Absolutely continuous spectrum of substitution trees

Matthias Keller,
Friedrich Schiller University Jena,

joint work with
Daniel Lenz and Simone Warzel

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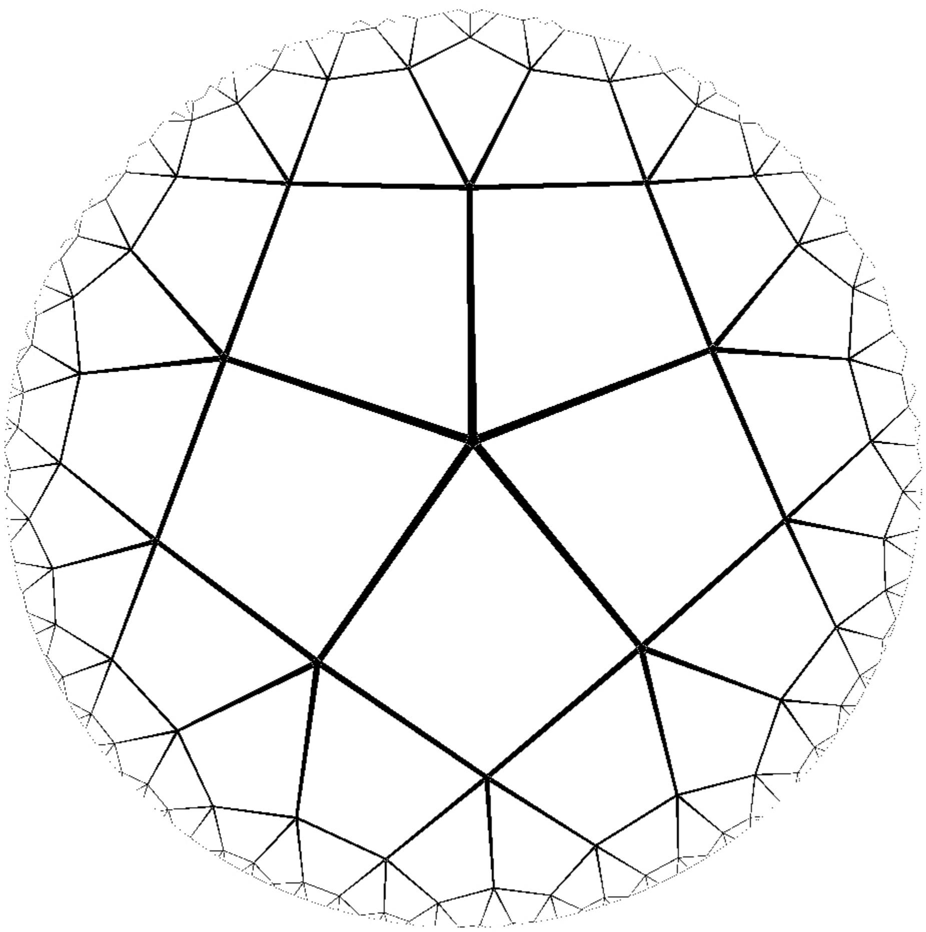
The plan

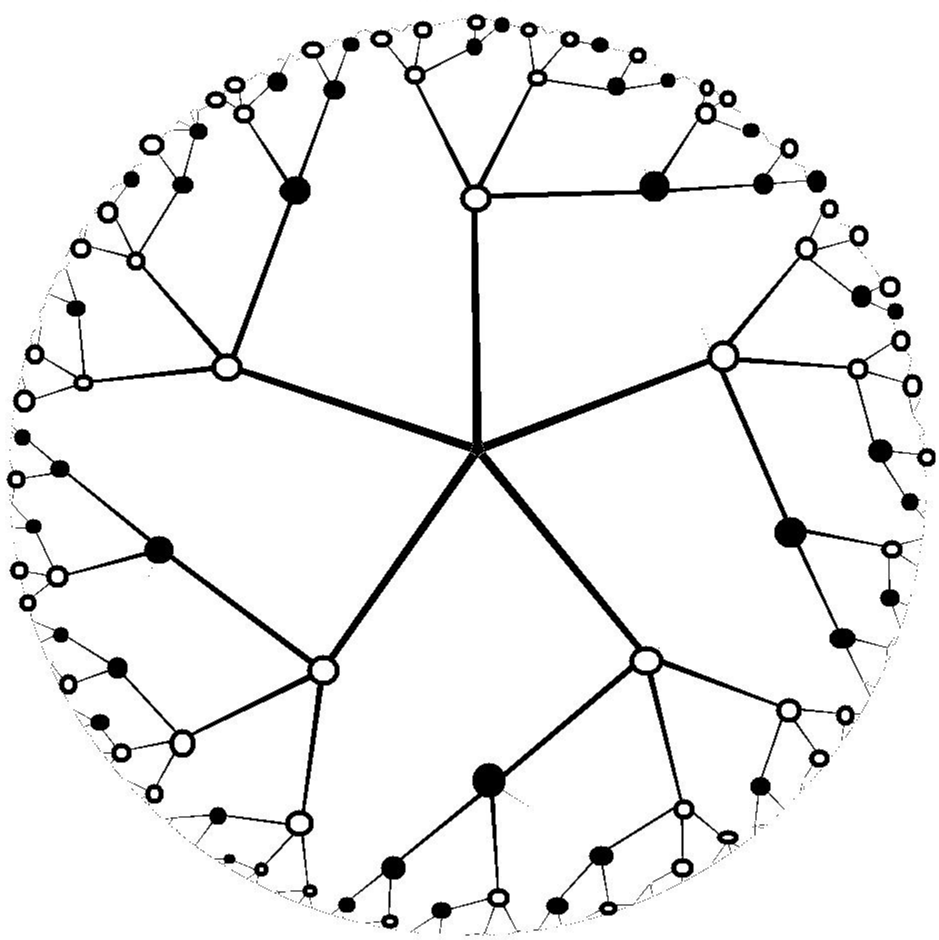
1. Substitution trees

2. Absolutely continuous spectrum

3. Stability of absolutely continuous spectrum

4. Ideas of the proof





Some references

Random walks

- Takacs '97
- Woess/Nagnibeda '02
- Krön '02, Krön/Teufl '04
- Mairesse '05

Random Schrödinger operators on trees

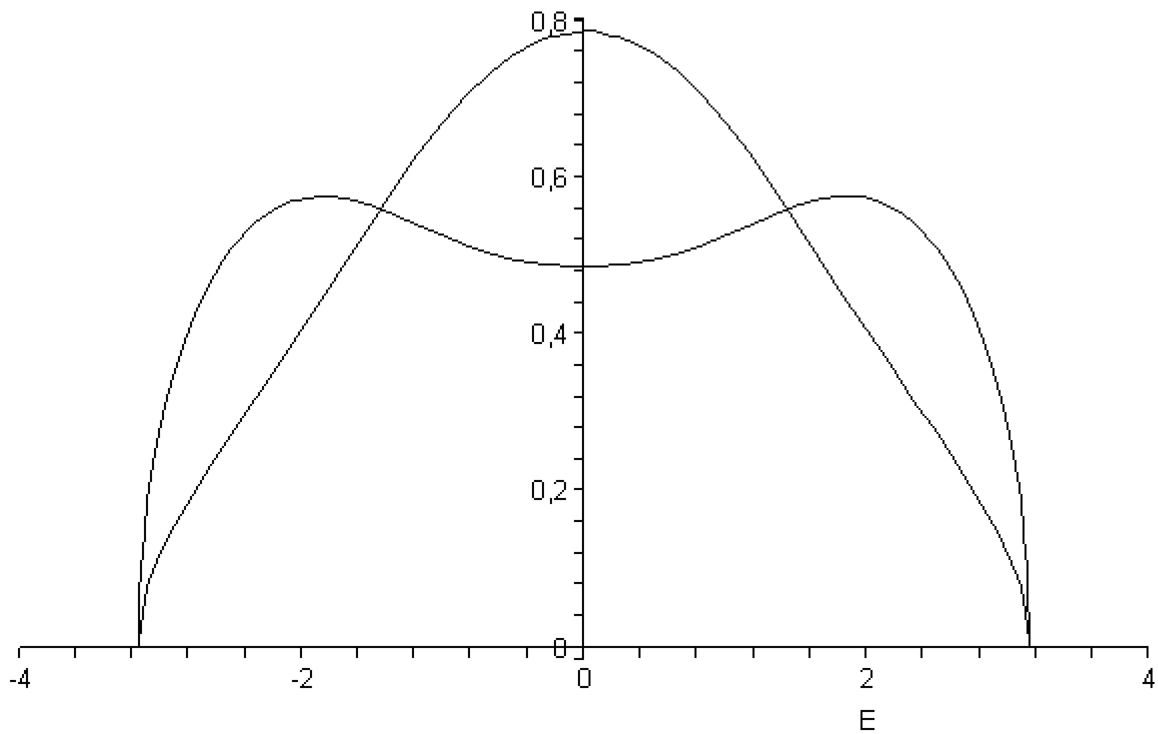
- Klein '94, '96, '98
- Aizenman/Sims/Warzel '06, '06, '06
- Froese/Hasler/Spitzer '07, '08, '09
- Halasan '09

Spectral theory of trees

- Aomoto '91
- Breuer '07, Breuer/Frank '09

Examples

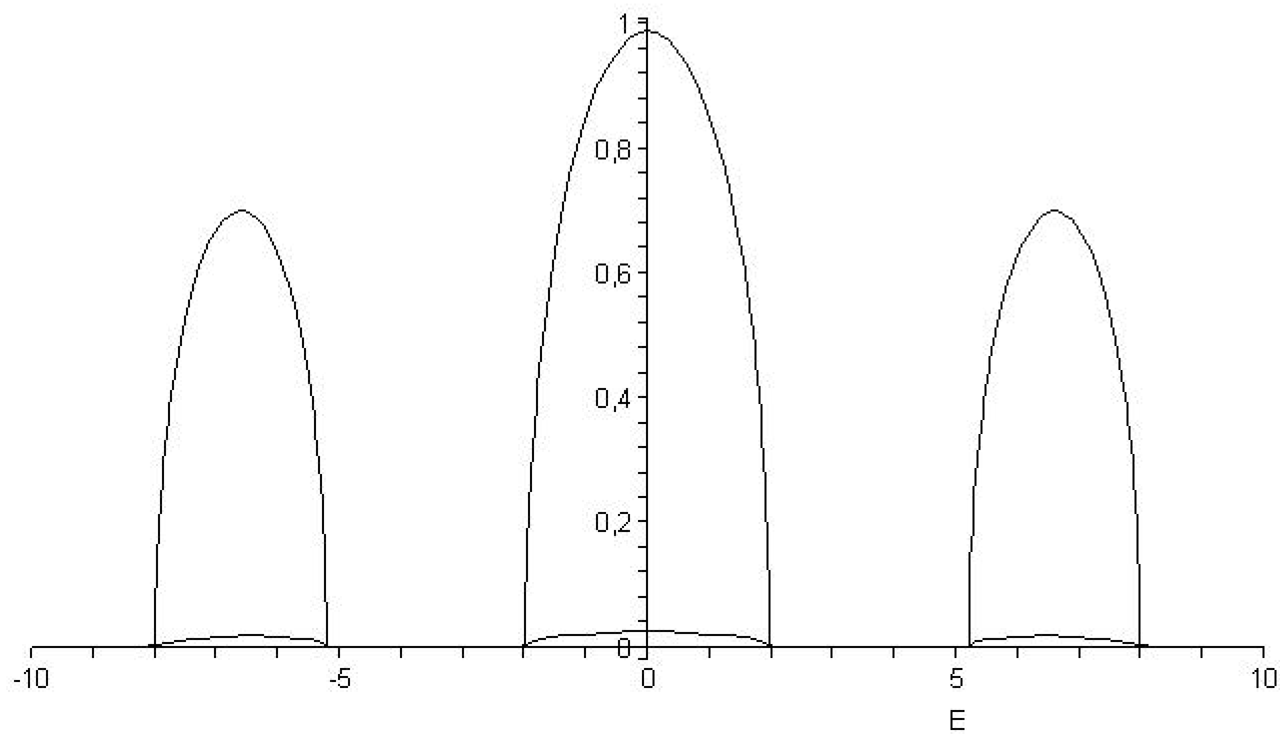
Let $\mathcal{A} = \{\circ, \bullet\}$ and $M = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$.



The imaginary parts of the Green functions of the roots $\circ$ and $\bullet$.

Examples

Let $\mathcal{A} = \{\circ, \bullet\}$ and $M = \begin{pmatrix} 1 & 42 \\ 1 & 1 \end{pmatrix}$.



The imaginary parts of the Green functions of the roots $\circ$ and $\bullet$.

Possible Extensions

- unbounded potentials ('large with small probability')
- small off-diagonal perturbations ('first passage percolation')
- random trees
  - percolation type models
  - multitype Galton Watson trees

Random potentials

Let $(\Omega, \mathbb{P})$ be a probability space. Let

$$v : \Omega \times V \to \mathbb{R}, \quad (\omega, x) \mapsto v_x^\omega$$

satisfy the following two assumptions:

(P1) For all $x, y \in V$ the random variables v_x and v_y are **independently distributed** if x, y have no common forward neighbors, i.e. $V_x \cap V_y = \emptyset$.

(P2) For all $x, y \in V$ the random variables $v|V_x$ and $v|V_y$ are **identically distributed** if x, y have the same label.

Random operators

$H^{\omega,\lambda} = \Delta + \lambda v^\omega$ on $\ell^2(V)$, $\lambda \geq 0$.

Theorem. (K., Lenz, Warzel) Let $K \subset \sigma(\Delta) \setminus \Sigma_0$ be compact, where Σ_0 is an explicitely given finite set. Then there exists $\lambda = \lambda_K > 0$ such that for all $v : \Omega \times V \to \mathbb{R}$ satisfying (P1) and (P2) and almost every $\omega \in \Omega$

$$K \subset \sigma_{\rm ac}(H^{\omega,\lambda})$$

and

$$K \cap \sigma_{\rm sing}(H^{\omega,\lambda}) = \emptyset.$$

Truncated Green functions

For an operator $H = \Delta + v$ on a tree and a vertex x, denote by H_x the restriction of H to the forward tree of x.

The truncated Green function

$$\Gamma_x(z, H) := \langle \delta_x, \frac{1}{H_x - z} \delta_x \rangle$$

for $z \in \mathbb{H} := \{w \in \mathbb{C} \mid \Im w > 0\}$, satisfies the **recursion formula**

$$\Gamma_x = \frac{-1}{z - v(x) + \sum_{y \in S_x} \Gamma_y}.$$