

**SIR GILBERT WALKER AND A CONNECTION BETWEEN  
EL NIÑO AND STATISTICS: FROM “TYPICAL CAMBRIDGE DON” TO  
RENAISSANCE MAN**

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**Reference:** Katz, R.W. (2002). *Statistical Science*, V. 17, pp. 97-112.

## OPENING QUOTE

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- **Sir Gilbert Walker (1925)**
  - ***“It is a natural supposition that there should be in weather free oscillations with fixed natural periods, and that these oscillations should persist except when some external disturbance produces discontinuous changes in phase or amplitude.”***

## OUTLINE

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**(1) Background**

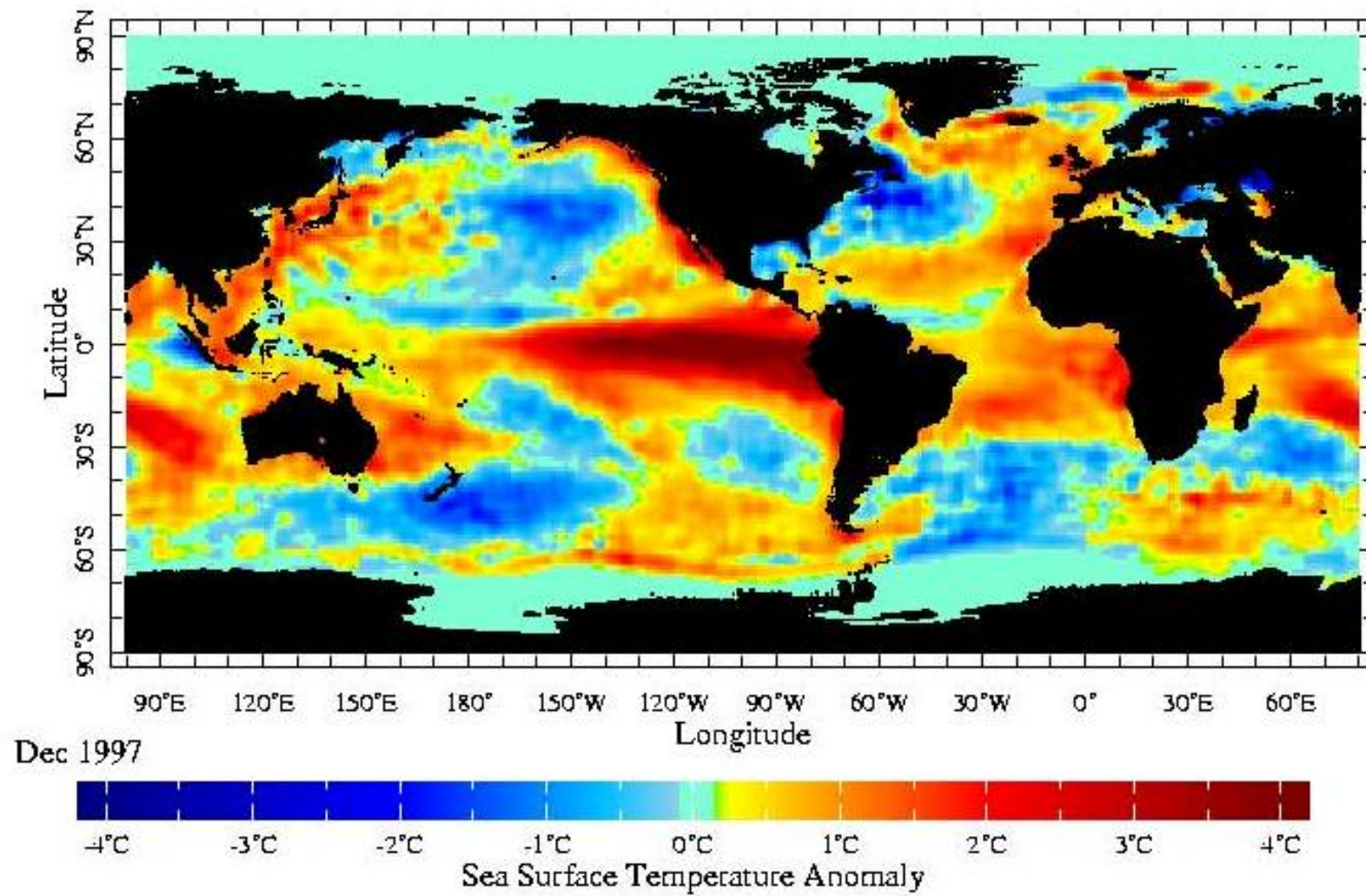
**(2) Sir Gilbert Walker**

**(3) Yule-Walker Equations**

**(4) Reaction to Walker's Work**

**(5) Present Situation**





## (1) BACKGROUND

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- **Quote About Cell Phones (Thierry Dutoit, 2004)**
  - ***“Every cell phone call solves the Yule-Walker equations every ten microseconds”***
- **Quote about ENSO phenomenon (Jacob Bjerknes, 1969)**
  - ***“The Walker Circulation . . . must be part of the mechanism of the still larger Southern Oscillation”***

- **El Niño - Southern Oscillation (ENSO) Phenomenon**

- **El Niño (“Christ Child”)**

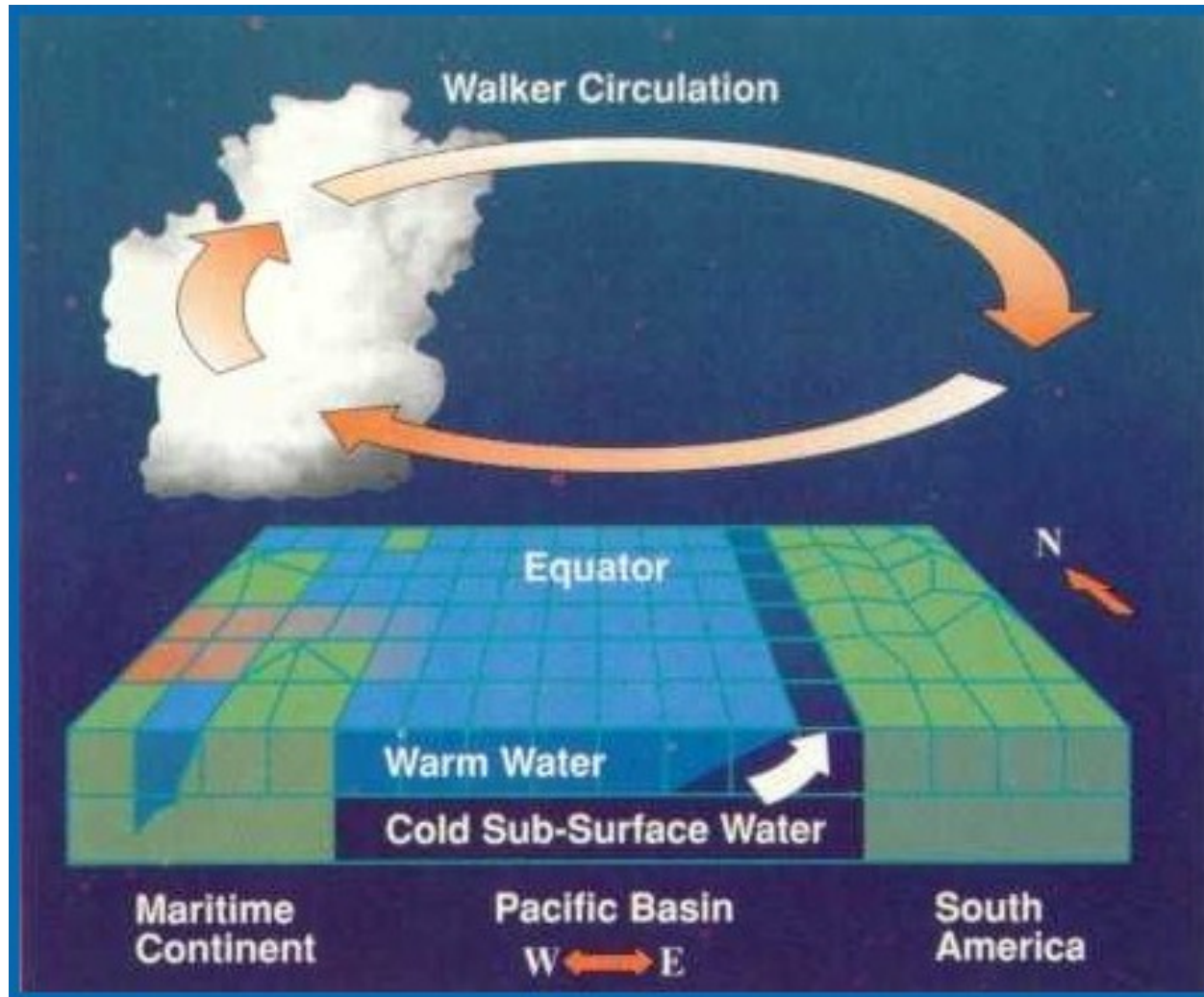
- **Teleconnections**

- **Centers of action / Pressure seesaw**

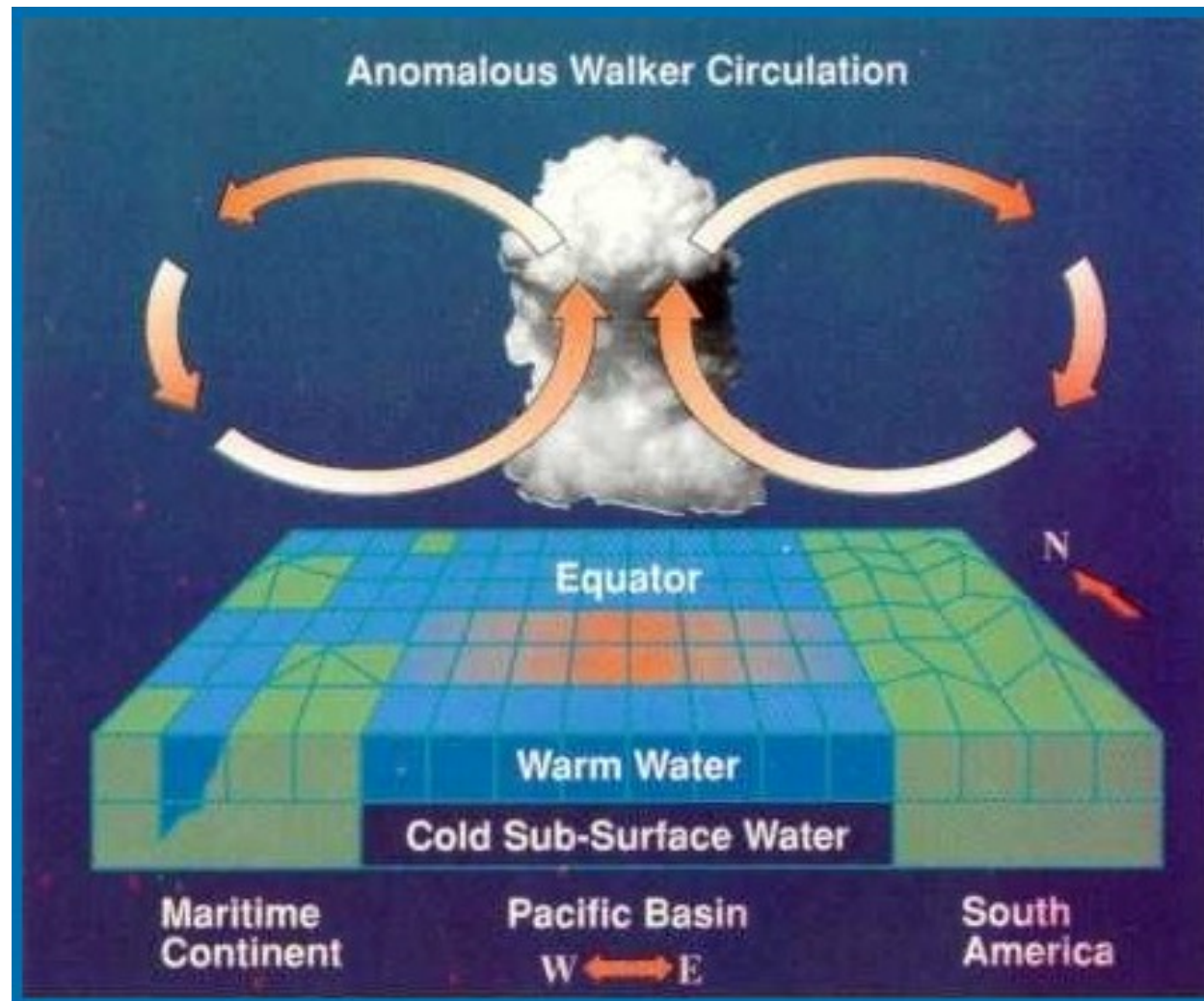
- Southern Oscillation (SO)**

- (Named by Walker)**

- Walker Circulation (Bjerknes, 1969)**







- **Quasiperiodic Behavior**

- $p^{\text{th}}$  - order autoregressive process [AR( $p$ )]

- “Yule-Walker Equations” (Kendall, 1949)

Yule (1927):      **AR(2)** model for sunspots

Walker (1931):    **AR(4)** model for Darwin pressure

Use today:        **Partial autocorrelations**  
(e. g., **R** statistical programming language)

**Speech compression**



## (2) SIR GILBERT WALKER

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- Training and Career of Sir Gilbert Walker

- 1868      Born (Lancashire, England)

- 1881      St. Paul's School

- “... spent two years on the classical side but was then ‘sent in disgrace’ to the mathematical side when he made some appalling linguistic blunder.”*

- 1886-1890**      Studied mathematics, Trinity College, Cambridge

“Senior Wrangler” in Part I of Mathematical Tripos

Developed an interest in boomerangs

G. I. Taylor (arrived at Trinity in **1905**):

*“The legend of his prowess in throwing boomerangs on the Cambridge Backs was still current”*
- 1890**            Health broke down – spent three subsequent winters in Switzerland & became an expert ice skater
- 1892**            Published first paper

**-- 1895-1903**    **Lecturer, Trinity College**

**Walker expressed “*difficulty that applied mathematicians experienced in those days at Cambridge in finding something to work on*”**

**“*a mathematician to his finger-tips*”**

**“*a typical Cambridge don and had never read a word of meteorology*”**

**Elected Fellow of Royal Society (1904)**

*“on the strength of his research in pure and applied mathematics . . . including original work in dynamics and electromagnetism before ever he turned his thoughts to meteorology”*

**-- 1903-1924**     **India Meteorological Department**

*“Walker soon realized the importance of the monsoon to India and at once saw the part the new branch of mathematics -- statistics, very active then under Pearson and others -- might play in scientific monsoon forecasting”*

- 1924**                    **Knighthood for work in India**
- 1924-1934**           **Imperial College, University of London  
(Professor of Meteorology)**
- 1934**                    **Retirement (lived in Cambridge 1934-1950)**
- 1950**                    **Published last paper**
- 1958**                    **Died (Surrey, England)**



- **Statistical Research of Walker**

- **Correlation and regression**

**Southern Oscillation (SO)**

**North Atlantic Oscillation (NAO)**

**North Pacific Oscillation (NPO)**

- **Multiple comparisons**

**Walker Test (Correlations & Periodogram)**

- **Harmonic analysis**

**“Period-hunting” (Cautioned against)**

### (3) YULE-WALKER EQUATIONS

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- AR Process

--  $X_1, X_2, \dots, X_t, \dots$  zero mean **AR( $p$ )** process

--  $p = 2$  case

$$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + a_t$$

$\varphi_k$   $k^{\text{th}}$  - order autoregression coefficient

$a_t$  zero mean, uncorrelated (“white noise”) process

- Yule-Walker Equations ( $p = 2$  case)

$$\rho_k = \varphi_1 \rho_{k-1} + \varphi_2 \rho_{k-2} \quad k \neq 0$$

$$\sigma_a^2 = (1 - \varphi_1 \rho_1 - \varphi_2 \rho_2) \sigma^2$$

$\rho_k = \text{corr}(X_t, X_{t-k})$        $k^{\text{th}}$  - order autocorrelation coefficient

$\sigma^2 = \text{var}(X_t)$       Process variance

$\sigma_a^2 = \text{var}(a_t)$       Error (or “innovation”) variance

- **Quasiperiods**

- **AR( $p$ ) process,  $p \geq 2$**

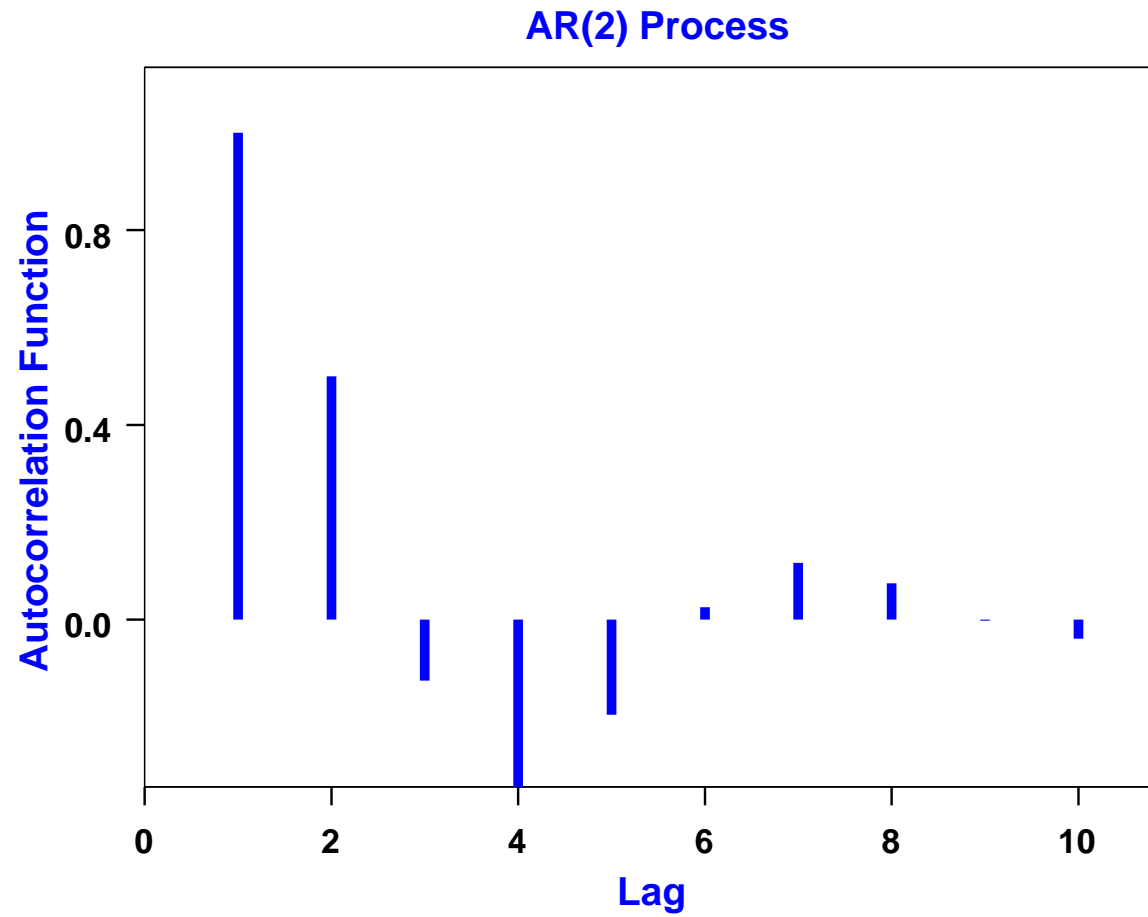
- Capable of producing quasiperiodic behavior**

- **Correlation quasiperiod**

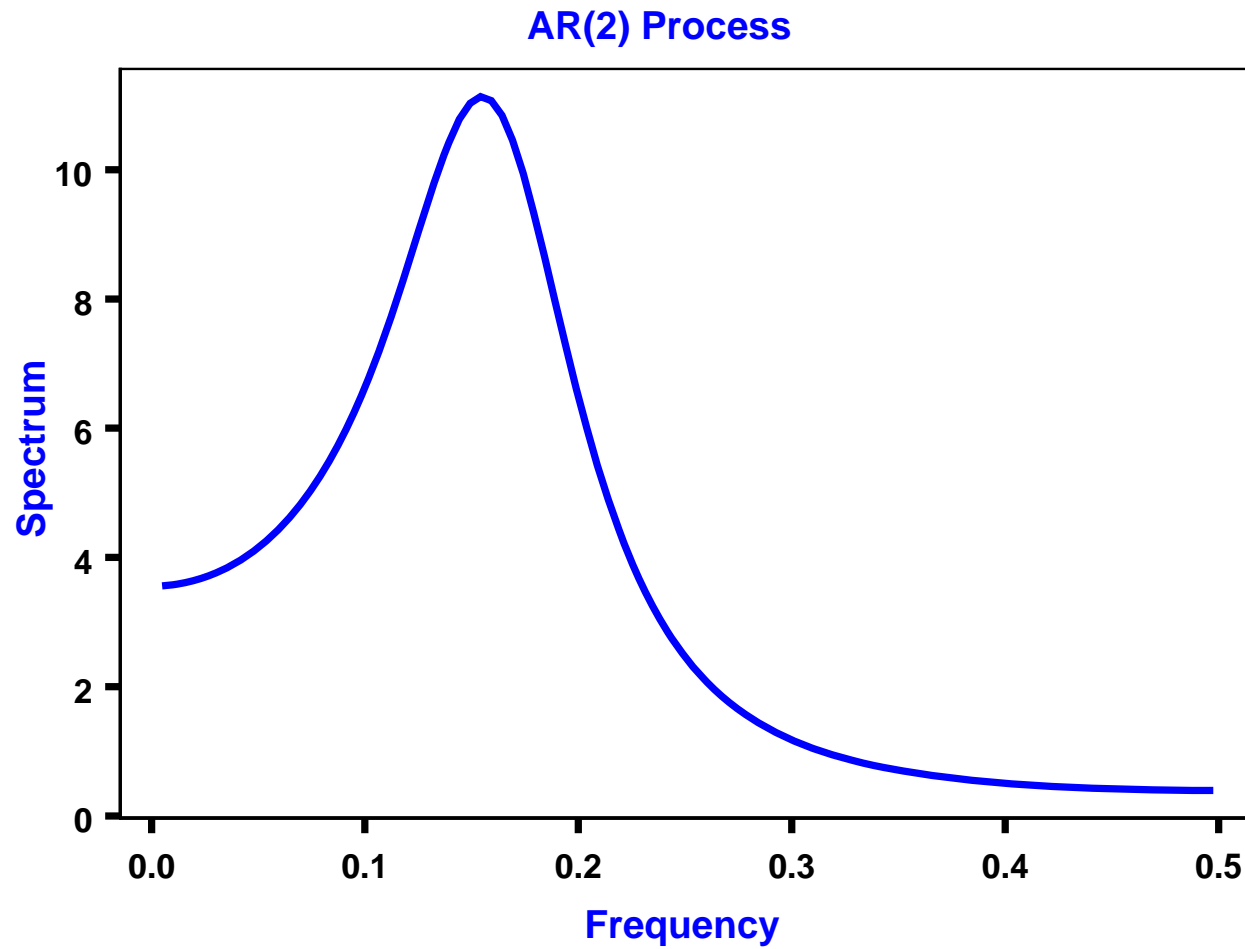
- Damped oscillation in autocorrelation function**

- **Spectral quasiperiod**

- Peak (Broad band) in spectrum**



**Hypothetical parameters**



**Hypothetical parameters**

- **Yule (1927) Model for Sunspot Numbers**

- **Sine wave randomly shifted in phase and amplitude**

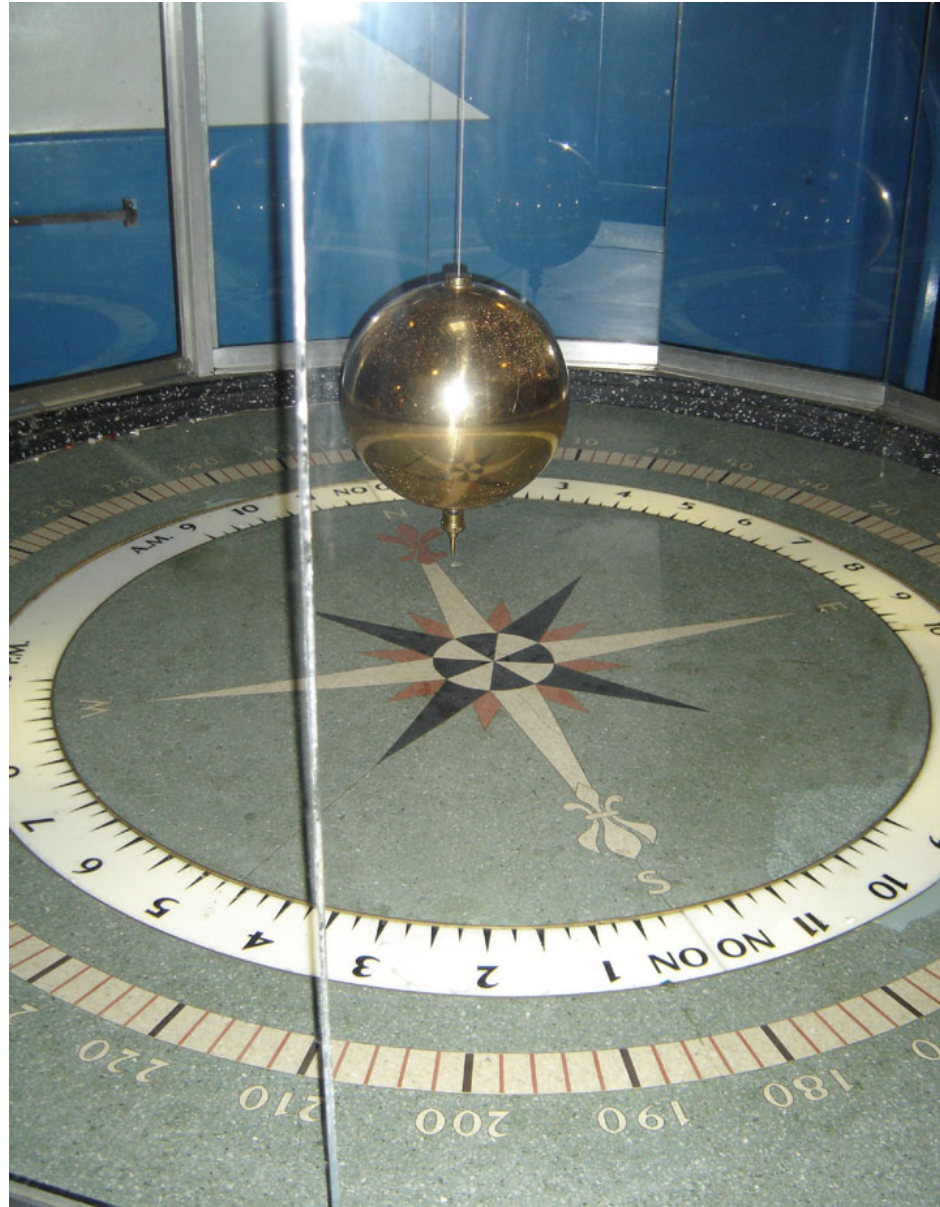
**Analogy to pendulum:**

*“Unfortunately boys get into the room and start pelting the pendulum with peas, sometimes from one side and sometimes from the other”*

- **AR(2) process**

**Fit by least squares to annual sunspot time series, 1749 – 1924**

**(Correlation quasiperiod  $\approx 11$  yrs.)**





- **Walker (1931) Model for Darwin Pressure**

- **Seasonal index of Southern Oscillation**

- **Derivation of Yule-Walker Equations**

Extended Yule's **AR(2)** model to general **AR(p)** process,  $p \geq 1$

- **Analysis of Darwin pressure**

Decided on **AR(4)** model

Correlation quasiperiod  $\approx 3$  yrs.

- **Reanalysis of Darwin Pressure**

- **Modifications**

- Correct for instrumental bias**

- Remove annual cycle in standard deviation**

- Parameter estimation by Yule-Walker Equations**

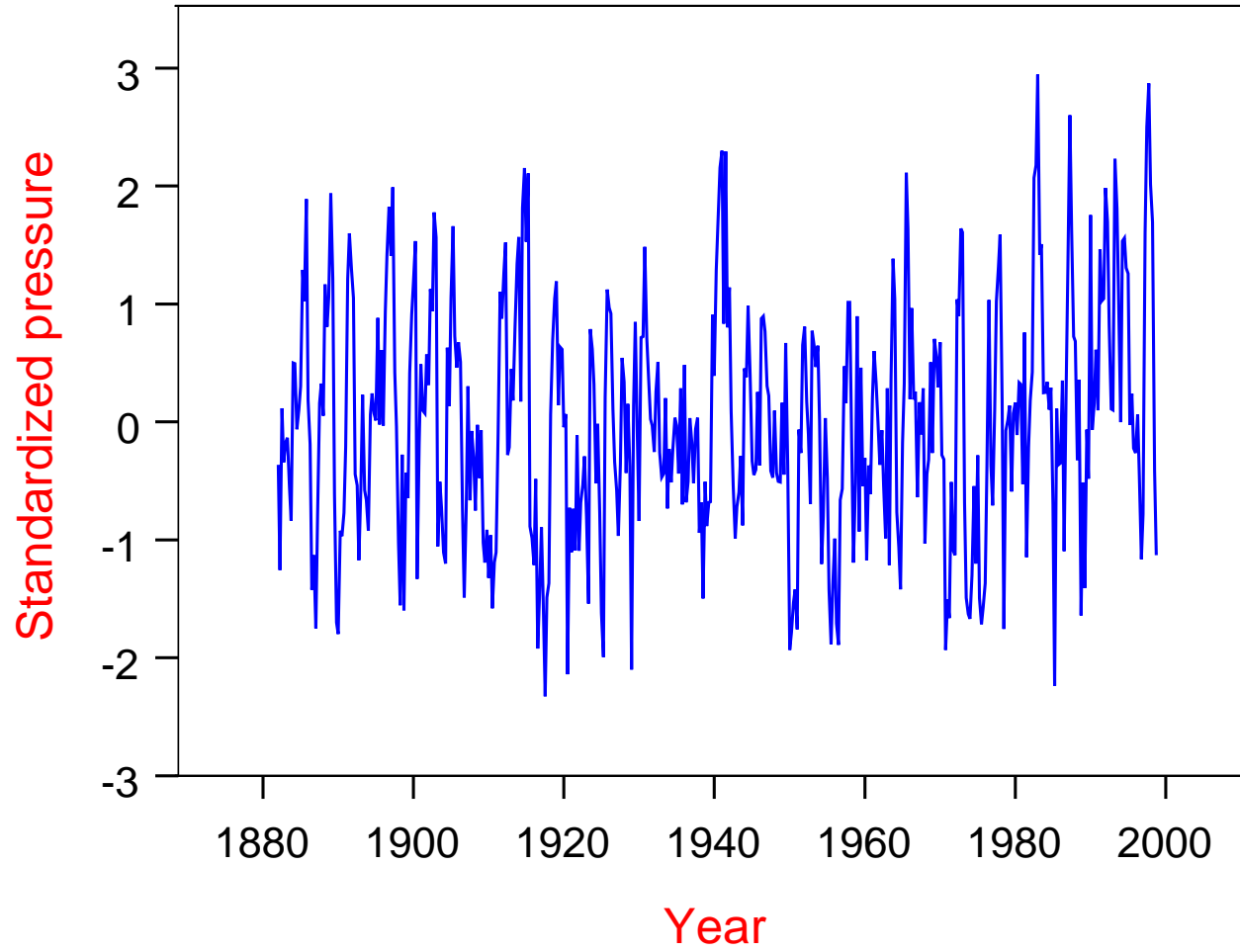
- **Results**

- Still only based on time period 1882 – 1926**

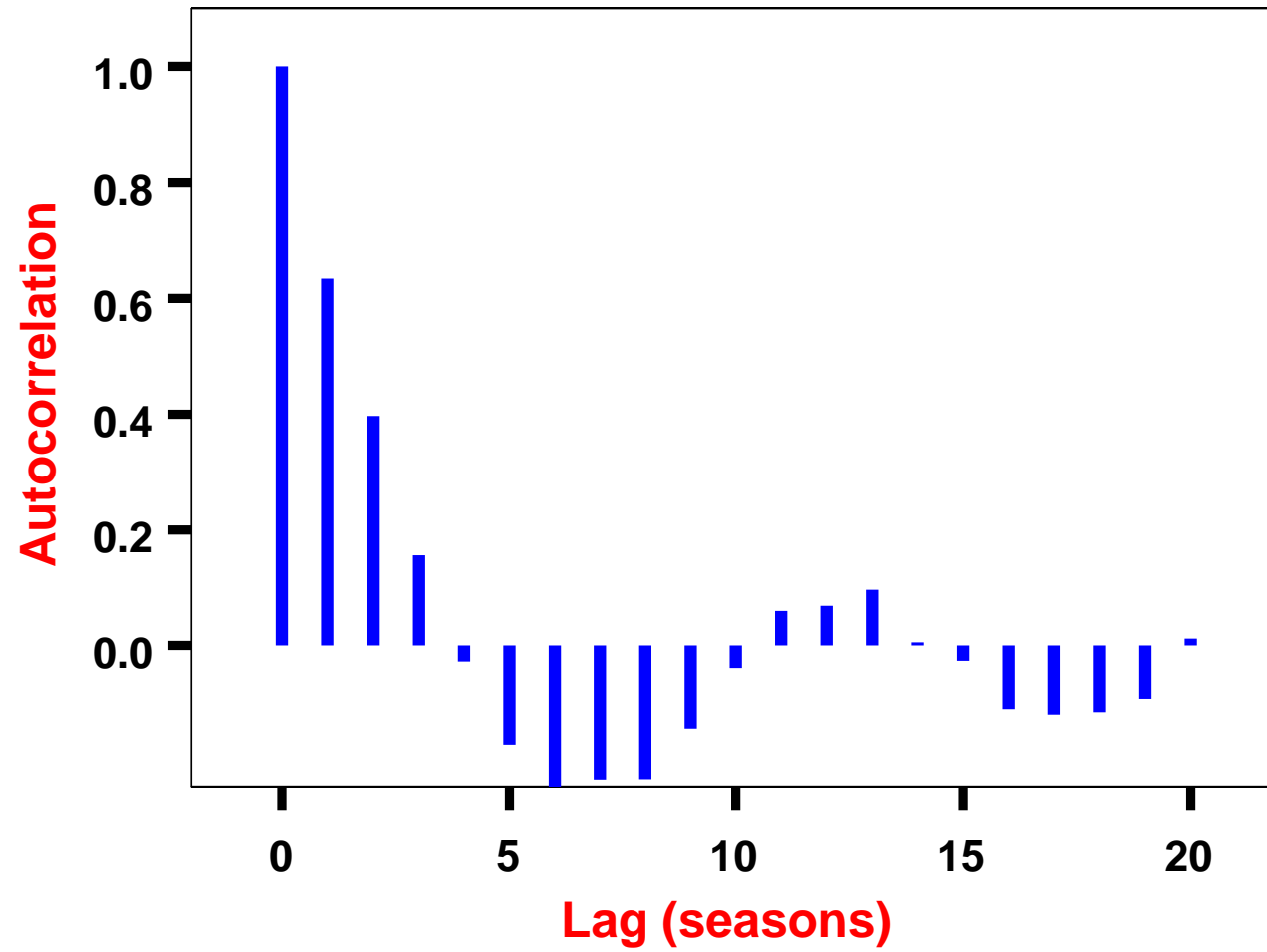
- Damped oscillation in autocorrelation function**

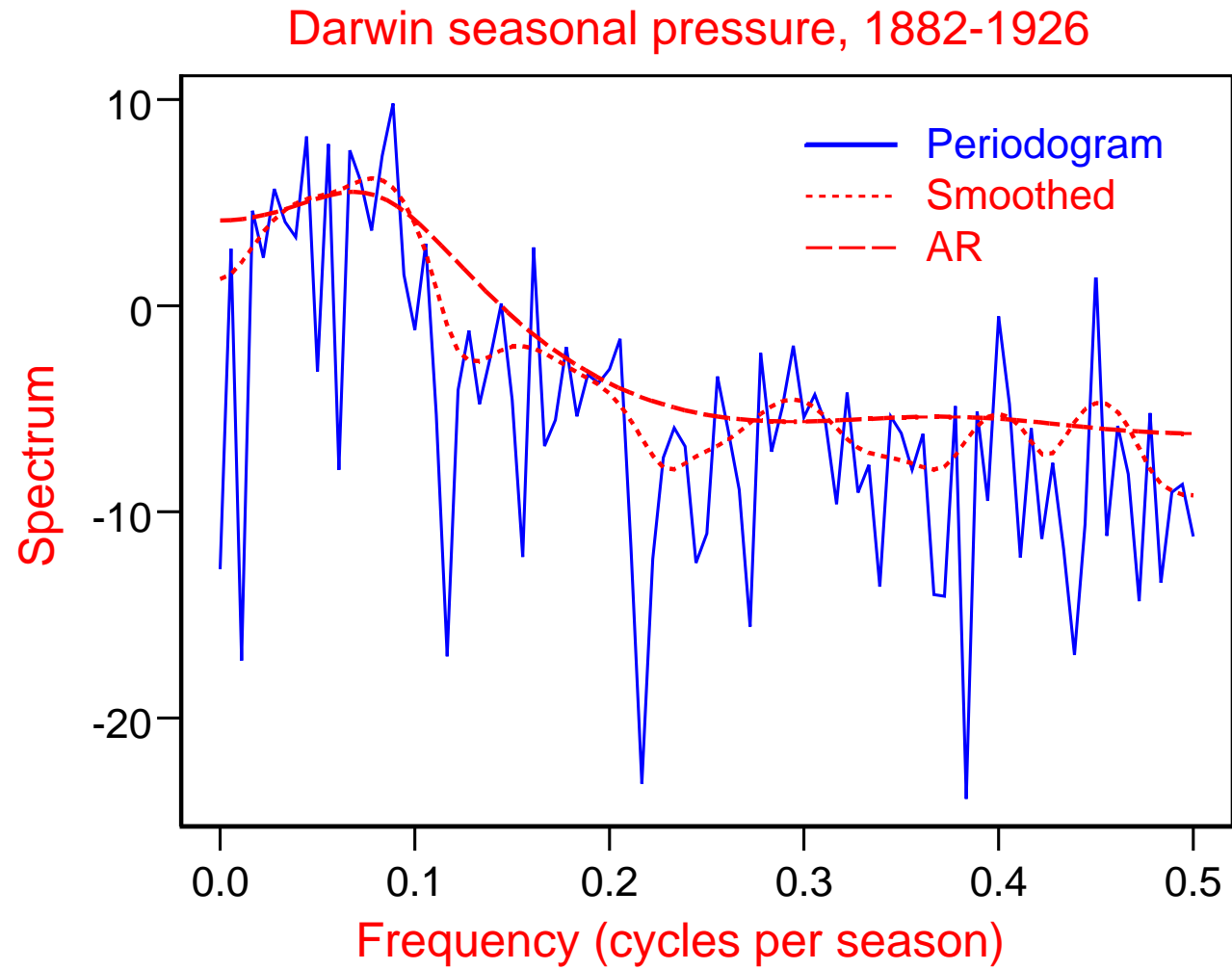
- AR(4) model gives spectral quasiperiod  $\approx 3 - 3.5$  yrs**

### Seasonal mean pressure at Darwin



### Darwin seasonal pressure, 1882-1926





## (4) REACTION TO WALKER'S WORK

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- Reaction in Statistics

- Wold (1938) book marked emergence of modern time series analysis

Cited only two applications of **AR** processes:

(i) Yule (1927) sunspots application

**AR(2)** model praised by Wold

(ii) Walker (1931) pressure application

Wold thought **AR(1)** model with *no* need for **AR(4)**:

*“this periodogram does not like that of the sunspots suggest a scheme of linear autoregression with a tendency to periodicity”*

- **Reaction in Meteorology**

- **General skepticism about statistics**

**Discussion of paper read by R. A. Fisher in 1922:**

*“no new meteorological fact had been discovered by means of correlation coefficients; certainly up to the present no practical forecasts had been obtained from correlation coefficients”*

- **Normand (1953)**

**About use of correlation in meteorology:**

*“meteorologists have not all accorded him whole-hearted thanks”*

-- Sheppard (1959)

**Obituary for Walker:**

***“Walker’s hope was presumably not only to unearth relations useful for forecasting but to discover sufficient and sufficiently important relations to provide a productive starting point for a theory of world weather. It hardly appears to be working out like that.”***



- **Physical explanation (Walker always sought)**

**Walker (1918) about relations between weather in different parts of world:**

***“I cannot help believing that we shall gradually find out the physical mechanism by which these are maintained”***

**Walker (1927) suggested that:**

***“variations in activity of the general oceanic circulation will be much more far reaching and important”***

**Walker (1936) recommended searching:**

***“for an explanation in terms of slowly changing features, such as ocean temperatures”***

## **(5) PRESENT SITUATION**

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- **Quasiperiodic Behavior**
  - **Consensus that ENSO phenomenon exhibits quasiperiodic behavior**
  - **Weakness of General Circulation Models**
    - Ability to reproduce correct quasiperiod?**
  
- **Nonlinear Dynamics**
  - **Quasiperiodic feature of ENSO viewed as fundamental**
  - **Source remains unclear**

## CLOSING QUOTE

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- **Sir Gilbert Walker (1927)**

-- ***“There is, to-day, always a risk that specialists in two subjects, using languages full of words that are unintelligible without study, will grow up not only, without knowledge of each other’s work, but also will ignore the problems which require mutual assistance.”***