Holographic Description of Dynamical Gauge Fields in Superconductors

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Condensed Matter, Black Holes and Holography

Based on

*O. Domènech, M. Montull, A. Pomarol, A. S. and P. J. Silva, JHEP **1008**, 033 (2010), arXiv:1005.1776*

M. Montull, O. Pujolas, A. S. and P. J. Silva, JHEP, arXiv:1202.0006

The holographic model **(Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)**

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + ... + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2\,, \qquad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

The use of AdS allows us to avoid no hair theorems **(Gubser, 2008)**

$\mathcal{O} \leftrightarrow \Psi$

$\Psi|_{z=0} = s =$ source of $\mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$

$A_\mu|_{z=0} = a_\mu =$ source of $\hat{J}_\mu$

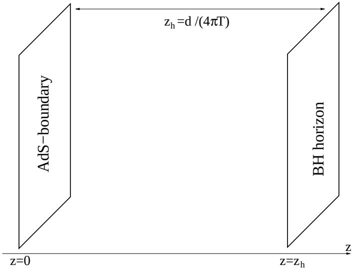
$$S_{matter} = \frac{1}{g^2}\int d^{d+1}x\,\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - \frac{1}{L^2}|D_M\Psi|^2\right)$$

$$J_\mu \equiv \langle \hat{J}_\mu \rangle = \frac{L^{d-3}}{g^2} z^{3-d}\mathcal{F}_{z\mu}|_{z=0}\,, \qquad \langle \mathcal{O} \rangle = \frac{L^{d-3}}{g^2} z^{1-d} D_z\Psi|_{z=0}$$

The normal phase, $\Psi = 0$

The system describes a conductor

(with non-zero conductivity)



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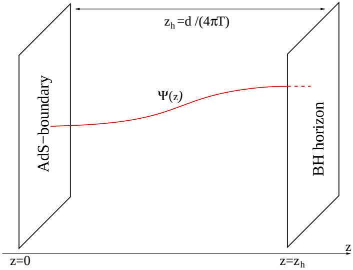
Superconducting phase $\Psi \neq 0$

no x^μ-dependence (homogeneous solutions) and $A_i = 0\,(i = 1, ..., d-1)$

$\mu \equiv A_t|_{z=0}$

$T < T_c = 0.03(0.05)\mu$ for $d = 3(4)$



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$A_\mu|_{z=0} = a_\mu$

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Non-homogeneous solutions with $A_i \neq 0$ have also been found.

(Albash, Johnson, 2008; Nakano, Wen, 2008; Maeda, Okamura, 2008; Hartnoll, Herzog, Horowitz, 2008; Montull, Pomarol, Silva, 2009; Keranen, Keski-Vakkuri, Nowling, Yogendran, 2009; Ge, Wang, Wu, Yang, 2010)

However, that (Dirichlet) boundary condition corresponds to a superfluid.

$\rightarrow$ non-dynamical a_i!

Dynamical a_μ in holography

- impose a **dynamical equation for a_μ**

$$J^\mu + \frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu} + J^\mu_{ext} = 0$$

Here, for generality, we have added a kinetic term for a_μ and a background external current J^μ_{ext}.

- Then we must add to S the following term

$$\int d^d x \left[-\frac{1}{4g_b^2}\mathcal{F}^2_{\mu\nu} + A_\mu J^\mu_{ext} \right]_{z=0}$$

- by using $J_\mu = \frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_{z\mu}|_{z=0}$

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^\mu \Big|_{z=0} + \frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^\mu_{ext} = 0$$

This is an AdS-boundary condition of the Neumann type.

▸ for $d = 2+1$ the gauge field is emergent, for $d \geq 3+1$ it is elementary

Insulator/superconductor model (Nishioka, Ryu, Takanagi, 2009; Horowitz, Way, 2010)

We compactify a spatial dimension: $\chi \sim \chi + 2\pi R$

We have two static metrics with symmetry IO$(d-1) \times U(1)$ or Poincaré$(d-2,1) \times U(1)$

Black Hole (deconfined) phase: a conductor

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + d\chi^2 + dy_{d-2}^2 + \frac{dz^2}{f(z)}\right]$$

$f(z) = 1-(z/z_h)^d$, $z_h = d/4\pi T$, **Favorable for $R > 1/2\pi T$** (at $\mu = 0$)

"Soliton" (confined) phase (Witten, 1998; Horowitz, Myers, 1998): an insulator

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2 + f(z)d\chi^2 + dy_{d-2}^2 + \frac{dz^2}{f(z)}\right]$$

$f(z) = 1-(z/z_0)^d$, $z_0 = dR/2$, **Favorable for $R < 1/2\pi T$** (at $\mu = 0$)

- the transition between them occurs at μ and/or T around $1/R$ (known as a Hawking-Page transition (1983))
- both phases exhibit SC bahavior: below $T \sim 1/R$ and increasing μ, one finds first a Soliton SC state and then (for $\mu \gtrsim 1/R$) a Black Hole SC

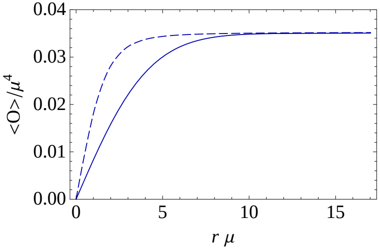
For the soliton (with no metric backreaction) $R_c \simeq \dfrac{1.81\ (1.70)}{\mu}$, for $d = 2{+}1\ (3{+}1)$

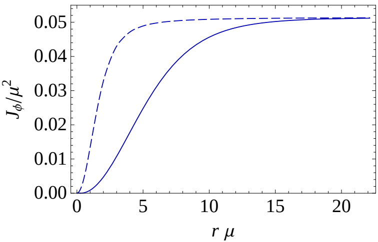
Vortices in the holographic insulator/superconductor transition

Vortex ansatz (for $d \geq 4$): $\quad \Psi = \psi(z,r)e^{in\phi}\ , \quad A_t = A_t(z,r)\ , \quad A_\phi = A_\phi(z,r)\ ,$

AdS-boundary conditions: $s = 0\ , \quad \mu = constant\ ,$

$a_\mu = A_\mu|_{z=0} = \frac{1}{2}Br^2$ **(Dirichlet boundary condition)**





Plots: $n = 1$, $R/R_c = 5$ and $B = 0$

- Solid lines: holographic model for $d = 3 + 1$.
- Dashed lines: GL model (with its parameters determined by fixing two observables predicted by the holographic model).

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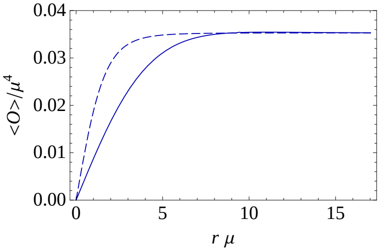
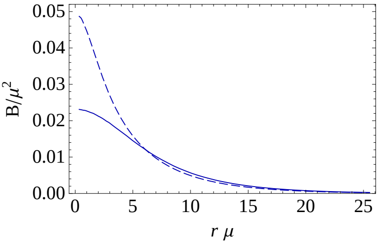
AdS-boundary conditions: $s = 0\,, \quad \mu = constant\,,$

$$\frac{L^{d-3}}{g^2} z^{3-d}\partial_z A_\phi\Big|_{z=0} + \frac{1}{g_b^2} r\partial_r\left(\frac{1}{r}\partial_r A_\phi\right)\Big|_{z=0} = 0\,, \quad (\text{for } J^\mu_{ext} = 0)$$

▸ The gauge field is emergent for $R \to 0$





Plots: The modulus of $\langle \mathcal{O} \rangle$ (up to L^{d-3}/g^2) and B versus r for $n = 1$.

- Solid lines: holographic model for $d = 3 + 1$, $R/R_c = 5$ and g_b chosen to satisfy $g_{GL}^{-1}(R = R_c) \simeq 1.7L/g^2$.
- Dashed lines: GL model (with its parameters determined by fixing three observables predicted by the holographic model).

Vortex ansatz (for $d \geq 4$): $\Psi = \psi(z,r)e^{in\phi}\,, \quad A_t = A_t(z,r)\,, \quad A_\phi = A_\phi(z,r)\,,$

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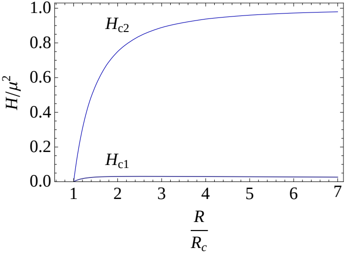

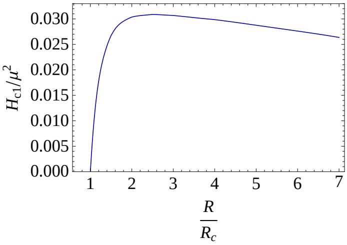

Plots: H_{c1} and H_{c2} versus R from holography for $d = 3+1$ and g_b chosen to satisfy $g_{GL}^{-1}(R = R_c) \simeq 1.7L/g^2$.

$H_{c1} < H_{c2}$ for every R, **so also in this phase the holographic superconductor is of Type II, like the known high-temperature superconductors**.

Important points

- We discussed a method to introduce a *dynamical gauge field* in holography and its application to superconductors

  In $d = 2+1$ it is emergent, in $d \geq 3+1$ it is elementary

- *The method is general* and can be used in several situations: e.g. we considered the (deconfined) AdS Black Hole and the (confined) AdS soliton

- We applied it to *vortices* showing the exponential damping of B far from the vortex core and the Meissner effect, as well as other properties which crucially rely on the dynamics of the gauge field

- We showed that the holographic superconductor in both phases is of *Type II*

more useful information can be found in arXiv:1005.1776 *and* arXiv:1202.0006

Extra slide 1: Dynamical a_μ, emergent or elementary?

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^{\mu}\Big|_{z=0} + \frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^\mu_{ext} = 0$$

$d = 3 + 1$ case

J_μ is logarithmically divergent:

$$\frac{1}{z}\mathcal{F}_{z\mu}\Big|_{z=0} = -\partial^\nu \mathcal{F}_{\nu\mu} \ln z\Big|_{z=0} + \dots$$

We can absorb the divergence in $\frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0}$ to define a renormalized electric charge g_0 in the normal phase ($\Psi = 0$):

$$\frac{1}{g_0^2} = \frac{1}{g_b^2} - \frac{L}{g^2}\ln z|_{z=0} + \textit{finite terms}$$

a_μ breaks conformal invariance
(the same is true for any $d > 4$).

$d = 2 + 1$ case

no divergence $\Rightarrow$
we can take $g_b \to \infty$
so $\frac{1}{g_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} \to 0$

In this case a_μ *does not* break conformal invariance and can be considered as an emerging phenomenon:
its kinetic term is induced by the dynamics.
(see also Witten, 2003)

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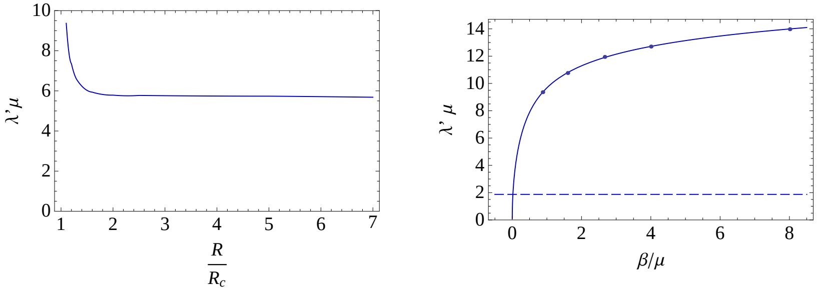
Extra slide 2: Emergent gauge field in the soliton for $R \to 0$

$\frac{1}{g_{2+1}^2} \propto R \ln(\beta z_0)$ can be finite for $R \to 0$ even if we remove the regulator, $\beta z_0 \to \infty$.
$\to$ in the limit we obtain an emergent gauge field



- **Plot on the left:** λ' as a function of R from holography for $d = 3+1$ and g_b chosen as before.
- **Plot on the right:** λ' as a function of β for $R/R_c = 1.1$. The dots are obtained from $a_\phi \overset{\text{large } r}{\simeq} n + a_1 \sqrt{r} e^{-r/\lambda'}$ while the solid line from $\lambda' = 1/\left(\sqrt{2} g_{GL} |\Phi_{GL}(r \to \infty)|\right)$ by computing separately g_{GL} and $\Psi_{GL}(r \to \infty)$
$\to$ the logarithmic running of g_{2+1} is reflected in a logarithmic running of λ'.

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