A methodology of optimal input design for aircraft parameter estimation

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Outline

- **Introduction :** problem statement
- **Experiment design :** aims and principles
- **A methodology of optimal input design**
- **Example**
- **Conclusion**

$$\begin{cases} \dot{x}(t, P) = f(x(t, P), P, u(t)), \quad x(0, P) = x_0, \\ y_m(t, P) = g(x(t, P), P), \end{cases}$$

- $u(t) \in \mathbb{R}^{\mathrm{m}}$: control,
- $x(t, P) \in \mathbb{R}^n$: state variables,
- $y_m(t, P) \in \mathbb{R}^p$: outputs,
- $P \in \mathbb{R}^q$: vector of parameters,
- $f, \ g$: nonlinear functions.

Measures :

$$y(t_i) = y_m(t_i, \bar{P}) + \nu(t_i),\ i = 1, 2, ..., N,$$

- $y(t) \in \mathbb{R}^p$: measured outputs,
- $\bar{P}$: true value of P,
- $\nu(t_i) \sim \mathcal{N}(0, \Sigma)$: measurement noise vector at time t_i,

Σ known diagonal matrix,

- N : total number of sample times.

Realistic practical constraint expressions :

- $|u_j(t) - u_{j_0}| \leq \mu_j \ \ \forall t,\ j = 1, 2, ..., m,$
- $|y_k(t) - y_{k_0}| \leq \eta_k \ \ \forall t,\ k = 1, 2, ..., p,$

- $\mu_j,\ \eta_k$: positive constants,
- $u_{j_0},\ y_{k_0}$: balance positions of u_j and y_k.

To select the most interesting experimental conditions in order to obtain simultaneously :

* the most relevant information with respect to parameters,

* the respect of the constraint set (inputs, outputs),

$\Longrightarrow$ improvement of the accuracy level on estimation of the model parameters,

$\Longrightarrow$ reduction of test number,

$\Longrightarrow$ reduction of test duration.

- To choose a cost function $j(\boldsymbol{\Xi})$

$\boldsymbol{\Xi}$: vector of experimental conditions to be optimized

  - inputs,
  - initial conditions.

- Definition of the set of admissible inputs $\mathbb{E}$.

- Optimization of the cost function with respect to $\boldsymbol{\Xi} \in \mathbb{E}$.

Scalar function of information matrix of Fisher $M(P, \Xi)$:

$$j(\Xi) = \Phi(M(P, \Xi)),$$

$* \ \phi(.)$: often chosen convex monotonous

$\longrightarrow$ here : the trace of $M^{-1}(P, \Xi)$

$$* \ M(P, \Xi) = \sum_{i=1}^{N} \left(\frac{\partial y_m(t_i, P)}{\partial P} \right)^T \Sigma^{-1} \left(\frac{\partial y_m(t_i, P)}{\partial P} \right),$$

$\longrightarrow \dfrac{\partial y_m(t_i, P)}{\partial P}$: sensitivities

$$\bullet \begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial x}{\partial P_j}\right) = \displaystyle\sum_{i=1}^{n} \left[\frac{\partial f}{\partial x_i} \frac{\partial x_i}{\partial P_j} \right] + \frac{\partial f}{\partial P_j}, \\ \dfrac{\partial y_m}{\partial P_j} = \displaystyle\sum_{i=1}^{n} \left[\frac{\partial g}{\partial x_i} \frac{\partial x_i}{\partial P_j} \right] + \frac{\partial g}{\partial P_j}, \quad j = 1, \ldots, q, \end{cases}$$

- finite differences.

Case of a nonlinear structure :

the optimal input depends on the values of the parameters to be estimated.

$\longrightarrow$ Classical approach : to fixe an initial parameter value P_0.

$\longrightarrow$ Obtention of P_0 : by wind tunnel measures (for following example).

STEP 1 Using of *dynamic programming*

• The test is divided into *stages*.

• Some possible constant control are applied over one stage time.

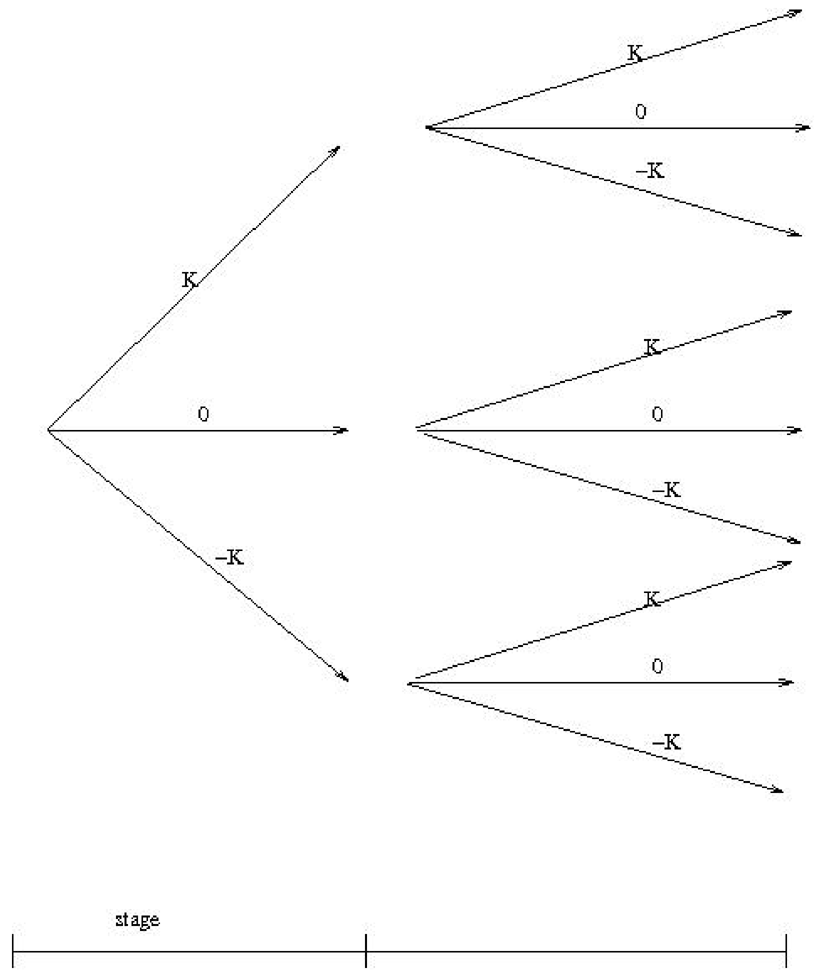
• The consequences of each control possibility are computed.

→ This technique discards any input among square wave sequence whose output trajectory exceeds constraint limits.

• Cost function value is computed for each input.

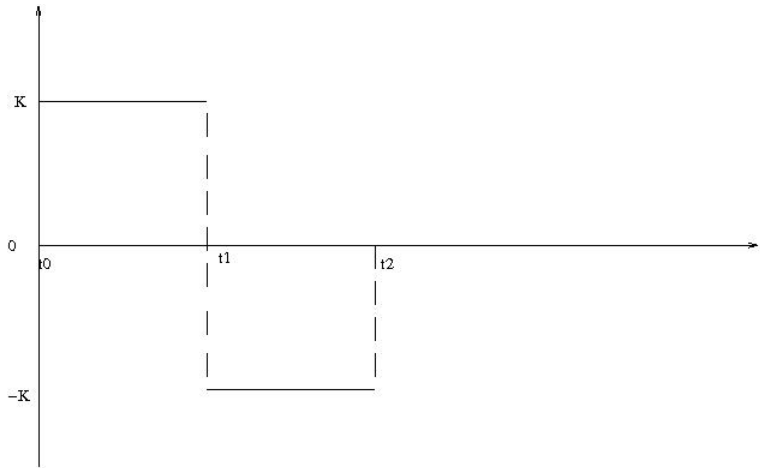
The developed optimal input method

- Suppose $|u(t)| < K,\ K > 0$



Step 1 : obtained input

An example of input obtained by the first step



$$u(t) = Kh(t) - 2Kh(t - t_1) + Kh(t - t_2), \quad \text{h : Heaviside function}$$

Optimal input method

- To search the most interesting input among "square wave " inputs :

$$u(t) = u_0 + \sum_{i=0}^{m} (a_i \varepsilon_i - a_{i-1} \varepsilon_{i-1}) h(t - t_i),$$

a_i and t_i are fixed

$$\Xi = (\varepsilon_0, ..., \varepsilon_m), \ \mathbb{E} = \{\Xi \in \mathbb{R}^{m+1} | \ \varepsilon_i \in \{-1, 0, +1\}, \ i = 0, ...m\}$$

$$j(\Xi) = \min_{\Xi \in \mathbb{E}} \ \mathrm{tr}(M(P, \Xi)^{-1})$$

$\Longrightarrow$ input with parameters : $\hat{\Xi}_1 = (\hat{\varepsilon}_{i_1}, \ i = 0, ...m),$

$$\rightarrow \hat{u}_1(t).$$

STEP 2 Approximation of previous function by a regular function

$$\tilde{u}(t) = u_0 + \sum_{i=0}^{m} \frac{\tilde{a}_i \varepsilon_i - \tilde{a}_{i-1}\varepsilon_{i-1}}{1 + \frac{exp(k(\tilde{t}_i - t))}{k}} \text{ with } k \to \infty,$$

$$j_k(\Xi) = \min_{\Xi \in \mathbb{E}} \operatorname{tr}(M(P, \Xi)^{-1})$$

$$\Xi = (\tilde{a}_i, \tilde{t}_i,\ i = 0, ..., m),$$

$\longrightarrow$ Optimization method based on a gradient algorithm with $\Xi_0 = \hat{\Xi}_1$

Example : longitudinal flight of an aircraft

$$
\begin{cases}
m\dot{V} = -mg\sin(\theta-\alpha) - \frac{1}{2}\rho S V^2 (C_x^0 + C_{x\alpha}(\alpha-\alpha_0) \\
\qquad + C_{x\delta_m}(\delta_m - \delta_{m_0})), \\
mV(\dot{\alpha}-\dot{\theta}) = mg\cos(\theta-\alpha) - \frac{1}{2}\rho S V^2 (C_{zD} + C_{z\dot{\alpha}}\frac{\dot{\alpha} l}{V}), \\
B\dot{q} = \frac{1}{2}\rho S l V^2 (C_{m\alpha}^{25}(\alpha-\alpha_0) + \frac{x_{25}-x_g}{l} C_{zD} + \frac{ql}{V}(C_{mq}^{25} \\
\qquad - \frac{x_{25}-x_g}{l} C_{m\alpha}^{25}) + C_{m\dot{\alpha}}^{25}\frac{\dot{\alpha} l}{V} + C_{m\delta_m}^{25}(\delta_m - \delta_{m_0}) + \frac{x_{25}-x_g}{l}, \\
\dot{\theta} = q,
\end{cases}
$$

$$
C_{zD} = C_z^0 + C_{z\alpha}(\alpha-\alpha_0) + (C_{zq} - \frac{x_{25}-x_g}{l} C_{z\alpha})\frac{ql}{V} + C_{z\delta_m}(\delta_m - \delta_{m_0}).
$$

- $x = [V, \alpha, q, \theta]^T$,
- $y_m = [V, \alpha, q, \theta]^T$,
- $u = \delta_m$, (elevator deflection angle)
- $P = (C_{z\dot{\alpha}}, C_{zq}, C_{m\dot{\alpha}}^{25}, C_{mq}^{25})$.

Example : longitudinal flight of an aircraft

This research was conducted at the ONERA Center of Lille.
Data : 4 stages, initial conditions known.
Experiments based on free flights of scaled models in a laboratory.

Step 1

$$u(t) = \delta_{m0} + \sum_{i=0}^{3} a(\varepsilon_i - \varepsilon_{i-1})h(t - t_i),$$

t_0, t_1, t_2, t_3 are fixed at 0s, 0.25s, 0.5s, 0.75s and $a = 1.6$ degrees

$$j(\Xi) = \min_{\Xi \in \mathbb{E}} \text{tr}(M(P, \Xi)^{-1}) \text{ with } \Xi = (\varepsilon_0, \varepsilon_1, \varepsilon_2, \varepsilon_3)$$

$$\rightarrow \hat{u}_1(t) = \delta_{m0} + ah(t - t_0) - 2ah(t - t_1) + 2ah(t - t_2) - 2ah(t - t_3).$$

Example : longitudinal flight of an aircraft

Step 2

$$\tilde{u}(t) = \delta_{m0} + \frac{\tilde{a}}{1 + \frac{exp(k(\tilde{t}_0 - t))}{k}} - 2\frac{\tilde{a}}{1 + \frac{exp(k(\tilde{t}_1 - t))}{k}}$$

$$+2\frac{\tilde{a}}{1 + \frac{exp(k(\tilde{t}_2 - t))}{k}} - 2\frac{\tilde{a}}{1 + \frac{exp(k(\tilde{t}_3 - t))}{k}}, \quad k = 100$$

$$\mathbb{E} = \{\tilde{a}, \tilde{t}_1, \tilde{t}_2, \tilde{t}_3\} \text{ with } \begin{cases} |\tilde{a}| \leq 1.6 \text{ degrees}, \\ 0.1 \leq \tilde{t}_1 \leq 0.4s, \\ 0.4 \leq \tilde{t}_2 \leq 0.7s, \\ 0.7 \leq \tilde{t}_3 \leq 1s, \end{cases}$$

$$j_k(\Xi) = \min_{\Xi \in \mathbb{E}} \text{ tr}(M(P, \Xi)^{-1}) \text{ with } \Xi = (\tilde{a}, \tilde{t}_1, \tilde{t}_2, \tilde{t}_3)$$

$$\rightarrow \hat{\tilde{u}}(t) \ \text{ with } \hat{\Xi} = (1.6,\ 0.2951,\ 0.4,\ 0.7563)$$

Example : longitudinal flight of an aircraft

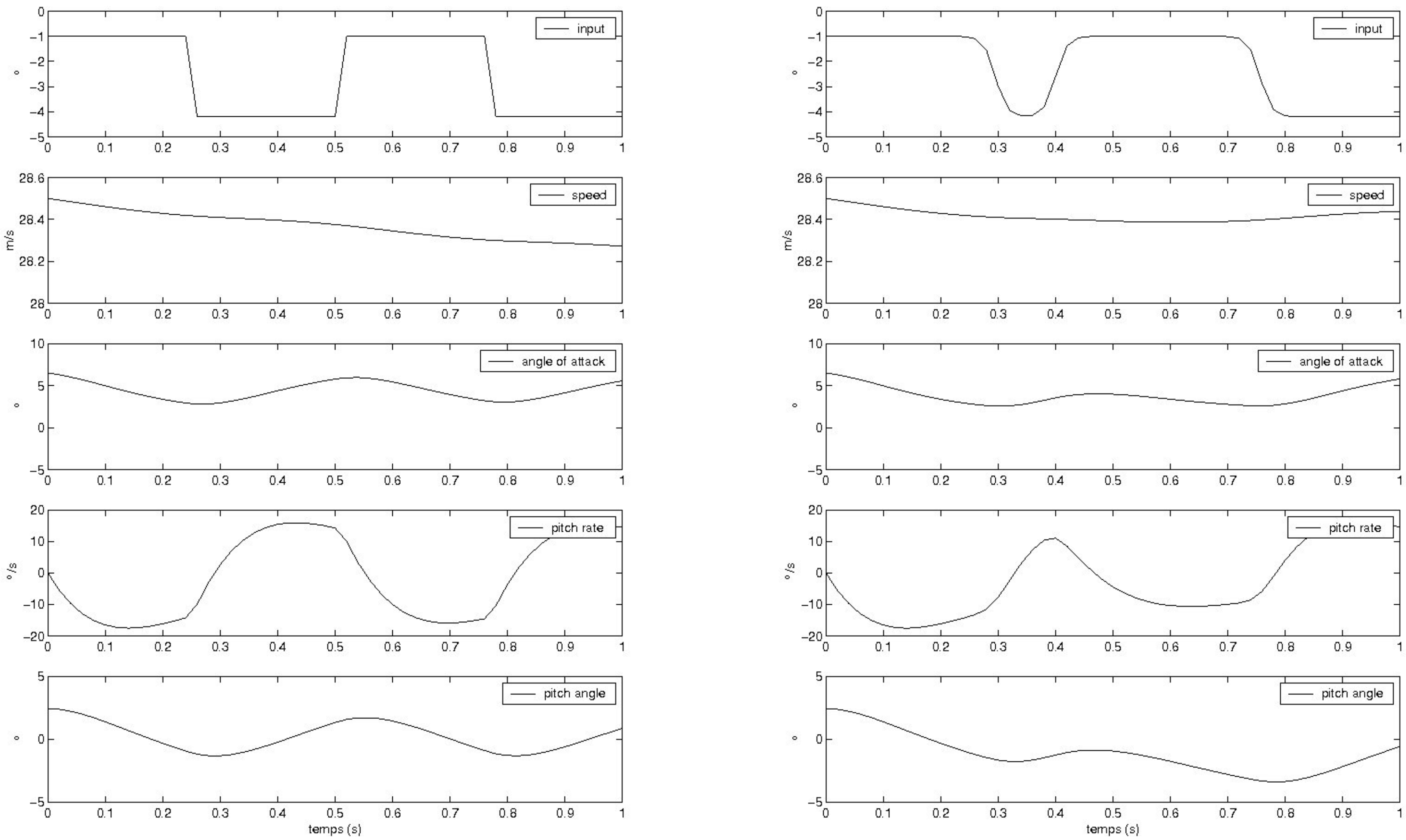


FIG. 1 – left : input obtained step 1, right : optimal input

Criterion values (4 stages)

	input 1	input 2	input 3	input 4
trace(M^{-1})	0.3101	0.0485	0.1010	1.7804

input 1 : obtained input by step 1,
input 2 : obtained optimal input by step 2,
input 3 : input square wave whose parameters are obtained by step 2,
input 4 : non-optimal classical input.

Estimation method : weighting method

- To improve the conditionning of the estimation problem

$$J(P) = \sum_{i=0}^{N} (y_m(t_i, P) - y(t_i))^T w_i (y_m(t_i, P) - y(t_i)).$$

$\longrightarrow$ Find $W = diag(w_0, \ldots, w_N)$. How find W ?

- Use LMI formulation :

where G_W is the Gauss-Newton approx. of the Hessian.

$$\begin{cases} \min_{w_0,\ldots,w_N,t} \quad t \\ \text{submitted to} \quad \begin{bmatrix} tI & G_W - I \\ (G_W - I)^\top & tI \end{bmatrix} > 0 \\ \qquad\qquad\qquad W > 0 \end{cases}$$

$\longrightarrow$ First constraint : $\rho(G_W - I) < t$.

$\longrightarrow$ Second constraint : positivity of W.

Relative errors on estimated parameters

	optimal input and J_{mc}	non-optimal input and $J_{mc} + LMI's$
$E_{C_{z\dot{\alpha}}}$	0.0287	0.0372
$E_{C_{zq}}$	0.0317	0.1035
$E_{C^{25}_{m\dot{\alpha}}}$	0.0249	0.0474
$E_{C^{25}_{mq}}$	0.0090	0.0543

$$J_{mc}(P) = \sum_{i=0}^{N} (y_m(t_i, P) - y(t_i))^T Q (y_m(t_i, P) - y(t_i))$$

column 2 : optimal input and minimisation of least-squares with $Q = \Sigma^{-1}$,

column 3 : non-optimal input and minimisation of least-squares with $Q = W$ (LMI's)

• Optimal input design in two steps : a good improvement on criterion values and on accuracy estimation of the model parameters.

→ A reasonable computation requirement.

• Optimal input design for systems with bounded uncertainties :

- Same aims as previous approach.
- To obtain an optimal input for a possible interval of parameters.

We consider :

$$\begin{cases} \dot{x}(t,p) = f(x(t,p),p,u(t)), \quad x(0,p) = x_0, \\ y_m(t,p) = g(x(t,p),p), \end{cases}$$

- $p \in P$: P an *a priori* box (cartesian product of intervals),

- $x_0 \in \mathbb{X}_0$: $\mathbb{X}_0$ an *a priori* box,

- $M(p,\Xi)$ is an *interval* matrix (each component of $M(p,\Xi)$ is an interval).

$\Longrightarrow$ Maximin optimization :

$$\hat{\Xi} = \boldsymbol{Arg} \max_{\Xi \in \mathbb{E}} (\min_{\boldsymbol{p} \in \boldsymbol{P}} \det \big(\boldsymbol{M} (\boldsymbol{p}, \Xi) \big)).$$

$\rightarrow$ This problem is feasible by the first step of the previous methodology.

$\rightarrow$ We have obtained some first encouraging results for previous example.

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