Split-plot experiments with factor-dependent whole pot sizes

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Reference and co-authors

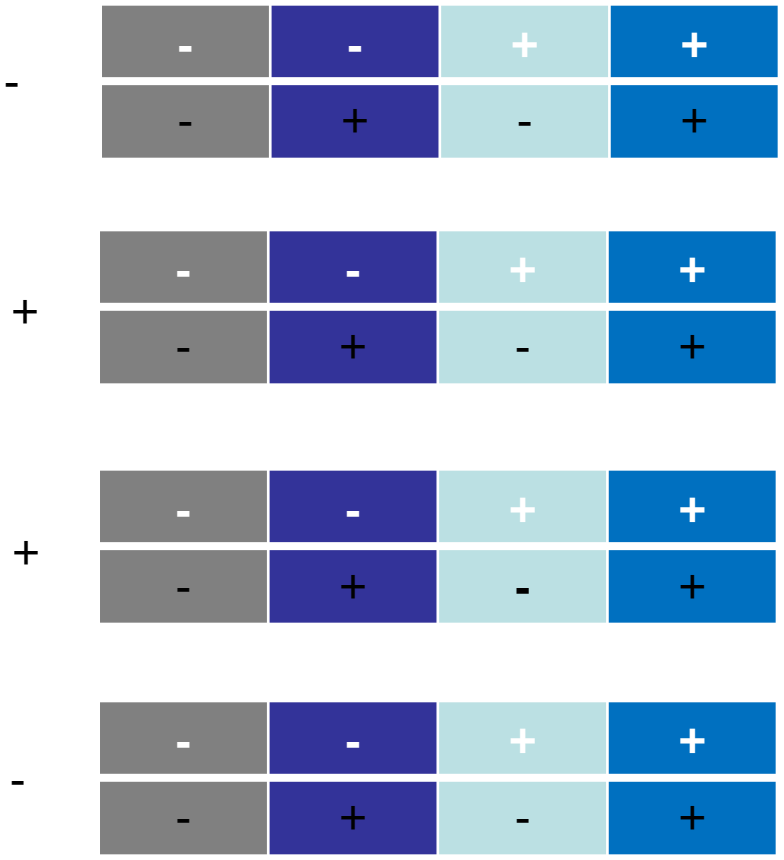
- Eric Schoen, Bradley Jones, Peter Goos
- *Journal of Quality Technology* **43** (2011): 66-78.

Overview

1. A simple split-plot design
2. How to construct D-optimal split-plot designs
3. Why this did not help for a project involving coffee cream production
4. A modified algorithm
5. Design alternatives
6. Recommendations

A simple split-plot design

- 4 whole plots
- 4 subplots per whole plot

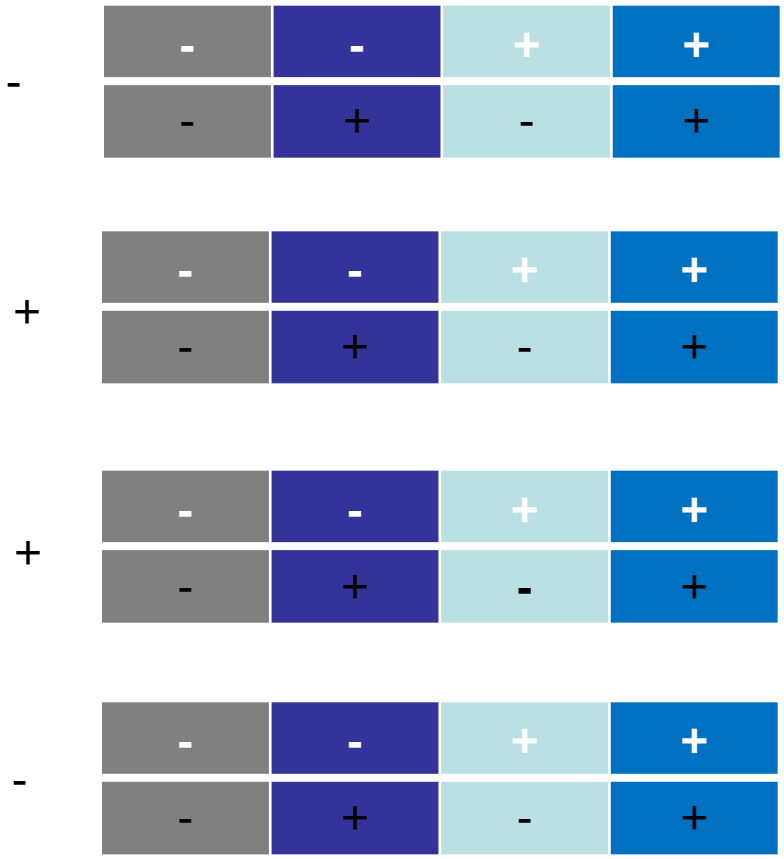


- 1 whole-plot factor at two levels, hard to change (HTC)
- 2 subplot factors, easy to change (ETC)

A simple split-plot design

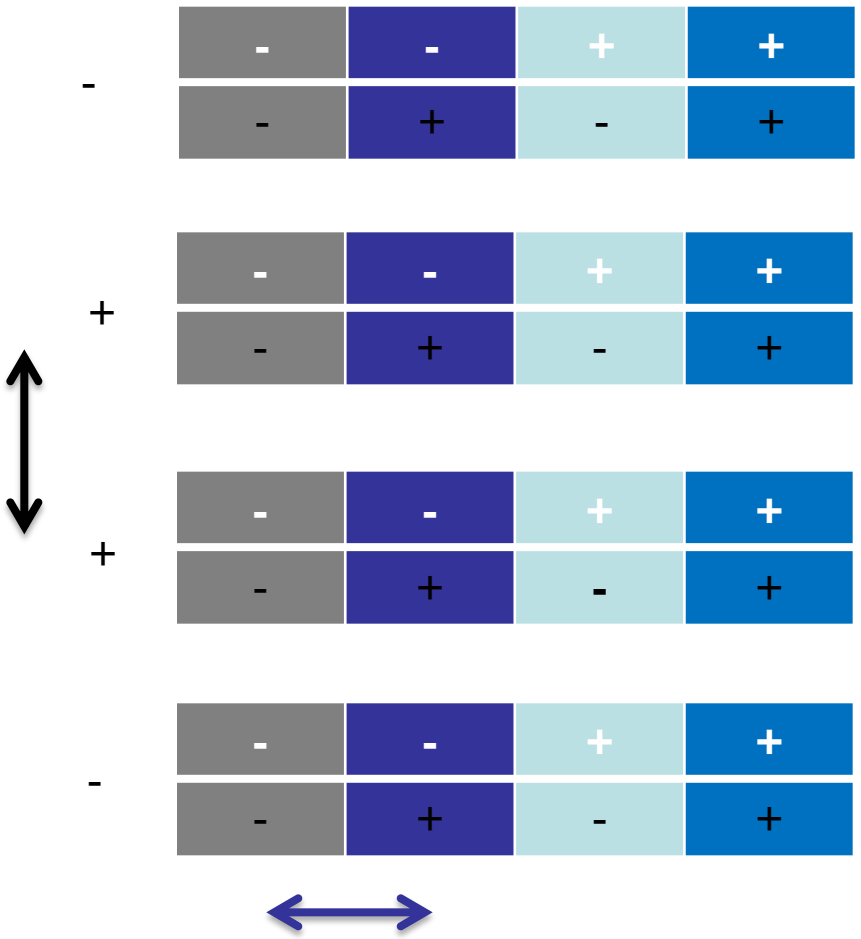
- Statistical model of response:

- 3 main effects
- 3 two-factor interactions



Random variation

- Variance component between whole plots
- Variance component within whole plots / between subplots
- Variance-covariance matrix of observations
  Σ is block diagonal



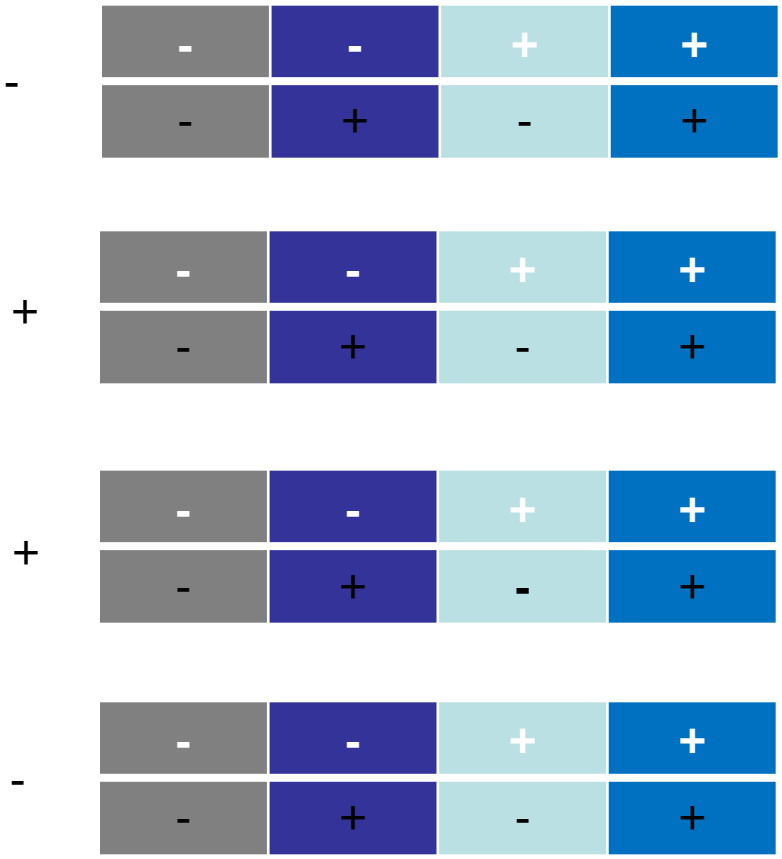
Random variation

- Model coefficients have information matrix

$$X'\Sigma^{-1}X$$

Layout dependent

Treatment dependent



Coffee cream project

- 2 HTC factors:

1. Number of process loops (1 x, 2 x)
2. Speed (slow, fast)

- 7 ETC factors:

  $3^5 2^2$

- Model:

1. Intercept
2. 9 linear terms
3. 5 quadratic terms
4. 36 product terms

- At least 51 observations required

Coffee cream project

- Budget:

6 whole plots ↔ weeks

3 weeks of 7 subplots

3 weeks of 14 subplots

- 63 observations

- Model:

1. Intercept
2. 9 linear terms
3. 5 quadratic terms
4. 36 product terms

- At least 51 observations required

Algorithmic design construction

- Goal: maximize

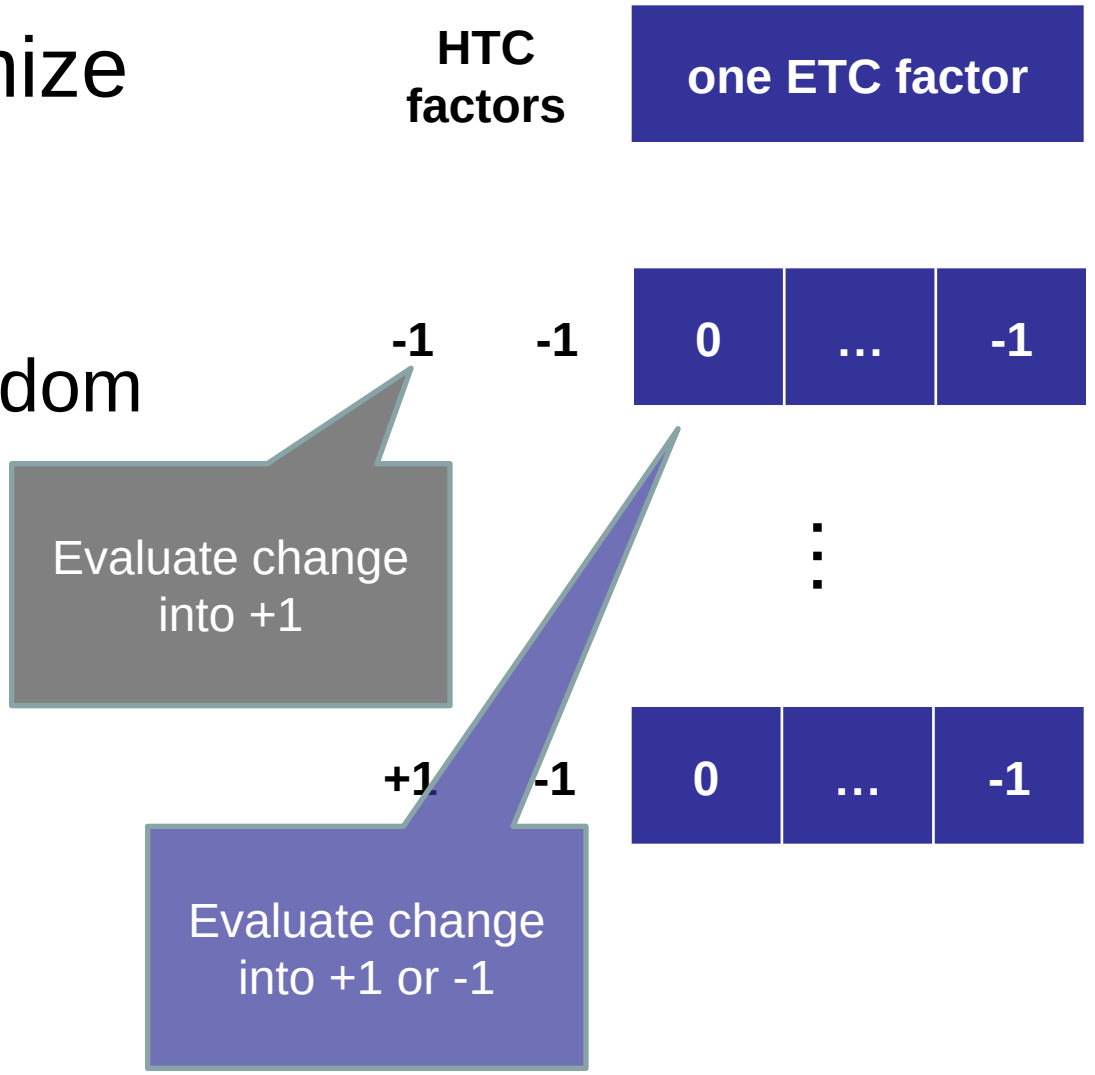
  $\det (X'\Sigma^{-1}X)$

- We have to make assumptions on Σ
- The assumptions do not enter this story
- Results are not very sensitive to the assumptions

Algorithmic design construction

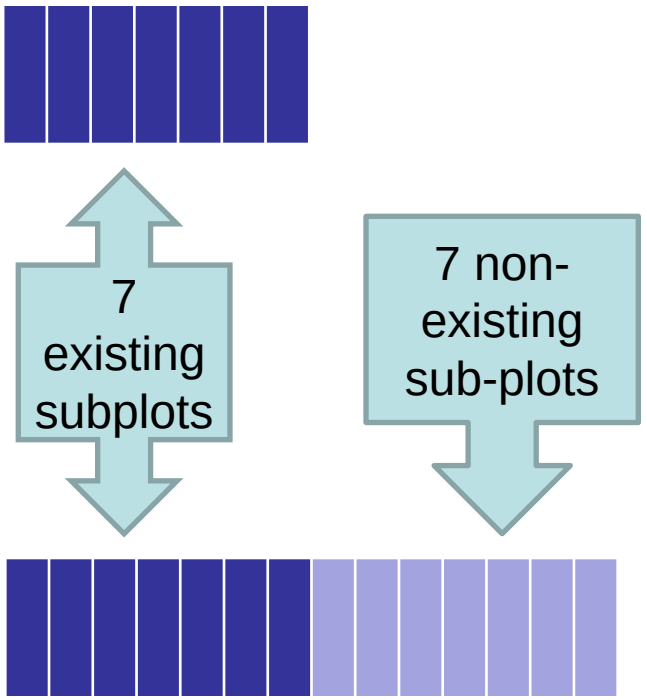
- Goal: maximize $\det (X'\Sigma^{-1}X)$

- First step: random start design
- Second step: exchange



A problem

- Speed is linked to whole-pot size.
- Change to high speed results in 7 extra subplots
- Settings of SP factors unknown
- Information matrix cannot be evaluated



A solution

speed	loop	
	1 x	2 x
slow	WP 1 WP 5	WP 3
fast	WP 2 WP 6	WP 4

speed	loop	
	1 x	2 x
slow	WP 1 WP 5	WP 3
fast	WP 2	WP 4 WP 6

- Do not exchange factor settings for whole-plot factors
- Consider all possible whole-plot designs
- There are two different designs for 63 runs, apart from relabeling ‘loop’

A solution

speed	loop	
	1 x	2 x
slow	WP 1 WP 5	WP 3
fast	WP 2 WP 6	WP 4

- $|(X'\Sigma^{-1}X)|^{1/p} = 2.202$
- WP design orthogonal for main effects

speed	loop	
	1 x	2 x
slow	WP 1 WP 5	WP 3
fast	WP 2	WP 4 WP 6

- $|(X'\Sigma^{-1}X)|^{1/p} = 2.193$
- Non-orthogonal WP design

70-run alternatives

speed	loop	
	1 x	2 x
slow	WP 1	WP 3
fast	WP 2 WP 5	WP 4 WP 6

- $|(X'\Sigma^{-1}X)|^{1/p} = 2.448$

speed	loop	
	1 x	2 x
slow	WP 1	WP 3
fast	WP 2 WP 5 WP 6	WP 4

- $|(X'\Sigma^{-1}X)|^{1/p} = 2.364$

WP contrasts in 63-run designs

speed	loop	
	1 x	2 x
slow	WP 1 WP 5	WP 3
fast	WP 2 WP 6	WP 4

speed	loop	s x l
3:3	4:2	3:3

speed	loop	
	1 x	2 x
slow	WP 1 WP 5	WP 3
fast	WP 2	WP 4 WP 6

speed	loop	s x l
3:3	3:3	4:2

WP contrasts in 70-run designs

speed	loop	
	1 x	2 x
slow	WP 1	WP 3
fast	WP 2 WP 5	WP 4 WP 6

speed	loop	s x l
4:2	3:3	3:3

speed	loop	
	1 x	2 x
slow	WP 1	WP 3
fast	WP 2 WP 5 WP 6	WP 4

speed	loop	s x l
4:2	4:2	4:2

Summary

N	orthogonal?	speed	loop	s x l	$\lvert(X'\Sigma^{-1}X)\rvert^{1/p}$
63	yes	3:3	4:2	3:3	2.202
63	no	3:3	3:3	4:2	2.193
70	yes	4:2	3:3	3:3	2.448
70	no	4:2	4:2	4:2	2.364

Recommendations

For factor-dependent whole plot sizes:

- Find an orthogonal WP design
- Fix the design and apply an exchange algorithm for the SP factors
- Randomize the order of whole plots and the order of subplots within each whole plot separately

The last slide

- Thank you!