KATHOLIEKE UNIVERSITEIT
LEUVEN


# Estimating the heterogeneity distribution of willingness-to-pay using individualized choice sets

Vishva DANTHUREBANDARA
Jie YU
Martina VANDEBROEK

# OUTLINE

- Willingness-to-pay
- Choice experiments
- Market heterogeneity
- Design issues
- Individualized designs
- Simulation results
- Conclusion

# WILLINGNESS-TO-PAY

*the willingness-to-pay (WTP) is the maximum amount a person is willing to pay to receive a good or to avoid something undesired*

# WILLINGNESS-TO-PAY



- **marketing**: how much are people WTP for an extra hour battery life of a laptop
- **transportation**: WTP for travel time reduction
  (VTTS = valuation of travel time savings or VAT = value of time)
- **health economics**: WTP for one day reduction in waiting time for an appointment with a general practitioner
- **environmental economics**: how much are people WTP for clean air

# WILLINGNESS-TO-PAY

KATHOLIEKE UNIVERSITEIT LEUVEN

## HOW TO ESTIMATE WTP ?

previously: *contingent valuation experiments*
(open-ended, closed ended, sequential bids)

nowadays: based on a utility model which expresses the utility of a product with attributes: $x_1, x_2, \ldots x_m$ and price $P$:

$$U = \beta_1 x_1 + \ldots + \beta_m x_m + \beta_p P + \epsilon$$

a change in $x_m$ should give a change in $P$ such that U is constant:

$$dU = \beta_m \, dx_m + \beta_p \, dP = 0$$

which leads to $WTP = \frac{dP}{dx_m} = -\frac{\beta_m}{\beta_p}$ (*marginal* WTP)

# CHOICE EXPERIMENTS

to estimate the utility parameters: **discrete choice experiments**

| | | | |
|---|---|---|---|
|  |  |  |  |
| Screen:15.4 inch<br>HD:80 GB<br>Battery:5h<br>Price:€650 | Screen:17 inch<br>HD:60 GB<br>Battery:6h<br>Price:€700 | Screen:15.4 inch<br>HD:120 GB<br>Battery:4h<br>Price:€600 | Screen:17 inch<br>HD:80 GB<br>Battery:6h<br>Price:€650 |

let respondents choose their preferred alternative in all choice sets presented to them

# CHOICE EXPERIMENTS



the probability that alternative $k$ is chosen from choice set $s$ is

$$p_{ks}(\boldsymbol{\beta}) = \frac{\exp\left(\beta_1 x_{1ks} + \ldots + \beta_m x_{mks} + \beta_p P_{ks}\right)}{\sum_{k=1}^{K} \exp(\beta_1 x_{1ks} + \ldots + \beta_m x_{mks} + \beta_p P_{ks})}$$

the likelihoodfunction is

$$\mathbf{L}(\boldsymbol{\beta}|\mathbf{X}, Y) = \prod_{n=1}^{N} \prod_{s=1}^{S} \prod_{k=1}^{K} (p_{ks}(\boldsymbol{\beta}))^{y_{ksn}}$$

the Fisher information matrix

$$\mathbf{I}_{FIM}(\boldsymbol{\beta}) = -\mathbf{E}_Y\left[\frac{\partial^2 \ln \mathbf{L}(\boldsymbol{\beta}|\mathbf{X}, Y)}{\partial \boldsymbol{\beta} \partial \boldsymbol{\beta}'}\right] = N \sum_{s=1}^{S} \mathbf{X}_s'(\mathbf{P}_s - \mathbf{p}_s \mathbf{p}_s')\mathbf{X}_s$$

# CHOICE EXPERIMENTS

**design problem**:

- what profiles to show
- what profiles to combine in a choice set

choose $\mathbf{X}$ such that $\mathbf{I}(\mathbf{X}, \boldsymbol{\beta})$ is large somehow

$\Rightarrow$ OK for the conditional logit model
(local + semi-Bayesian D-optimal)

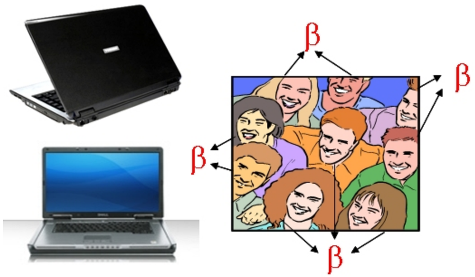
# MARKET HETEROGENEITY



## Conditional logit

Homogeneous market

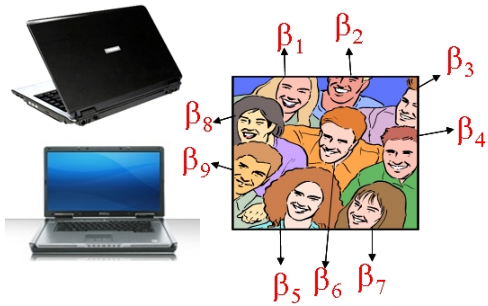
$$U_{ksn} = \mathbf{x}'_{ks}\boldsymbol{\beta} + \varepsilon_{ksn}$$




## Mixed logit

Heterogeneous market

$$U_{ksn} = \mathbf{x}'_{ks}\boldsymbol{\beta}_n + \varepsilon_{ksn}$$
$$\boldsymbol{\beta}_n \sim \mathcal{F}(\boldsymbol{\mu}_{\beta}, \Sigma_{\beta})$$



# MARKET HETEROGENEITY



in the mixed logit model, we have for given $\boldsymbol{\beta}_n$

$$p_{ksn}(\boldsymbol{\beta}_n) = \frac{\exp\left(\mathbf{x}'_{ks}\boldsymbol{\beta}_n\right)}{\sum_{k=1}^{K}\exp\left(\mathbf{x}'_{ks}\boldsymbol{\beta}_n\right)}$$

with $\boldsymbol{\beta}_n$ randomly drawn from $\mathcal{F}(\boldsymbol{\mu}_\beta, \boldsymbol{\Sigma}_\beta)$, the total likelihood, unconditional on $\boldsymbol{\beta}_n$ is

$$\mathbf{L}(\boldsymbol{\mu}_\beta, \boldsymbol{\Sigma}_\beta) \; = \prod_{n=1}^{N}\int\prod_{s=1}^{S}\prod_{k=1}^{K}(p_{ksn}(\boldsymbol{\beta}_n))^{y_{ksn}} \; f(\boldsymbol{\beta}_n \mid \boldsymbol{\mu}_\beta, \boldsymbol{\Sigma}_\beta)\; d\boldsymbol{\beta}_n$$

# MARKET HETEROGENEITY

KATHOLIEKE UNIVERSITEIT LEUVEN

## HIERARCHICAL BAYES ESTIMATION:

with $\boldsymbol{\beta}_n \sim N(\boldsymbol{\mu}_{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}})$ and uninformative priors for $\boldsymbol{\mu}_{\boldsymbol{\beta}}$ and $\boldsymbol{\Sigma}_{\boldsymbol{\beta}}$ use Gibbs sampling to take iterative draws from the joint distribution of $\boldsymbol{\mu}_\beta$, $\boldsymbol{\Sigma}_\beta$ and all $\beta_n$:

- the conditional distribution of $\boldsymbol{\mu}_\beta$, given $\beta_n$ and $\boldsymbol{\Sigma}_\beta$:
  draw $\boldsymbol{\mu}_{\boldsymbol{\beta}}^{j+1}$ from $N(\bar{\boldsymbol{\beta}}^j, \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^j / N)$
- the conditional distribution of $\boldsymbol{\Sigma}_\beta$ given $\boldsymbol{\mu}_\beta$ and $\beta_n$
  draw $\boldsymbol{\Sigma}_\beta^{j+1}$ from $IW\left(q + N, \frac{qI + NS^j}{q + N}\right)$
- the conditional posterior for $\boldsymbol{\beta}_n$ conditional on $\boldsymbol{\mu}_{\boldsymbol{\beta}}$ and $\boldsymbol{\Sigma}_\beta$:
  use *Metropolis-Hasting random walk*

# DESIGN ISSUES



**Fisher information matrix:**

$$\mathbf{I}_{FIM}(\boldsymbol{\mu}_{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}}) = -\mathbf{E}_Y \left[ \frac{\partial^2 \ln \mathbf{L}(\mathbf{y}_n^S)}{\partial \boldsymbol{\theta}\, \partial \boldsymbol{\theta}'} \right] \quad \text{with} \quad \boldsymbol{\theta} = (\boldsymbol{\mu}_{\boldsymbol{\beta}}; \boldsymbol{\Sigma}_{\boldsymbol{\beta}})'$$

for the (panel) mixed logit model:

$$\mathbf{L}\left(\boldsymbol{\mu}_{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}}\right) = \prod_{n=1}^{N} \int \left( \prod_{s=1}^{S} \prod_{k=1}^{K} p_{ksn}(\boldsymbol{\beta}_n)^{y_{ksn}} \right) f\left(\boldsymbol{\beta}_n | \boldsymbol{\mu}_{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}}\right) d\boldsymbol{\beta}_n$$

$\Rightarrow$ depends on the responses $y_{ksn}$

$\Rightarrow$ compute $\mathbf{I}_{FIM}(\boldsymbol{\mu}_{\boldsymbol{\beta}}, \boldsymbol{\Sigma}_{\boldsymbol{\beta}})$ by simulating observations $y_{ksn}$

$\Rightarrow$ computationally very intensive!
(some results by Bliemer & Rose, 2010)

# DESIGN ISSUES

**Individually adapted sequential Bayesian designs = IASB**

For each respondent $n$:

- Assume a prior distribution $\pi(\boldsymbol{\beta}_n) = N(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$
- Show $S_1$ semi-Bayesian D-optimal choice sets for a conditional logit model to respondent $n$
- Update prior information with the choices made

$$q(\boldsymbol{\beta}_n) = \frac{\mathbf{L}(\boldsymbol{\beta}_n|\mathbf{X}_n^{S_1}, \mathbf{y}_n^{S_1})\pi(\boldsymbol{\beta}_n)}{\int \mathbf{L}(\boldsymbol{\beta}_n|\mathbf{X}_n^{S_1}, \mathbf{y}_n^{S_1})\pi(\boldsymbol{\beta}_n)d\boldsymbol{\beta}_n},$$

# DESIGN ISSUES



- For $i = S_1 + 1$ to $S_1 + S_2$
  - look for next choice set $\mathbf{x}_{i+1}$ such that $D_B$ is minimal:

$$D_B = \int \det \left[\mathbf{I}_{GFIM}\left(\boldsymbol{\beta}_n | \mathbf{X}_i, \mathbf{x}_{i+1}\right)\right]^{-1/p} q(\boldsymbol{\beta}_n) d\boldsymbol{\beta}_n$$

    with

$$\mathbf{I}_{GFIM}\left(\boldsymbol{\beta}_n | \mathbf{X}\right) = \mathbf{I}_{FIM}\left(\boldsymbol{\beta}_n | \mathbf{X}\right) + \boldsymbol{\Sigma}_0^{-1}$$

  - update the prior with the choice made

Finally, estimate the mixed logit model by HB estimation to get final estimates for $\boldsymbol{\mu}_\beta$, $\boldsymbol{\Sigma}_\beta$ and all $\beta_n$

# DESIGN ISSUES

This approach works very well to estimate the heterogeneity distribution of the utility parameters!

Because $WTP_m = -\frac{\beta_m}{\beta_p}$ problems are guaranteed if one wants to derive the heterogeneity distribution $\mathcal{F}(\boldsymbol{\mu}_\omega, \boldsymbol{\Sigma}_\omega)$ of the WTP values from these results!

How to adapt this procedure to estimate the heterogeneity distribution $\mathcal{F}(\boldsymbol{\mu}_\omega, \boldsymbol{\Sigma}_\omega)$ of the WTP values?

# SIMULATION RESULTS

**MODEL IN WTP-SPACE**: rewrite the random utility model:

$$\begin{aligned} U_{ksn} &= \beta_{1n}x_{1ksn} + \beta_{2n}x_{2ksn} + .... + \beta_{mn}x_{mksn} - \beta_{pn}P_{ksn} + \varepsilon_{ksn} \\ &= \beta_{pn}\frac{\beta_{1n}}{\beta_{pn}}x_{1ks} + .... + \beta_{pn}\frac{\beta_{mn}}{\beta_{pn}}x_{mksn} - \beta_{pn}\frac{\beta_{pn}}{\beta_{pn}}P_{ksn} + \varepsilon_{ksn} \\ &= \beta_{pn}WTP_{1n}x_{1ksn} + .... + \beta_{pn}WTP_{mn}x_{mksn} - \beta_{pn}P_{ksn} + \varepsilon_{ksn} \end{aligned}$$

the parameters are now $\boldsymbol{\omega}_n^* = [WTP_{1n}, WTP_{2n}, \ldots, WTP_{mn}, \beta_{pn}]$

# SIMULATION RESULTS



the probability that an alternative is chosen from a choice set and the likelihood are similar as before

some changes are needed:

- to adapt the design criterion
- to adapt the MH procedure in the HB estimation

# SIMULATION RESULTS

design setting:

- 12 choice sets: S1 = 4 and S2 = 8
- three alternatives per choice set
- four attributes with two levels
- 100 respondents

# SIMULATION RESULTS

assume $\pi(\boldsymbol{\omega}_n^*)$ is $N(a\,\bar{\boldsymbol{\omega}}, \sigma\,\boldsymbol{\Sigma})$ with

$$\bar{\boldsymbol{\omega}} = [-0.8 \;\; 1.3 \;\; 1.8 \;\; -2.3]$$

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.5 & 0.0 & 0.0 & -0.1 \\ 0.0 & 0.5 & 0.0 & -0.1 \\ 0.0 & 0.0 & 0.5 & -0.1 \\ -0.1 & -0.1 & -0.1 & 0.1 \end{pmatrix}$$

and with $a$ small = low accuracy & $a$ large = high accuracy,

$\sigma$ small = low heterogeneity & $\sigma$ large = high heterogeneity

# SIMULATION RESULTS

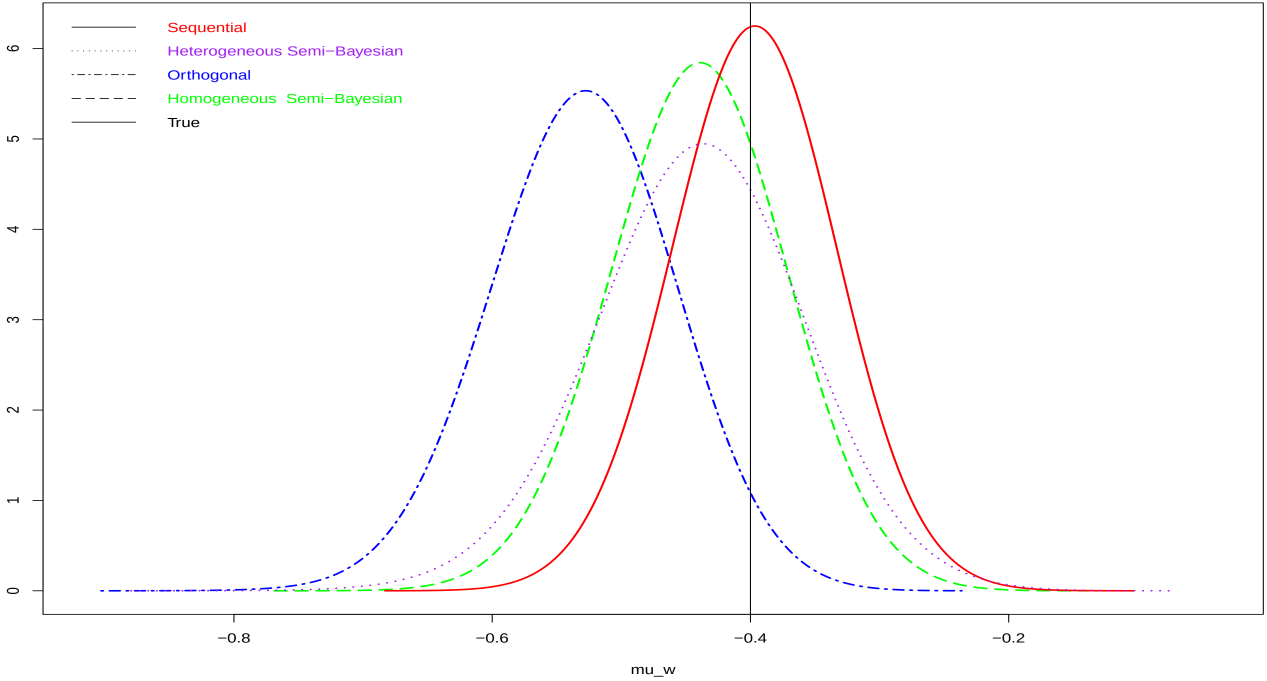
compare the IASB design with:

- a nearly orthogonal design (close to a utility neutral design)
- a semi-Bayesian *homogeneous* D-optimal design: constructed for the conditional logit model and the same prior distribution $\pi(\boldsymbol{\omega}^*)$
- a semi-Bayesian *heterogeneous* D-optimal design: constructed for the conditional logit model and the same prior distribution $\pi(\boldsymbol{\omega}^*)$: $\min_{X_i} \mathbf{D}_B(X_1, \ldots X_{i-1}, X_i)$

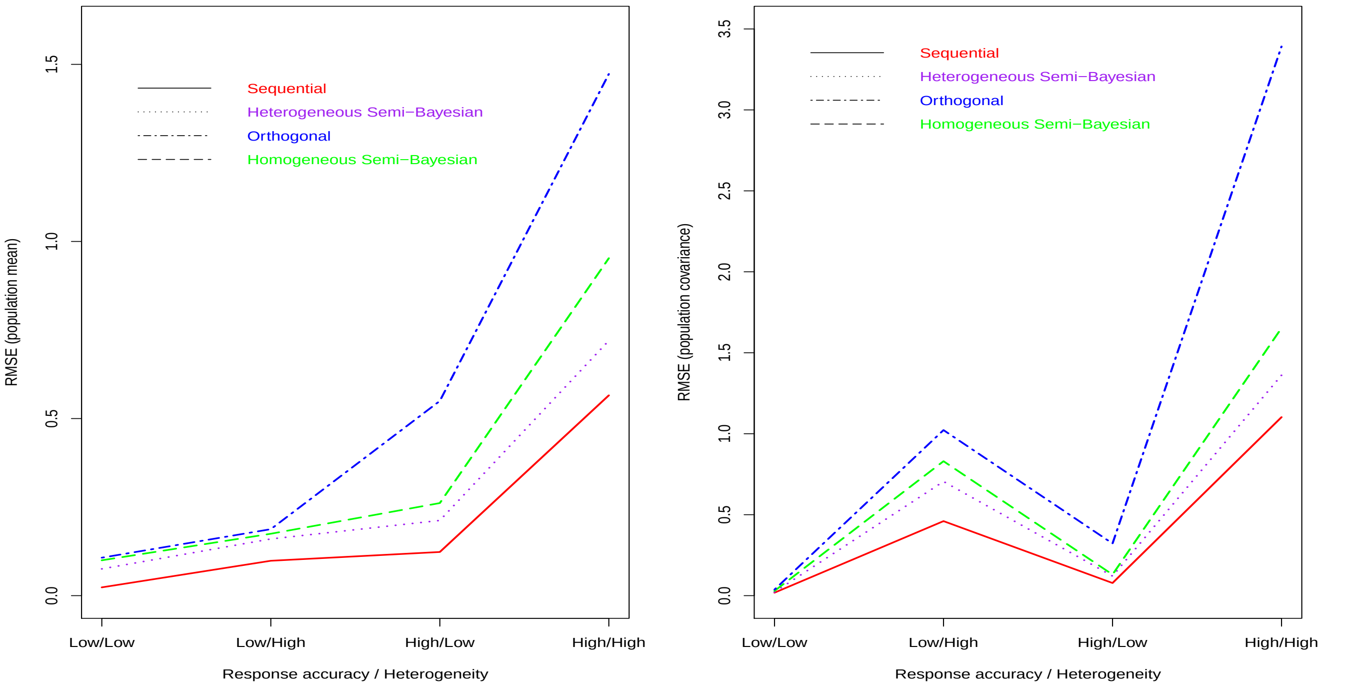
# SIMULATION RESULTS

KATHOLIEKE UNIVERSITEIT LEUVEN

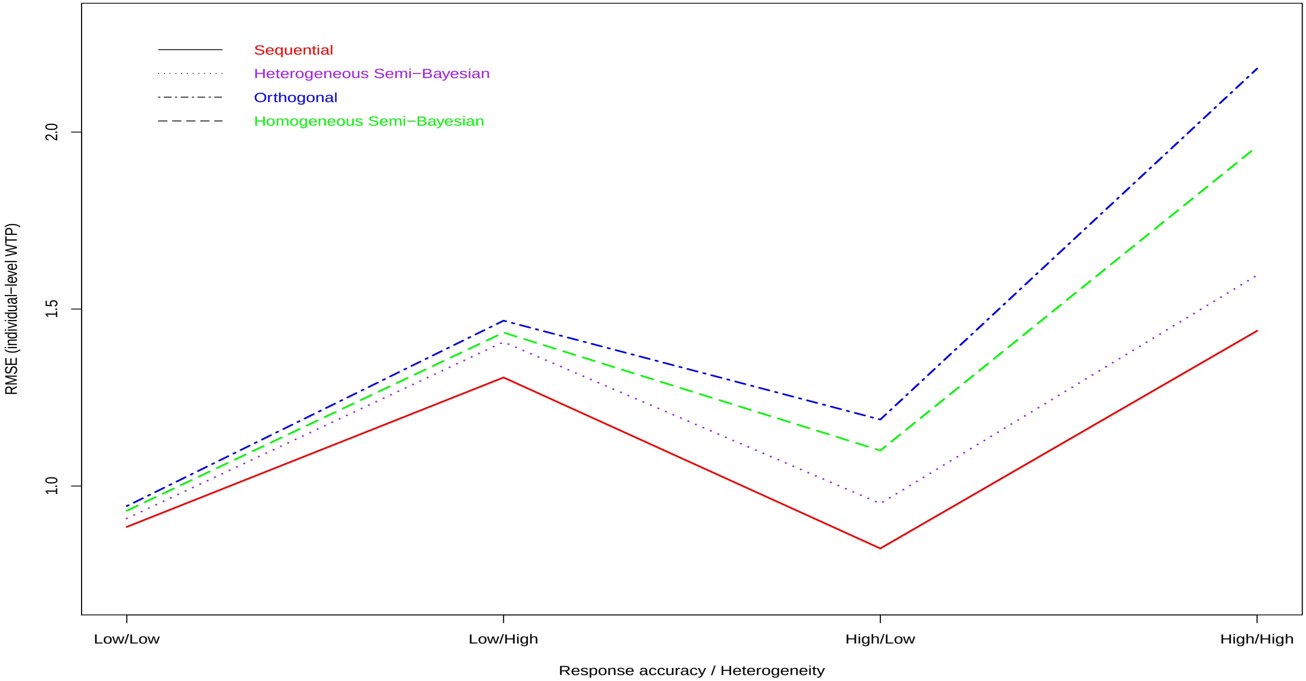
## low accuracy, low heterogeneity: the posterior distribution of $\boldsymbol{\mu}_{\omega_1}$



# SIMULATION RESULTS



# SIMULATION RESULTS




KATHOLIEKE UNIVERSITEIT LEUVEN

# SIMULATION RESULTS

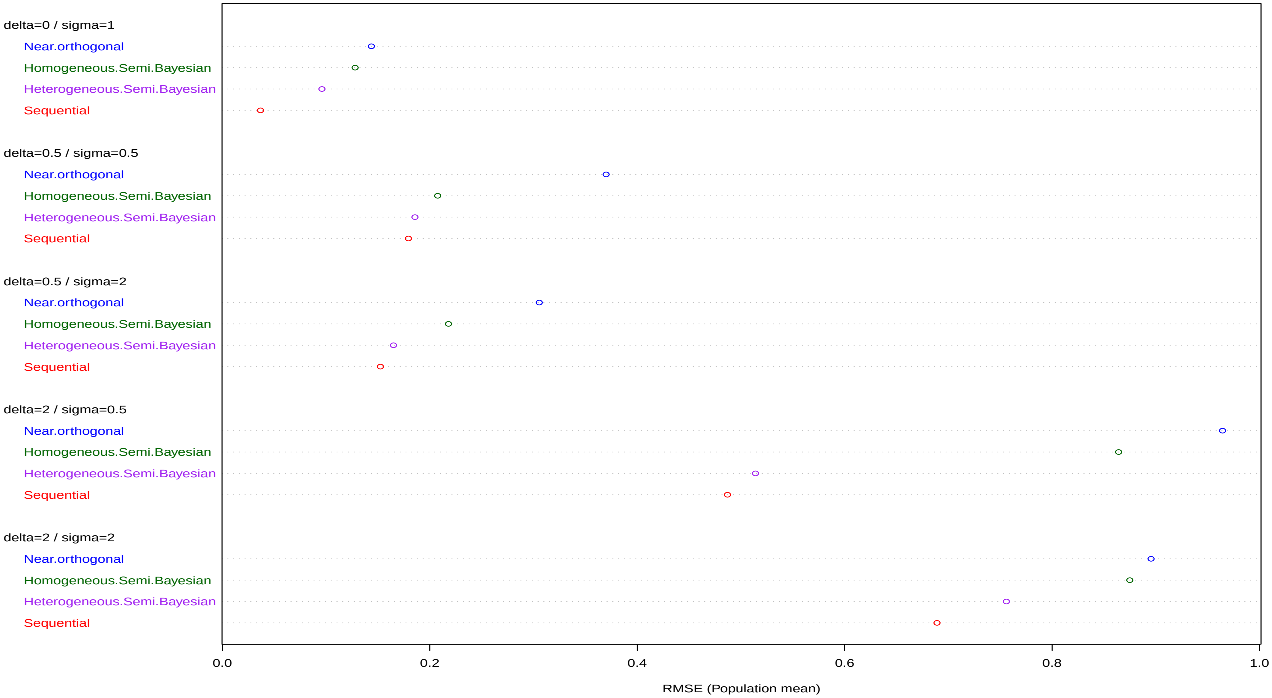
KATHOLIEKE UNIVERSITEIT LEUVEN

**robust to misspecification of the prior?**

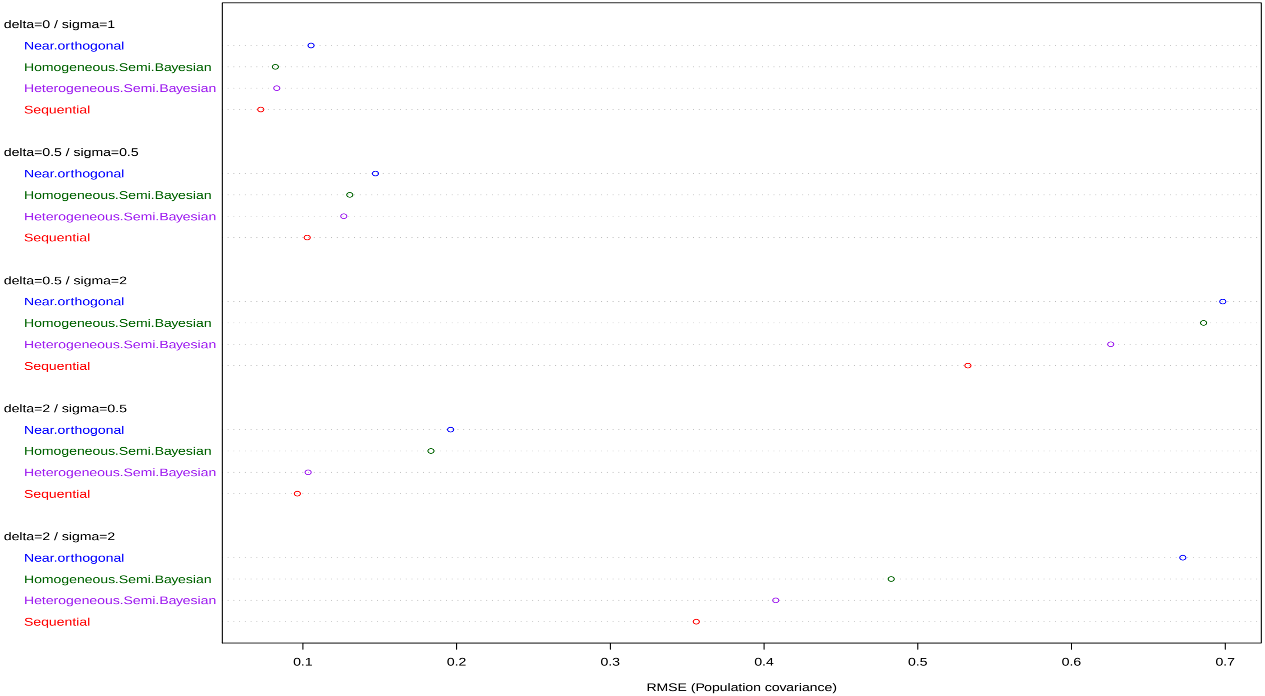
- use same prior $N(\mu_\omega, \Sigma_\omega)$ as before to generate the designs
- use $N(\mu_\omega + \delta \mathbf{1}_p, \sigma\Sigma_\omega)$ to generate the data
- $\delta = 0.5$ and $\delta = 2$ ($\delta = 0$: correctly specified prior mean)
- $\sigma = 0.5$ and $\sigma = 2$ ($\sigma = 1$: correctly specified covariance matrix)

- results for the low accuracy + high heterogeneity scenario

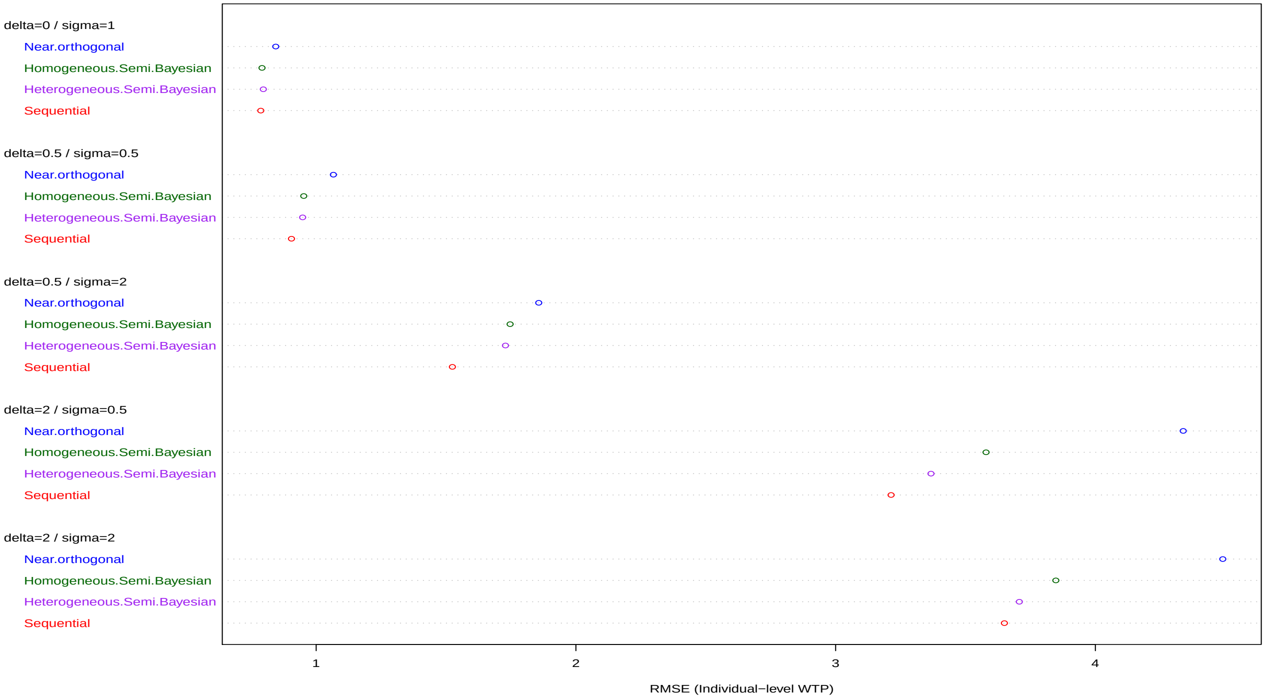
# SIMULATION RESULTS



# SIMULATION RESULTS



# SIMULATION RESULTS



# CONCLUSIONS



IASB works very well

- for estimating individual level parameters
- for estimating population level parameters
- is more robust to misspecification than other approaches

# REFERENCES

**Individually adapted sequential Bayesian conjoint-choice designs in the presence of consumer heterogeneity**
by J. YU, P. GOOS, M. VANDEBROEK

accepted for publication in the

**International Journal of Research in Marketing**

**Sequential choice designs to estimate the heterogeneity distribution of willingness-to-pay**
by V. DANTHUREBANDARA, J. YU, M. VANDEBROEK

accepted for publication in

**Quantitative Marketing and Economics**