Inverse Problems in Science and Engineering

Isaac Newton Institute for Mathematical Sciences

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Inversion of Pressure Transient Testing Data

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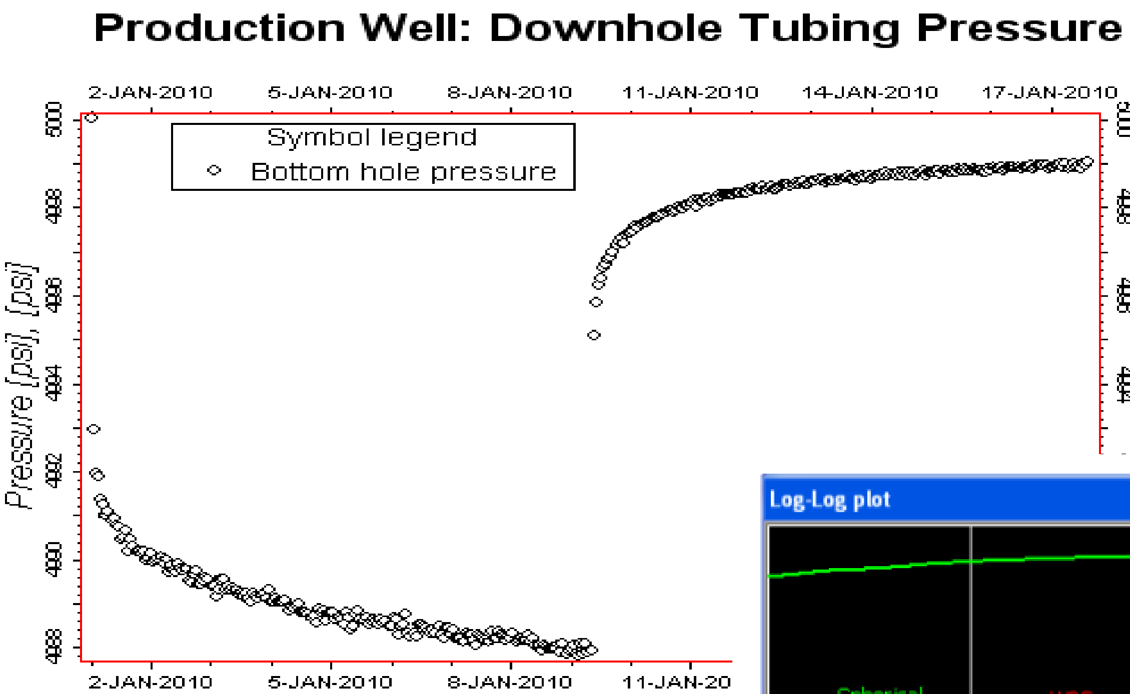
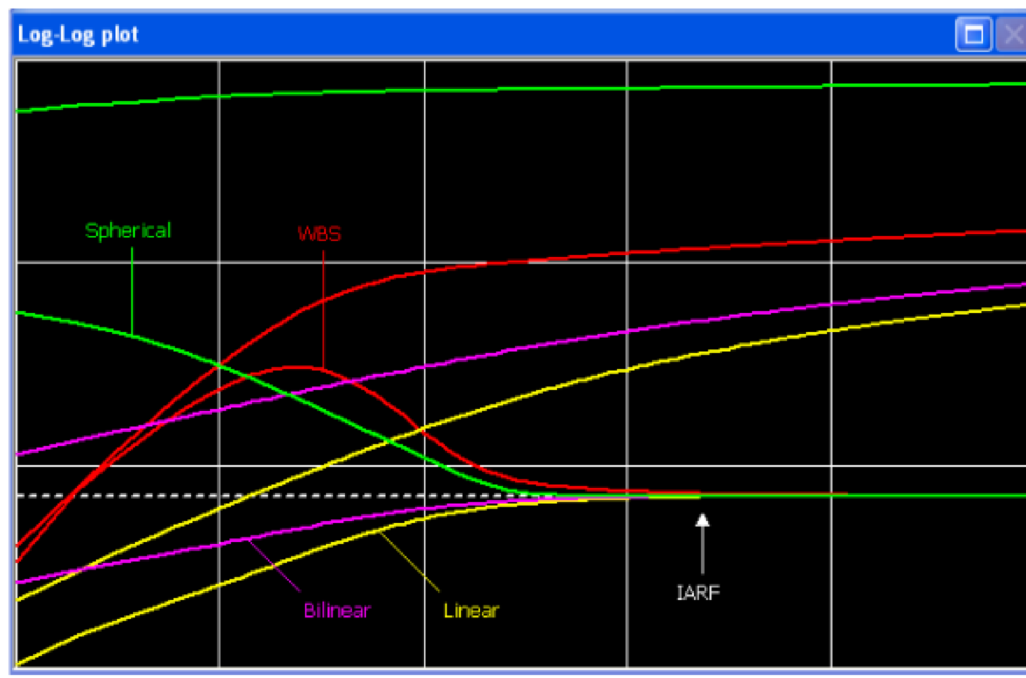
Outline

- What is a pressure-transient test?
  - What information can be obtained from the data.
- Simple prior model.
- Gradient-free methods.
- Gradient based methods.
  - Adjoint gradients necessary.
  - Separation of prior/likelihood.
  - Uncertainty?
- Gradient based sampling.
  - Langevin method.

Pressure Transient Testing (PTT)

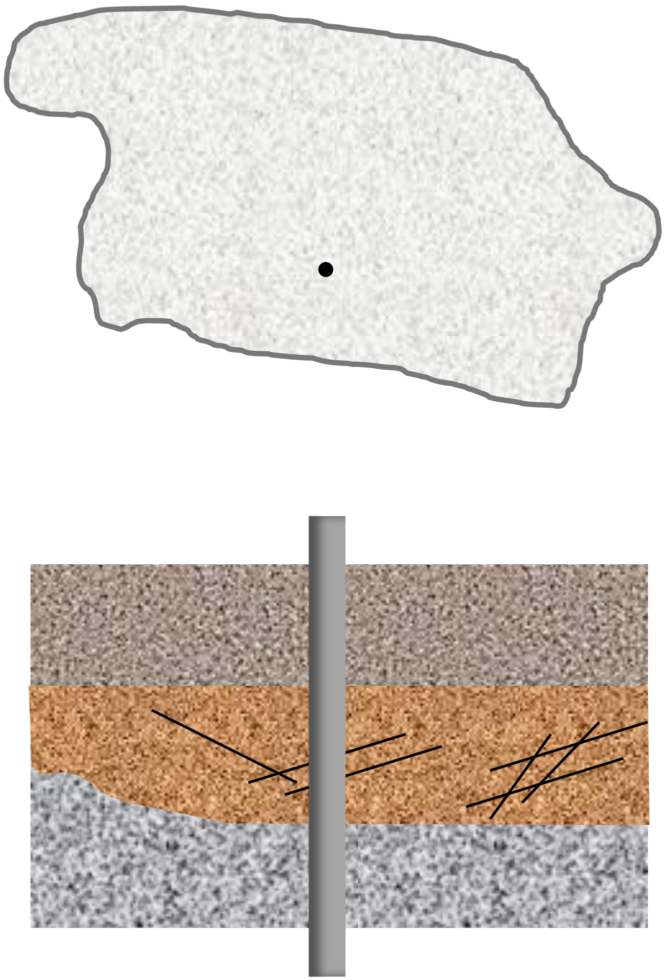


- Pressure response during draw-down and build-up
- Response is typically matched to analytical models

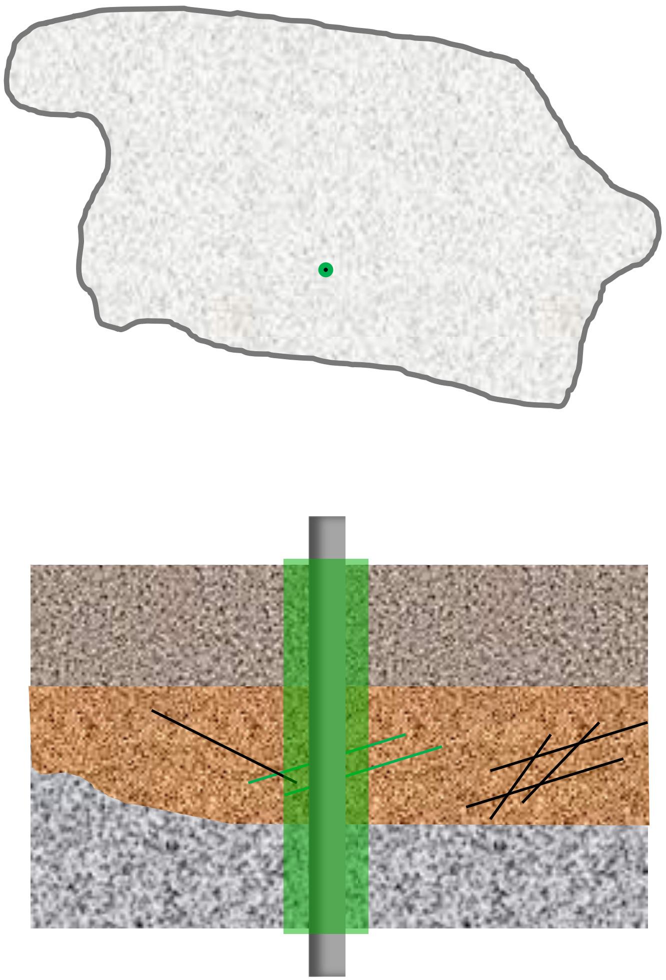


- In-situ, direct measurement of permeability
- Time scale is much shorter than for production history matching.

Resolution of inversion by data source



Cores/Logs



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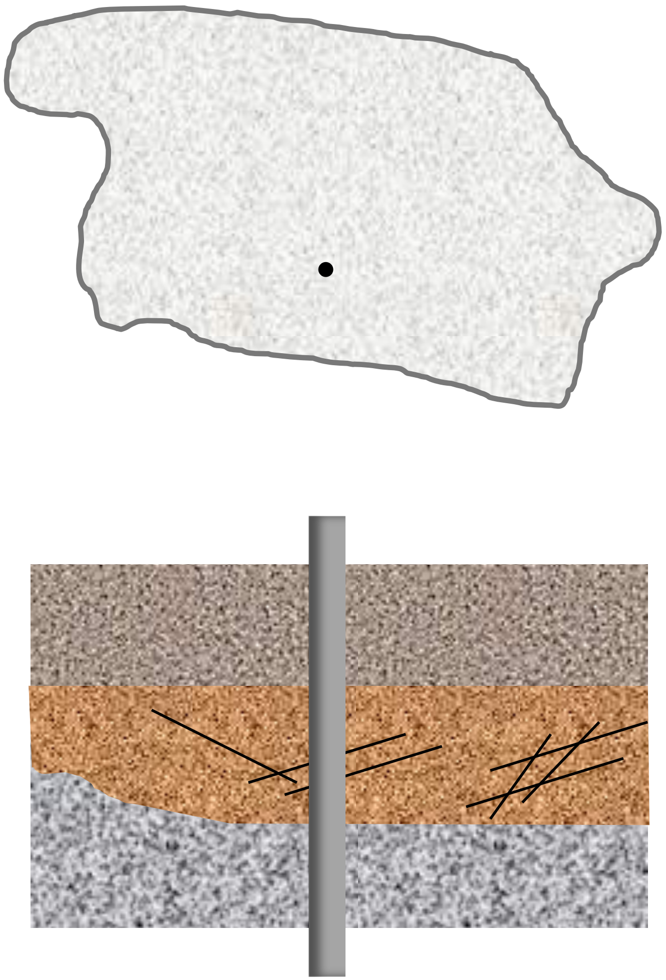
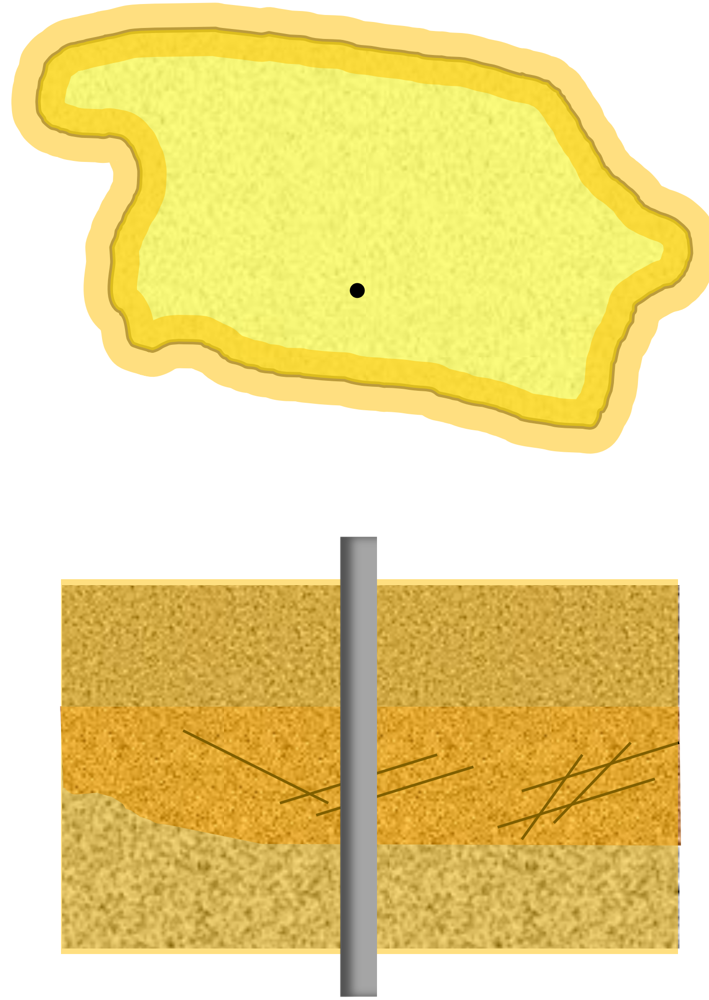


Seismic



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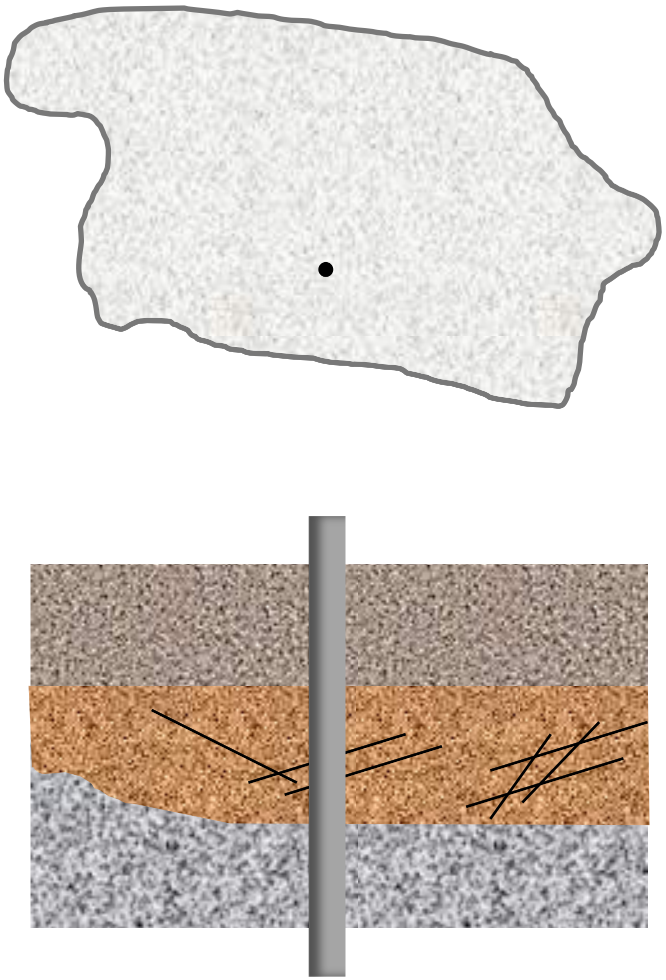
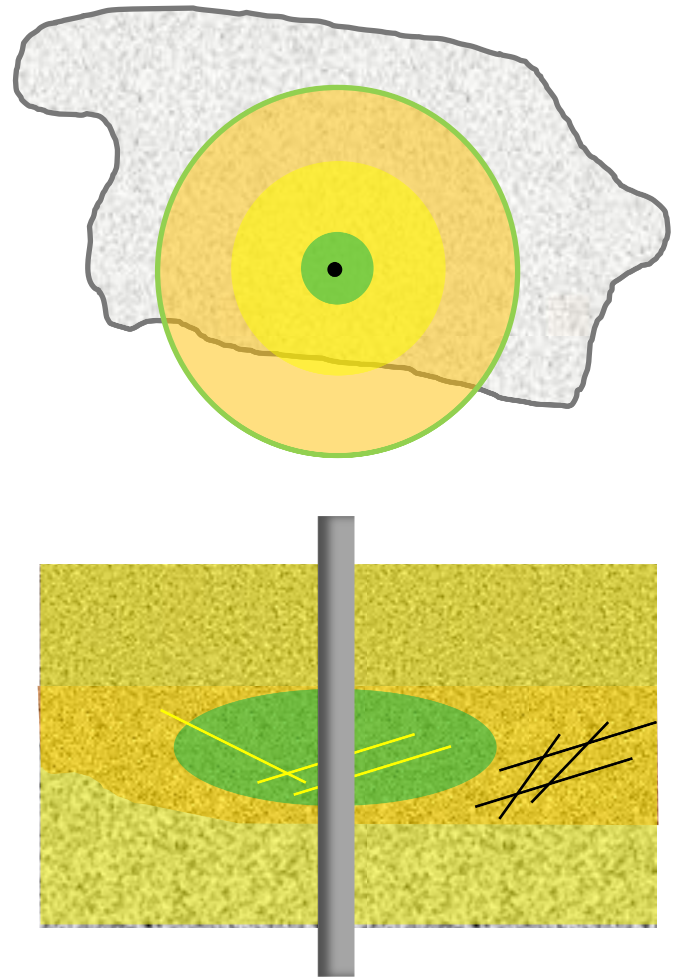
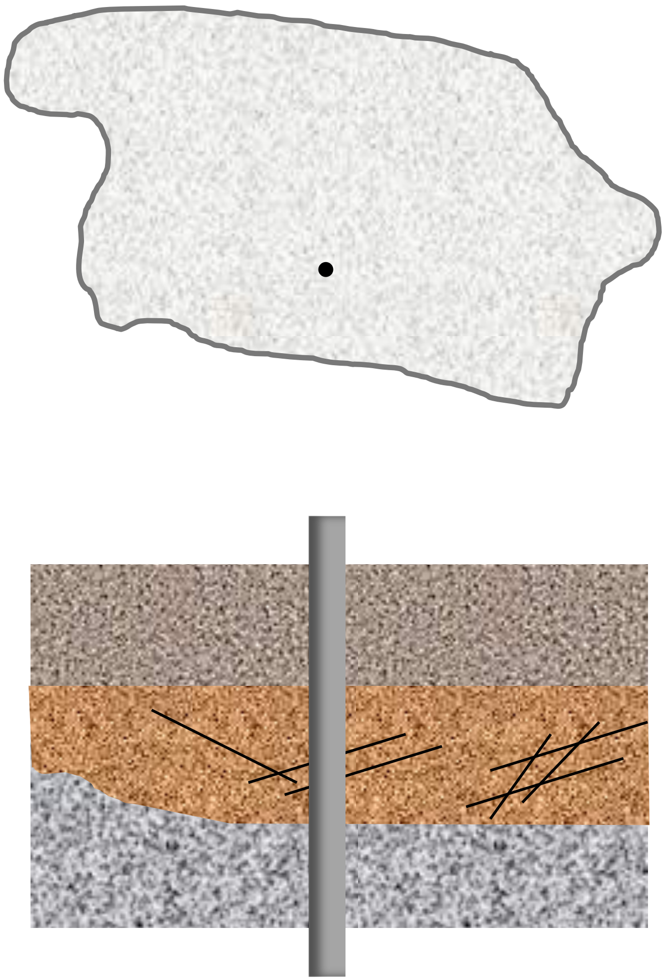


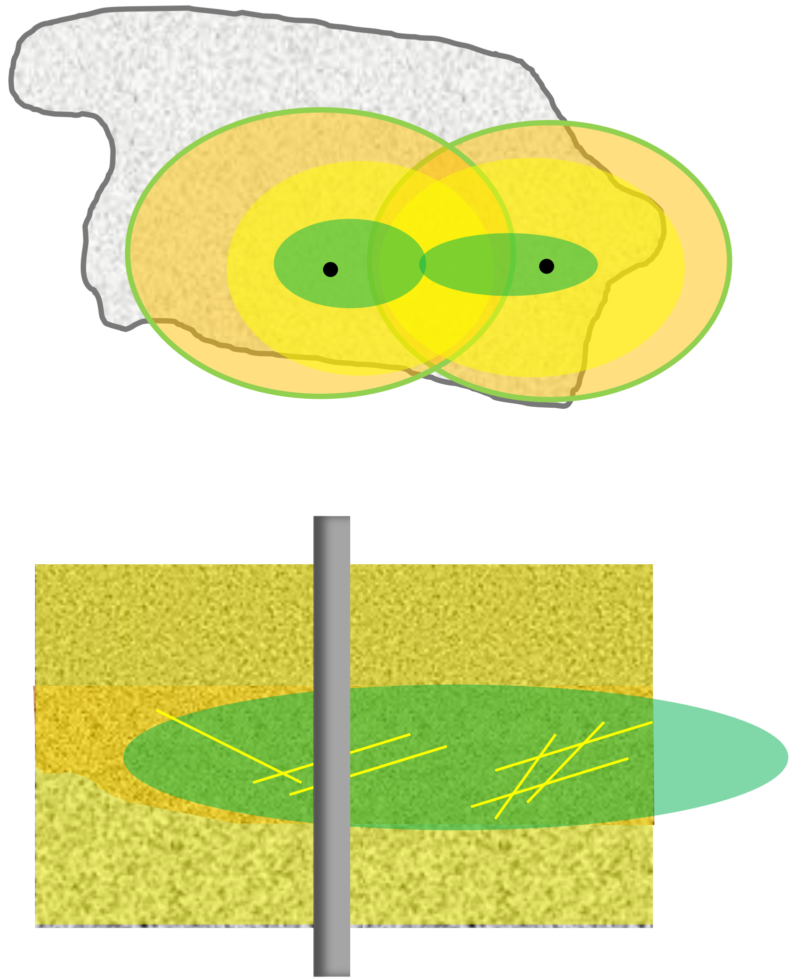
Single well PTT



Resolution of inversion by data source



Multiwell interference PTT



Confidence in property by data source

To determine lithology:

- Cores, cuttings, well-logs are most direct, therefore most reliable.
- Velocity is closely related to lithology, seismic is also useful.
- PTT measures permeability (and porosity), less closely related to lithology and so less useful.

To determine permeability:

- PTT is direct, in-situ measurement – most reliable.
- Measured from cores in labs, or from lithology – less reliable.
- Velocity is weakly related to permeability, chiefly via lithology – seismic is least reliable.

Confidence in property by data source

To determine lithology:

CORES

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SEISMIC

- Velocity is closely related to lithology, seismic is also useful.

PTT

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PTT

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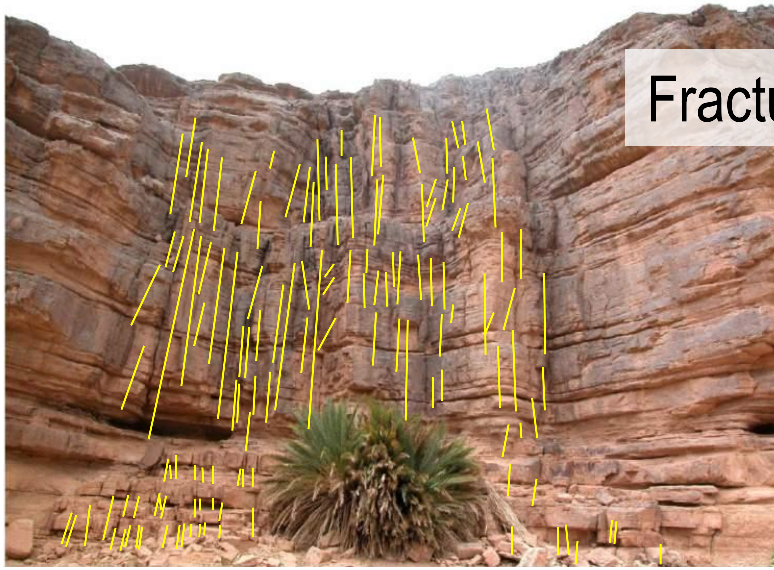
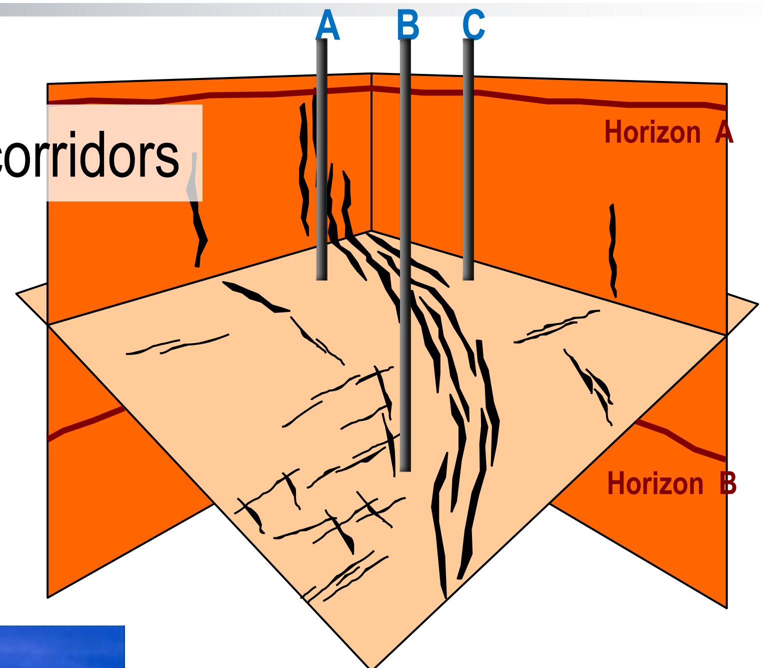
Reservoir Features



Fracture corridors



Pinch outs

Sub-seismic faults and lithology changes

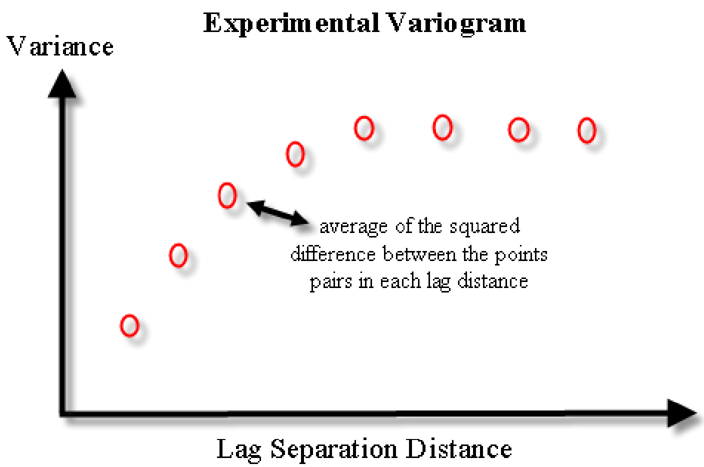
Bayesian model for PTT inversion

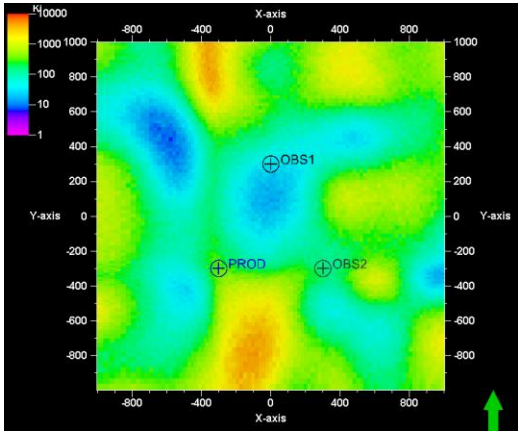
- Never enough data for exact inversion
  - Symmetries – radial or vertical.
  - Upscaling – small scale features can't be detected.

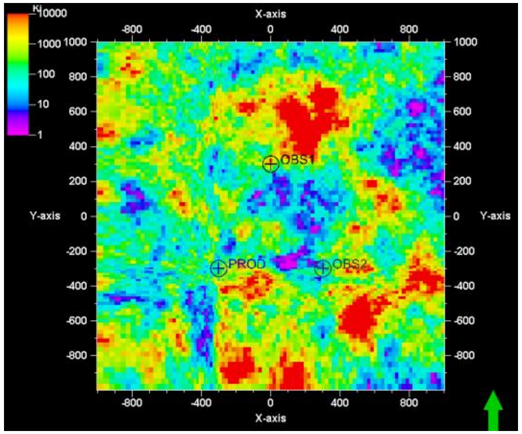
$$\underbrace{\pi[\mathbf{u}|\mathcal{M}]}_{\substack{\text{What we know}\\\text{after the test}\\\text{(Posterior)}}} \propto \underbrace{\pi_m[\mathcal{M}|\mathbf{u}]}_{\substack{\text{How likely}\\\text{is a set of}\\\text{parameters to give}\\\text{the data}\\\text{(Update)}}} * \underbrace{\pi_0[\mathbf{u}]}_{\substack{\text{What we}\\\text{'knew' before}\\\text{the test (Prior)}}}$$

Prior model

- In absence of specific geological model or training images, use simple pixel-based Gaussian model.
- Variogram :







Prior model - approximation

$$\pi(\mathbf{u}) \propto \exp\left(-\mathbf{u}^T \mathbf{A} \mathbf{u}\right)$$

$$\pi[u(\mathbf{x})] \propto \exp\left(- \int u(\mathbf{x}) \mathcal{L} u(\mathbf{x})\, \mathrm{d}\mathbf{x}\right)$$

$$\mathcal{L}u = a_0 u - a_1 \nabla^2 u + a_2 \nabla^4 u$$

$$\mathcal{L}u = \frac{4\pi\rho^2\sqrt{1-S^2}}{\sigma^2 \cos^- 1(S)} \left(\rho^4 \nabla^4 u - 2S\rho^2 \nabla^2 u + u\right)$$

- Tikhonov regularization
- Variogram approximated with few parameters
  - Can be rewritten in terms of physically relevant parameters – log-permeability variance, correlation length(s)
- Rational approximation in Fourier space
- Replacing $\mathbf{A}$ with a **sparse** approximant.

Approaches to inversion

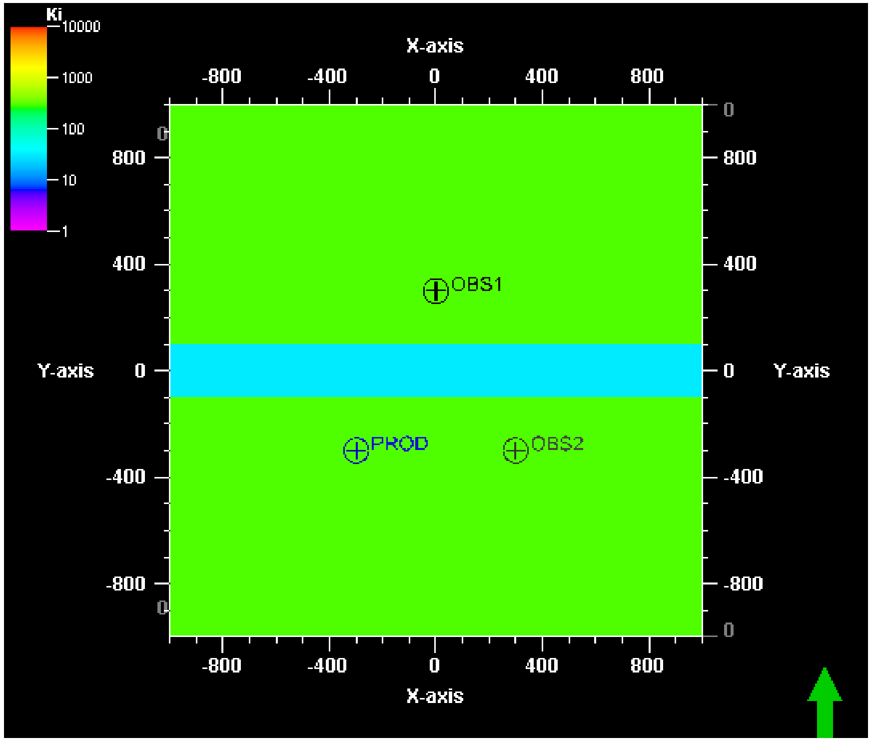
Derivative-free methods

- Naïve MCMC
  - Curse of dimensionality.
- Filtering methods
  - Data assimilation – efficient simulation.
  - How bad is Gaussian approximation (for EnKF)?
  - Unlike other applications, here geological uncertainty is **not** time-dependent.

Derivative-based methods

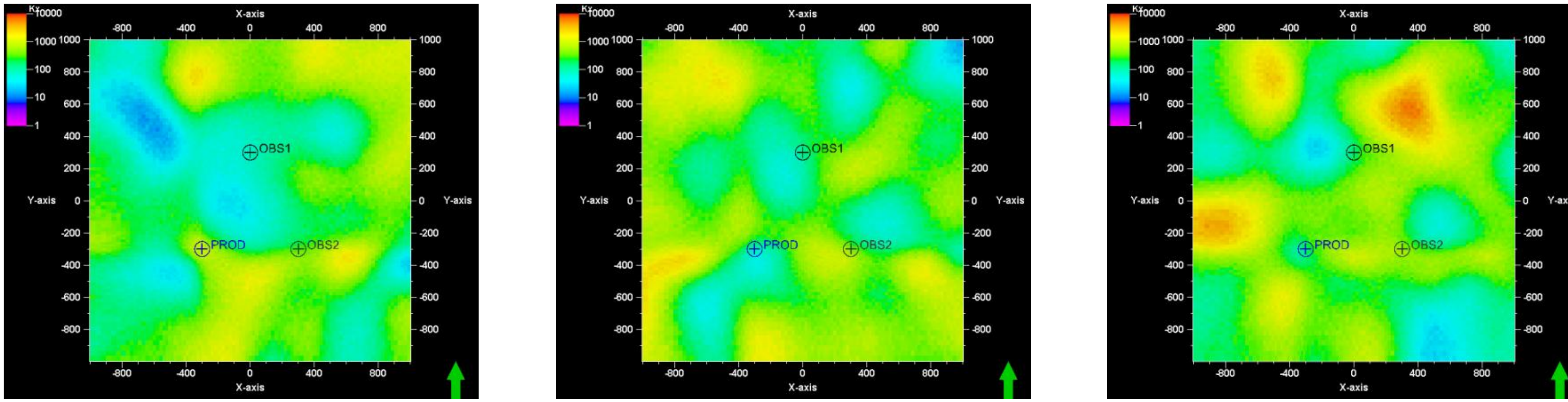
- Can find (locally) most likely parameters for any posterior distribution.
- How do we find the derivatives?
- What about uncertainty?
- Can we produce samples?

Application of EnKF

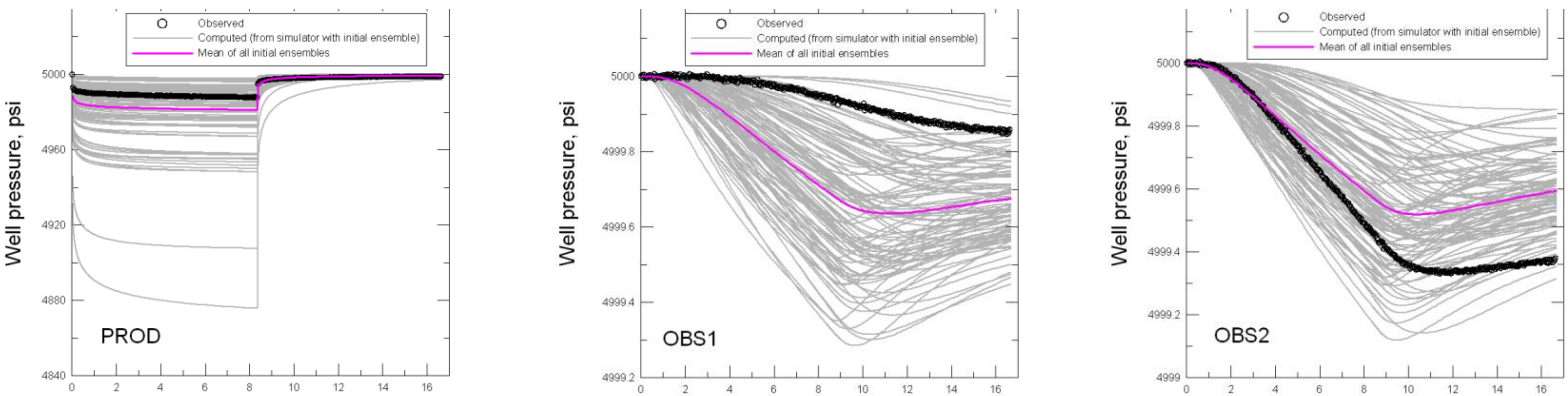


- Truth model
- One producer, two observation wells.
- Band of low permeability separates one of the observation wells from the producer.
- Draw-down (producing) followed by build-up (not-producing).

Application of EnKF

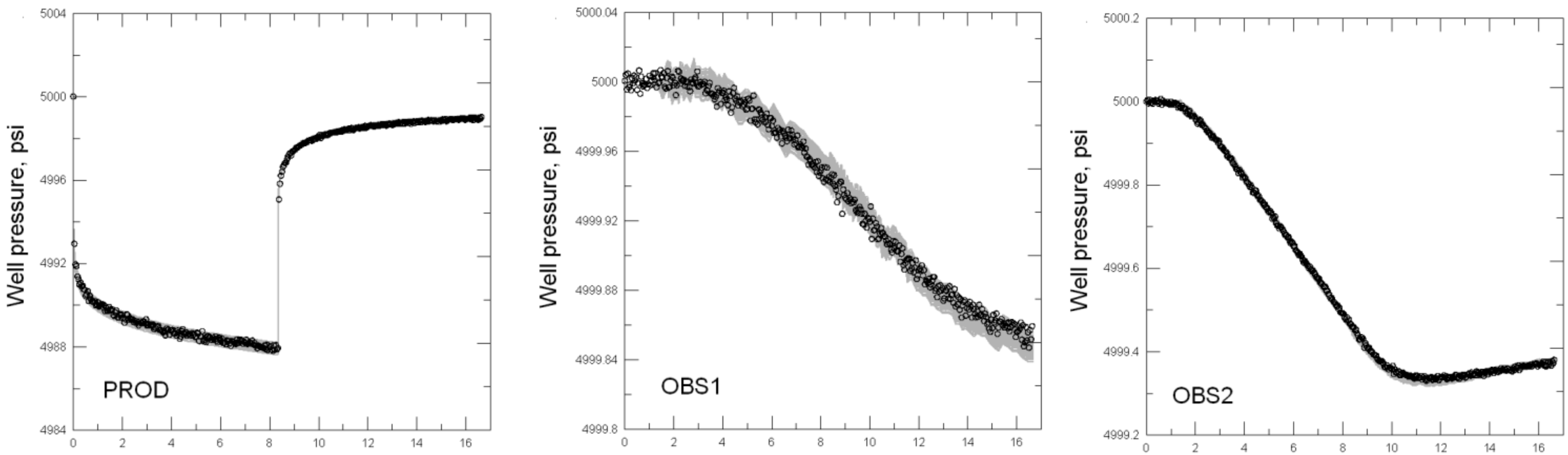


- 3 of 100 ensemble members: Gaussian variogram, range (l) = 500ft.



- Simulated well pressure response for prior ensemble.

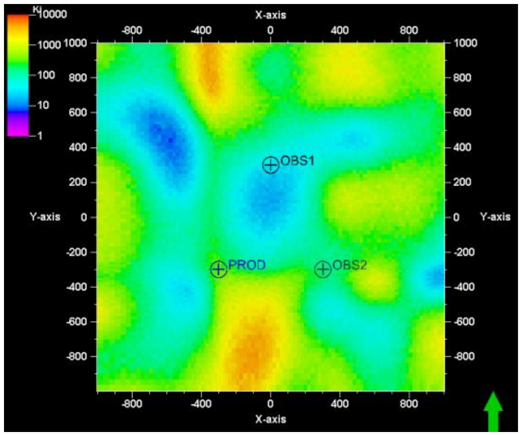
EnKF

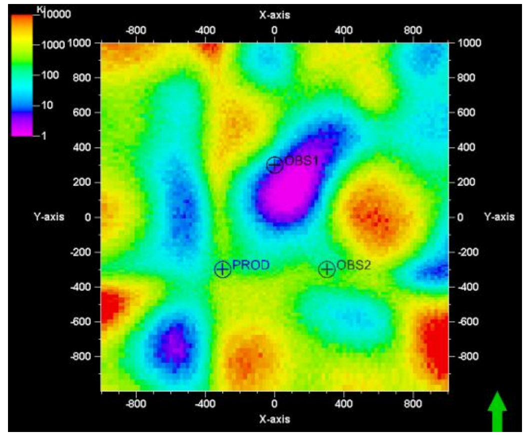


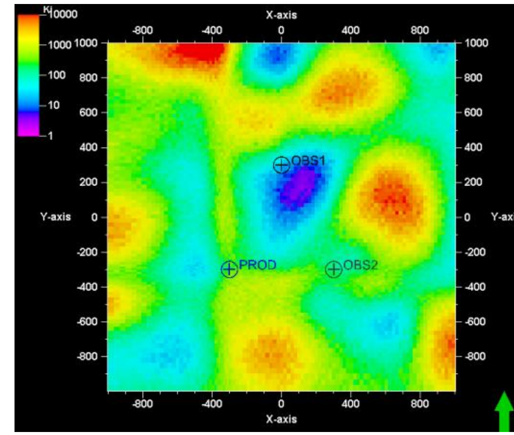
- Exact history match during assimilation.

EnKF

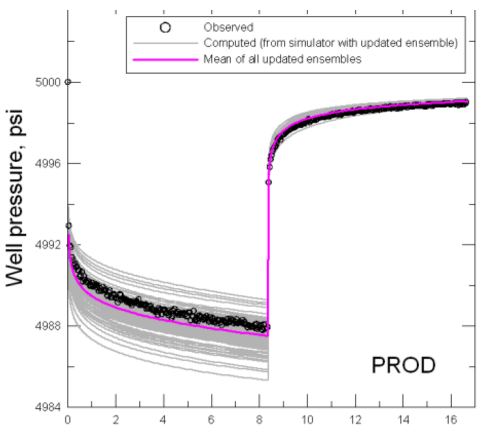
Samples from posterior distribution.

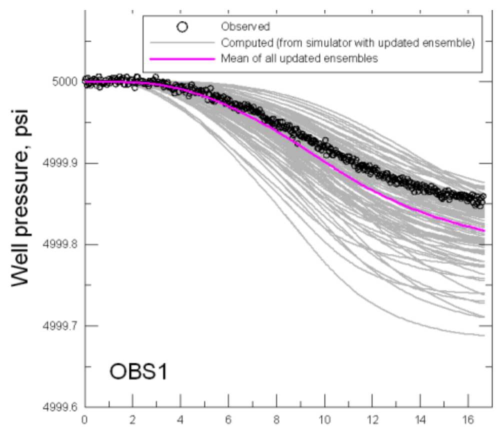


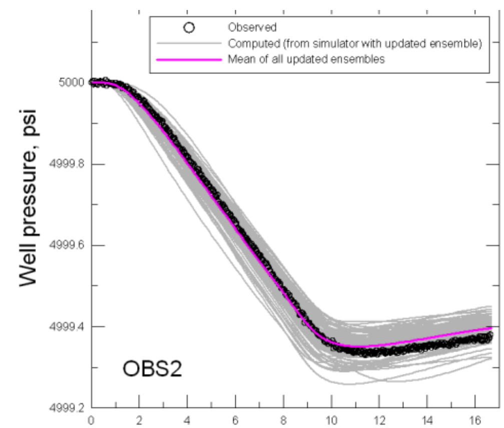




- 3 of 100 updated ensemble members: Gaussian variogram, l = 500ft.



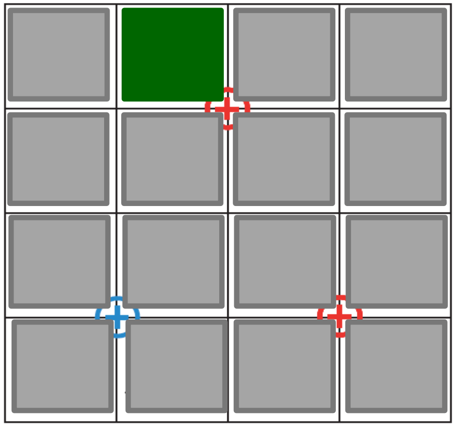


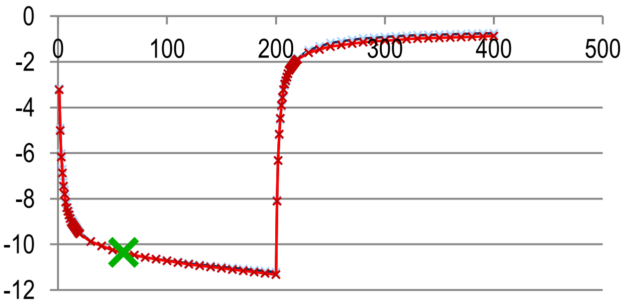


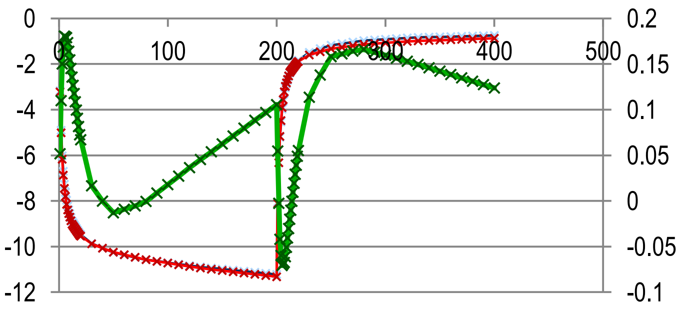
- Simulated well pressure response for updated ensemble & mean.

Calculating derivatives

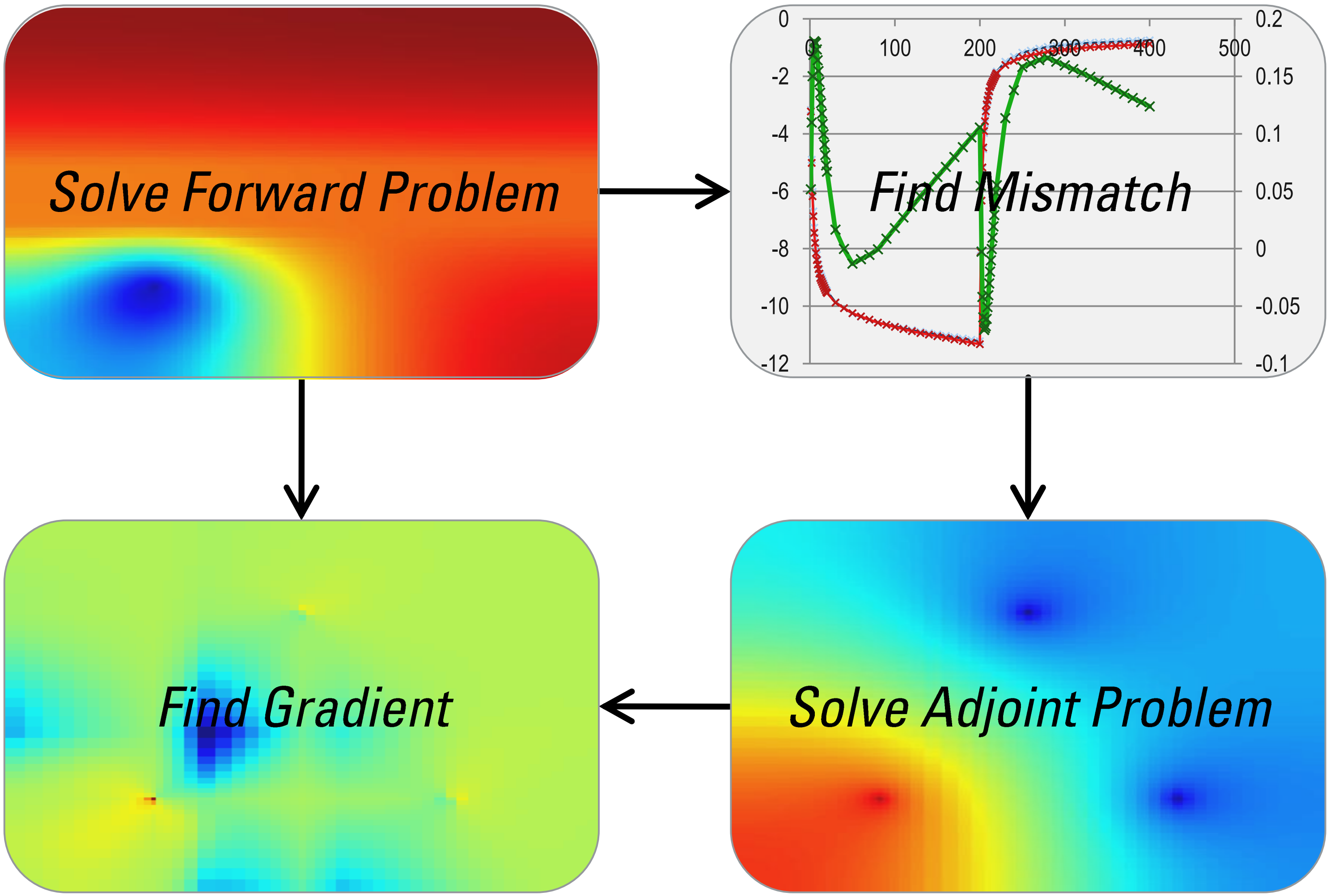
- Direct, finite difference
  - Expensive, extra simulation for each parameter.
  - Inaccurate
- Adjoint method for Jacobian
  - Extra simulation for each measurement.
- Adjoint method for gradient
  - One extra simulation, adjoint wrt all measurements at once.
  - Only obtain the gradient – not the Jacobian.







Adjoint method for gradients



Separation of prior and likelihood

$$H[\mathbf{u}] \;=\; I[\mathbf{u}] \;+\; R[\mathbf{u}]$$

log-posterior log-likelihood log-prior

- Steepest descent:

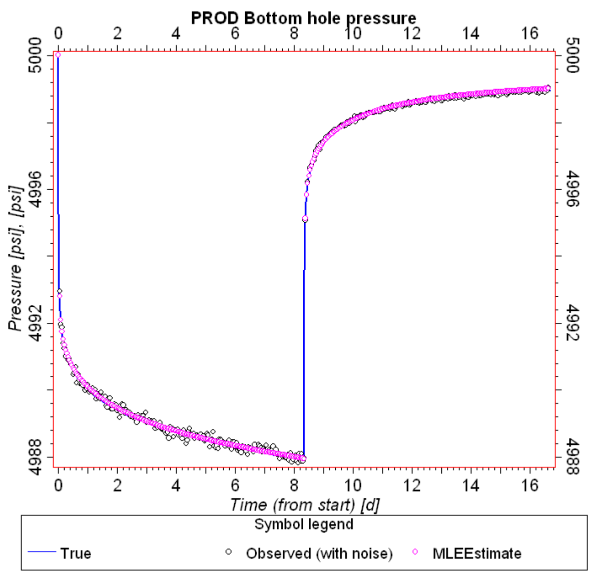
$$\mathbf{u}_{n+1} = \mathbf{u}_n - \alpha \nabla_u H[\mathbf{u}_n] \equiv \frac{\partial \mathbf{u}}{\partial \tau} = -\nabla_u H$$

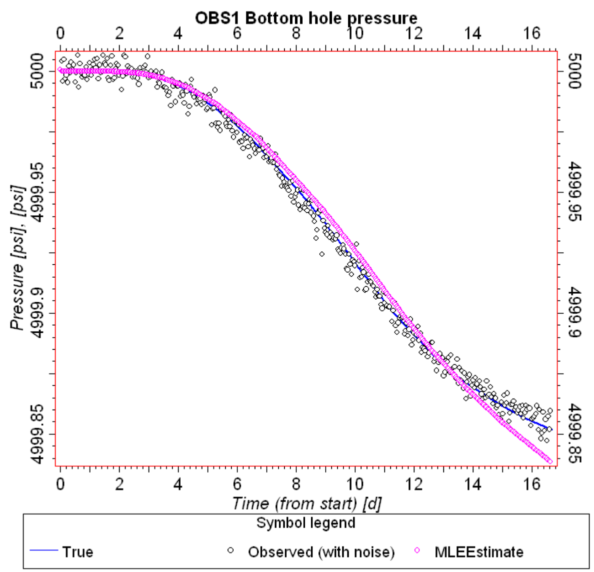
- **Explicit** Euler scheme – conditional stability analogous to limit on α
- Prior is typically stiff – strong correlation between adjacent cells
- Semi-implicit scheme (separating prior and likelihood):

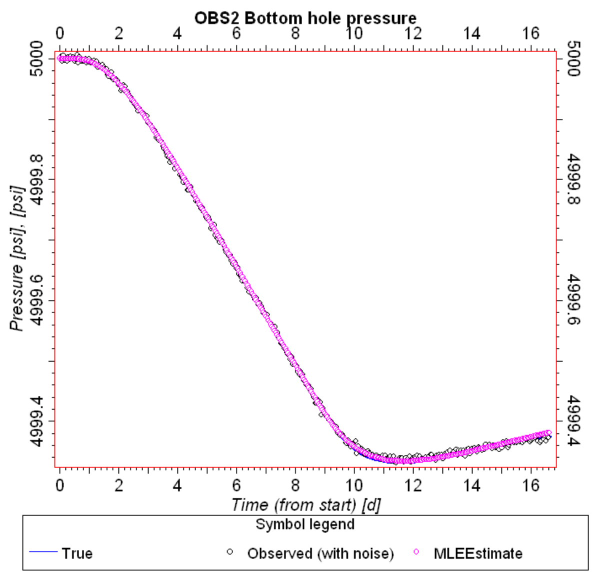
$$\mathbf{u}_{n+1} = \mathbf{u}_n - \Delta\tau \left(\nabla_u I[\mathbf{u}_n] + \theta \nabla_u R[\mathbf{u}_{n+1}] + (1-\theta)\nabla_u R[\mathbf{u}_n]\right)$$

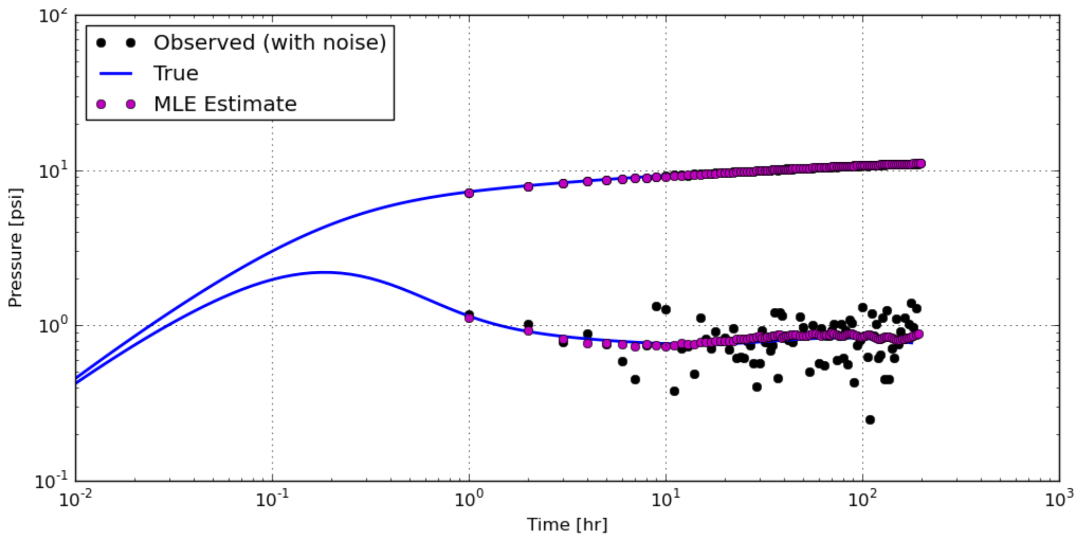
- Sparse representation of prior is useful

Steepest descent/ Adjoint gradient History match



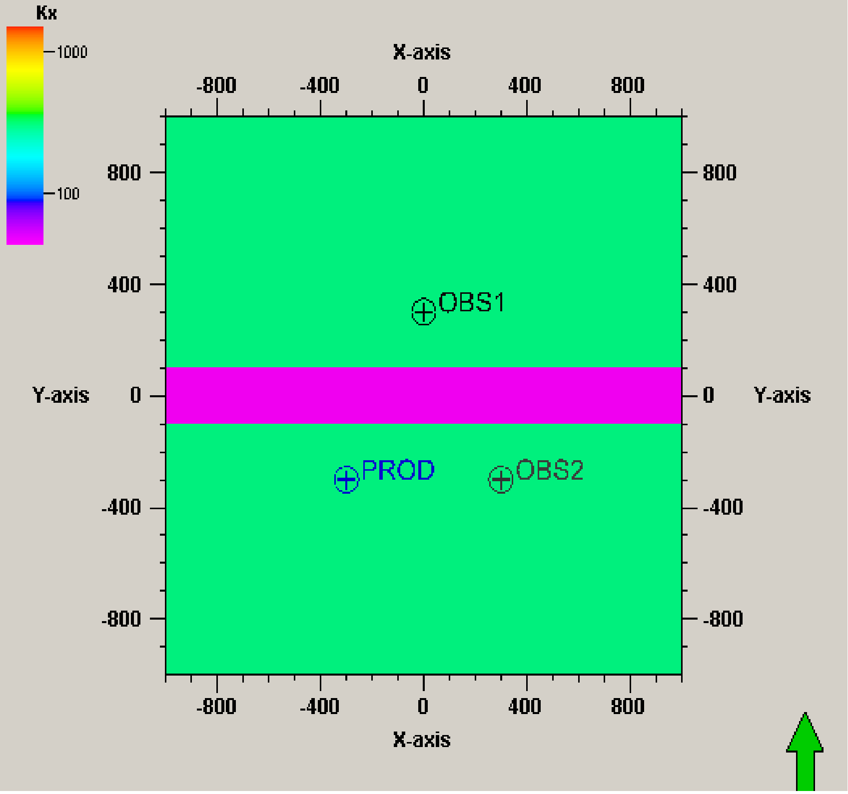




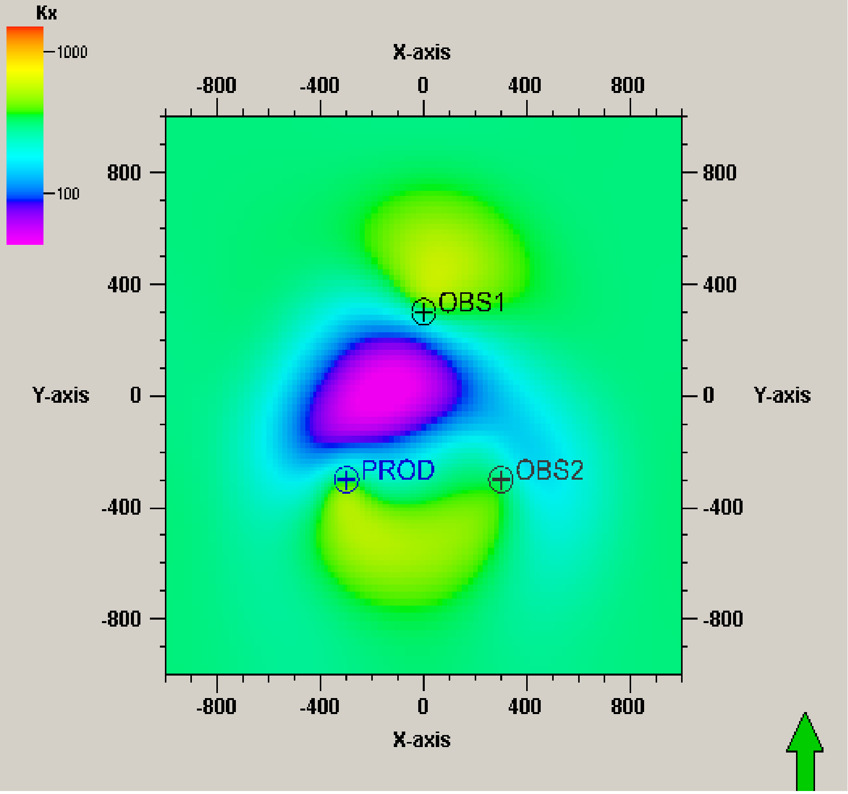


MAP estimate not generally a good sample

True permeability field



Output permeability field



Uncertainty - Langevin sampling method

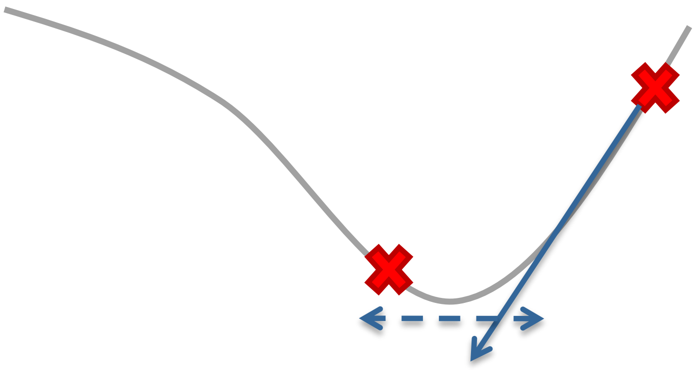
- Steepest descent

$$\frac{\partial u}{\partial \tau} = -\frac{\partial H}{\partial u}$$

- Stochastic differential equation

$$\frac{\partial u}{\partial \tau} = -\frac{\partial H}{\partial u} + \sqrt{2}\eta(\mathrm{x}, \tau)$$

- Steepest descent with noise – magnitude related to step size.



Langevin sampling method

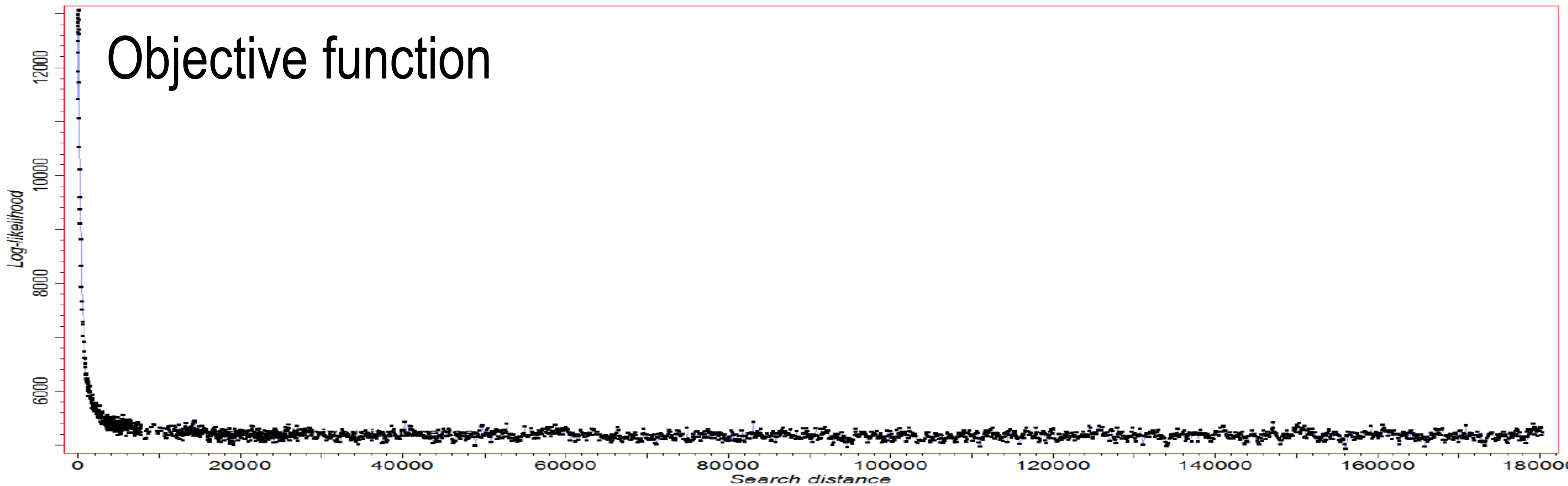
- Fokker-Planck – assured convergence in distribution, but not necessarily fast.
- Need stochastic analogue of 'line search'
  - Construct MCMC using Metropolis-Hastings
  - Langevin gives sample distribution.

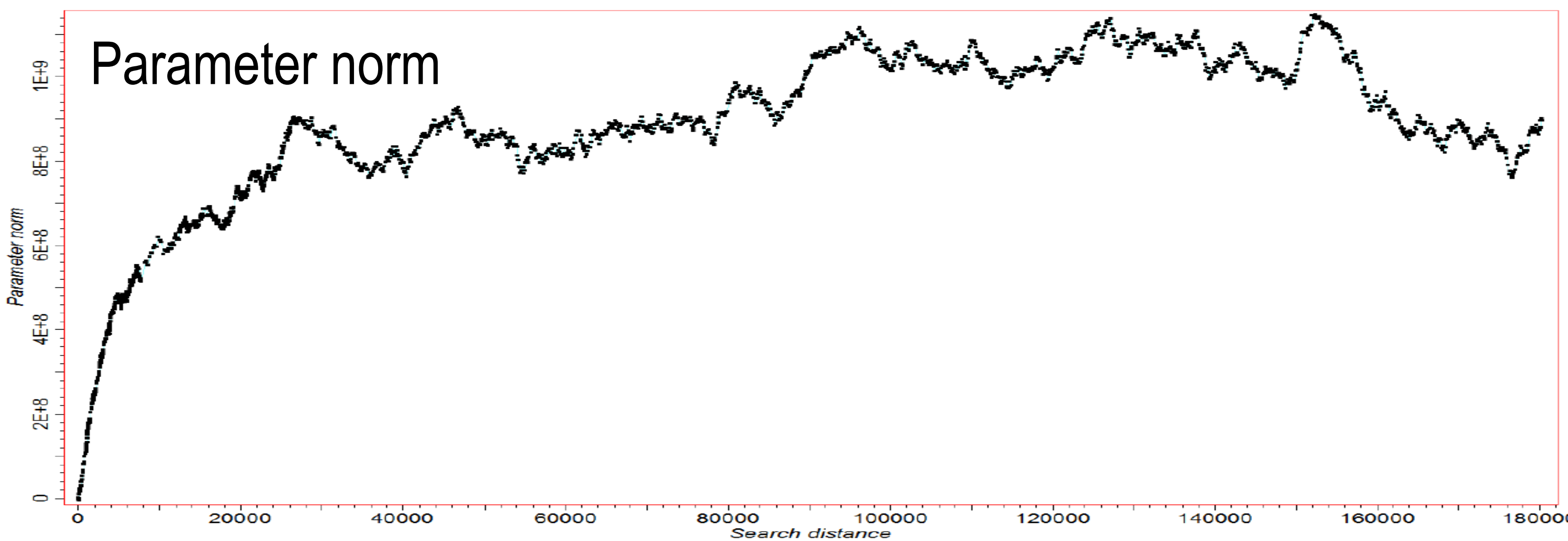
$$p_{\text{accept}} = \frac{\pi[u'_n|\mathcal{M}]Q[u_n;u'_n]}{\pi[u_n|\mathcal{M}]Q[u'_n;u_n]}$$

- Semi-implicit scheme again important for acceptance of large steps.
  - For sampling, accuracy is important too.
  - Need trapezoidal scheme (θ=½)

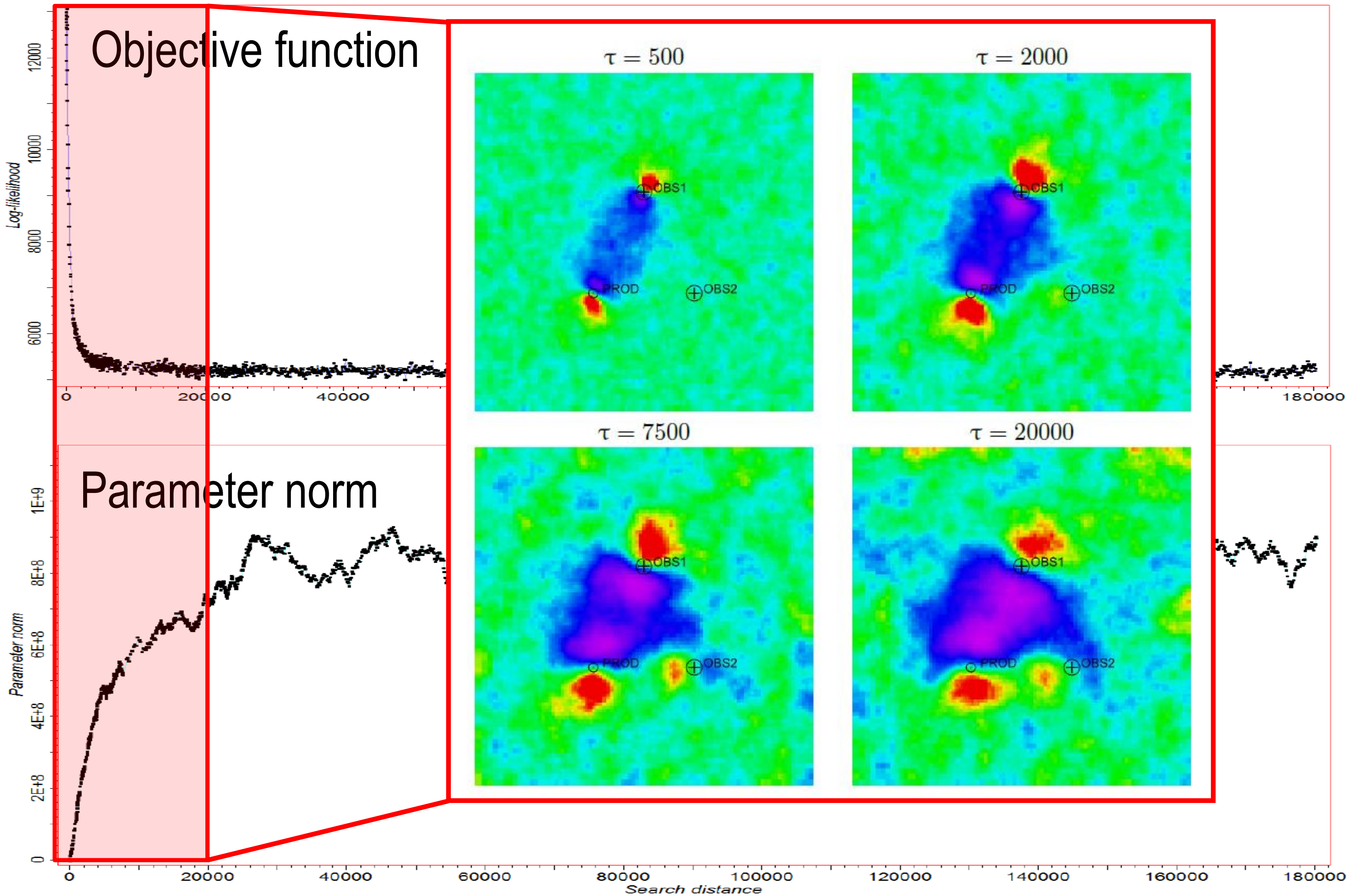
$$\mathbf{u}_{n+1} = \mathbf{u}_n - \Delta\tau\left(\nabla_u I[\mathbf{u}_n] + \tfrac{1}{2}\nabla_u R[\mathbf{u}_{n+1}] + \tfrac{1}{2}\nabla_u R[\mathbf{u}_n]\right) + \sqrt{2\Delta\tau}\,\eta(\mathbf{x})$$

Langevin sampling





Langevin sampling



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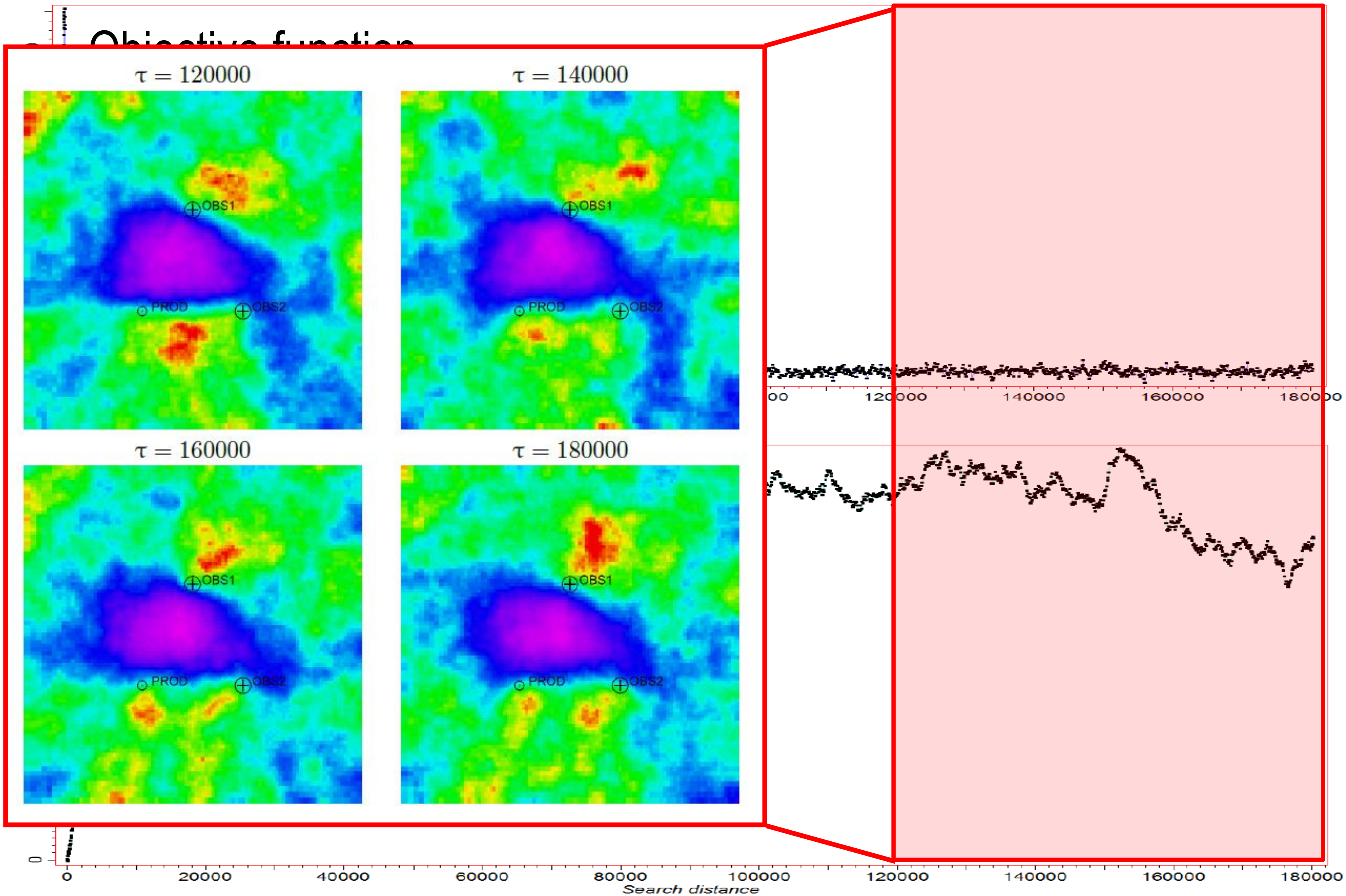

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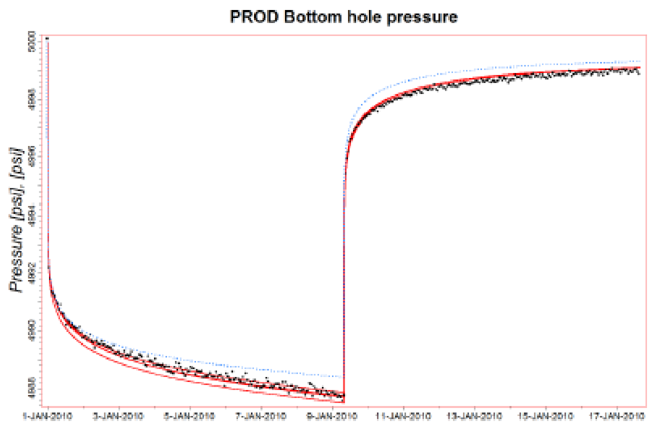

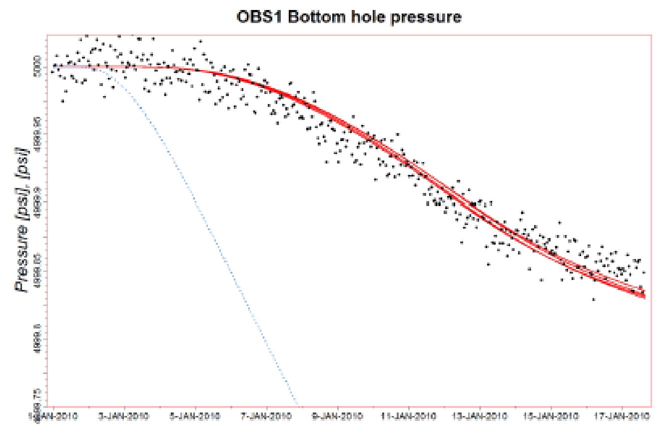

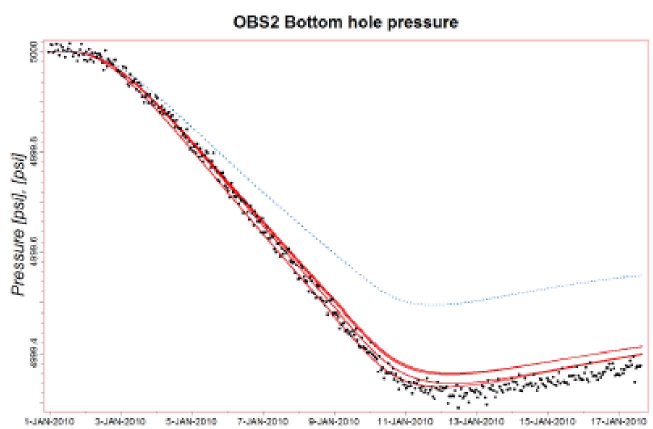

Simulated well responses from samples after convergence.

Conclusions

- PTT can be used to identify reservoir features, but significant uncertainty will remain.
- EnKF can give a poor match to PTT data.
- Gradient based methods
  - Can be made numerically inexpensive,
  - Provide good match with data.
  - Quantifying uncertainty still a challenge.
- Langevin method allows sampling but mixing rate is slow
  - Use second derivative information from quasi-Newton to improve implicit treatment of nonlinear term.
  - Optimized step-length selection.
  - Parallel sampling of ensemble.
  - Parameter reduction?