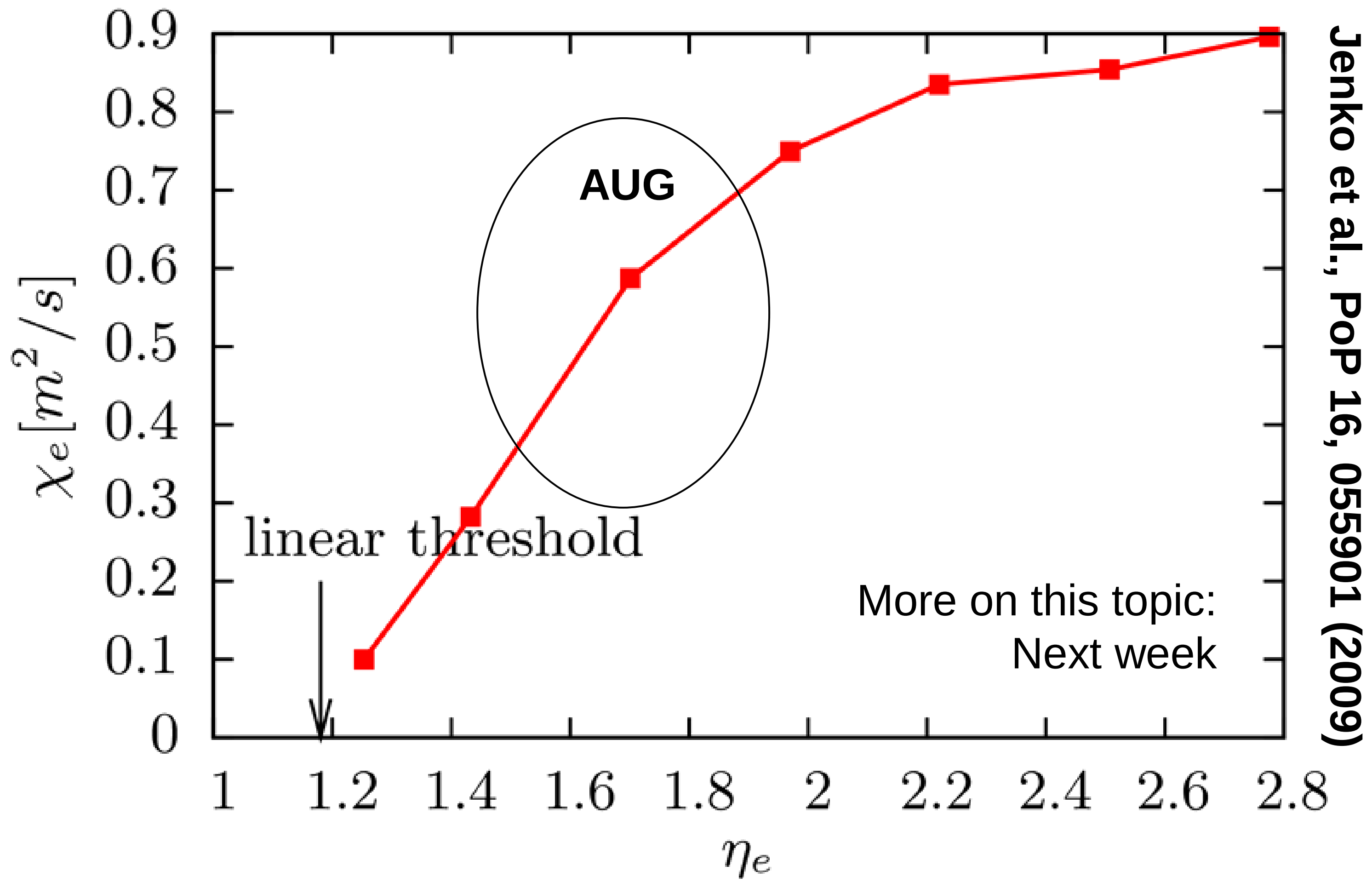
In the core, linear growth rates tend to peak at $k_x = 0$; here, they peak at large k_x values.

Transport spectrum from GENE



In contrast to the linear growth rate spectrum, the transport spectrum peaks at low k_x values.

Electron heat transport in edge barriers



ETG turbulence is able to explain the residual electron heat transport in H-mode edge plasmas.

Summary and outlook

- Recent surprises and advances concerning the nonlinear saturation, dissipation, and multi-scale properties of plasma turbulence

  (see also the posters on nonlocal effects by S. Brunner & T. Görler)

- More investigations targeted at improving our understanding of fundamental issues in plasma turbulence are certainly called for

- In the end, all of this is bound to have important practical consequences concerning the efficient transport modeling of ITER plasmas