Gravitational Anomaly and Transport

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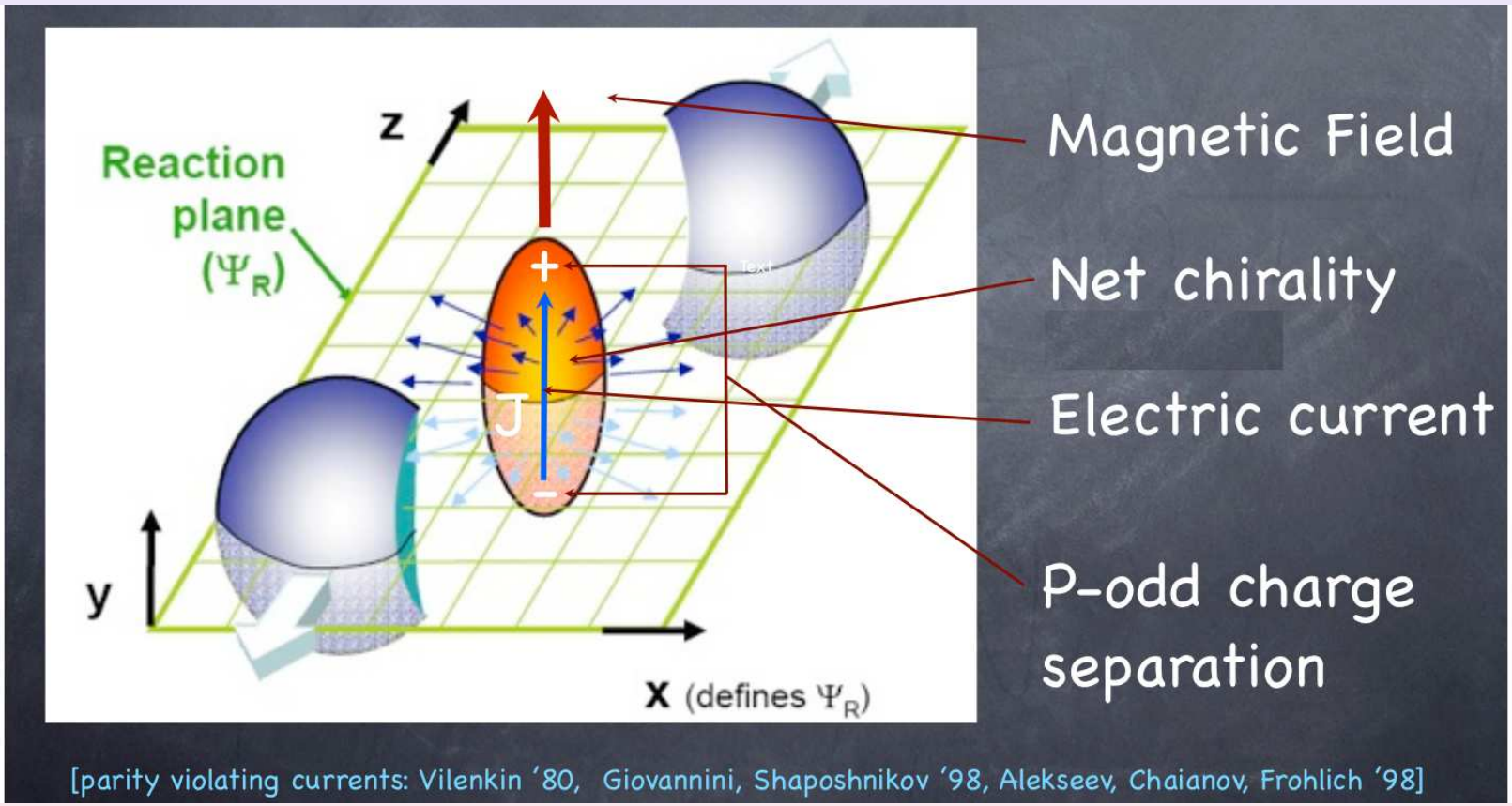
The Chiral Magnetic Effect

[Kharzeev, McLerran, Warringa '07]



Strong Magnetic field induces a P-odd charge separation $\Longrightarrow$

$\Longrightarrow$ Electric current: $\vec{J} = \sigma^{\mathcal{B}} \vec{B}$.

Kubo Formulas

- Constitutive relation for the current [see Minwalla's Lectures]:

$$J^\mu = \underbrace{nu^\mu}_{\text{Ideal Hydro}} + \underbrace{\sigma^{\mathcal{B}} B^\mu + \sigma^{\mathcal{V}} \omega^\mu}_{\text{Anomalous}} + \cdots .$$

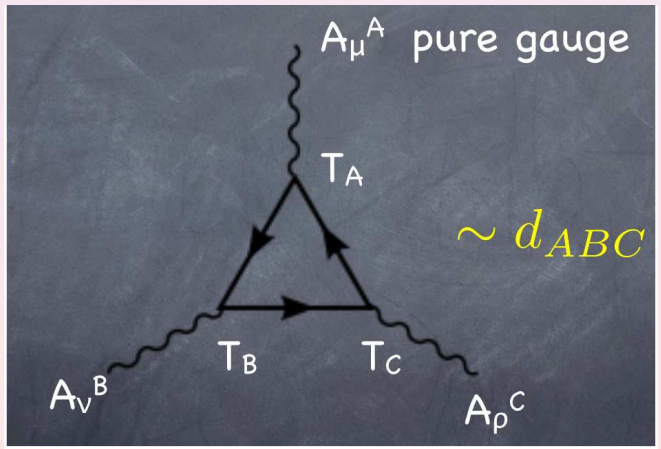
- Chiral Magnetic and Vortical Conductivities [Kharzeev'09, Amado'11]

$$\sigma^{\mathcal{B}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a J^b \rangle|_{\omega=0} , \quad \sigma^{\mathcal{V}} = \lim_{k_c \to 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a T^{0b} \rangle|_{\omega=0} .$$

- 1-loop calculation [Kharzeev, Warringa '09], [Megías et al. '11]

$$(\sigma^{\mathcal{B}})_{AB} = \frac{1}{4\pi^2} \underbrace{d_{ABC}}_{\text{Chiral Anom Coef}} \mu^C$$

$$d_{ABC} = \frac{1}{2} \mathrm{tr}(T_A \{T_B, T_C\})$$



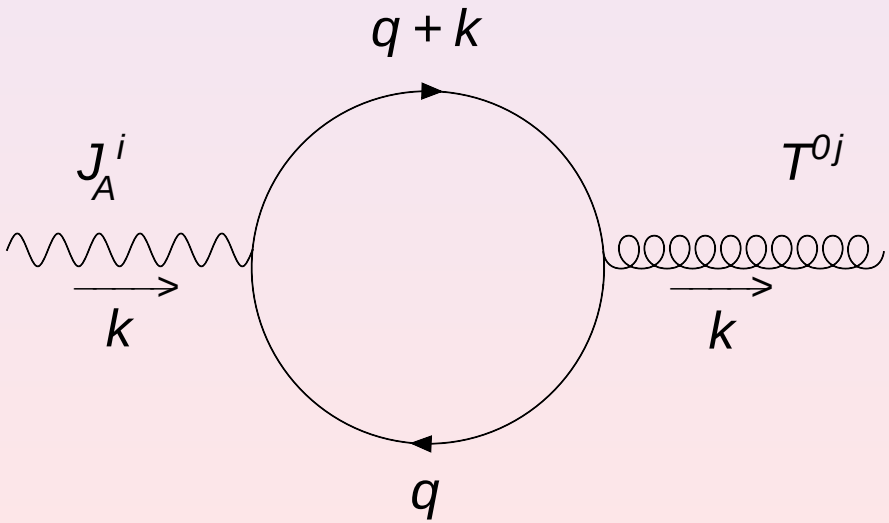
Chiral Magnetic Conductivity induced by the Chiral Anomaly.

Chiral Vortical Conductivity

- Theory of N Free Chiral fermions:

$$J_A^i = \sum_{f,g=1}^{N} (T_A)^g{}_f \bar{\Psi}_g \gamma^i \mathcal{P}_+ \Psi^f \,, \quad T^{0j} = \frac{i}{2} \sum_{f=1}^{N} \bar{\Psi}_f (\gamma^0 \partial^j + \gamma^j \partial^0) \mathcal{P}_+ \Psi^f \,.$$

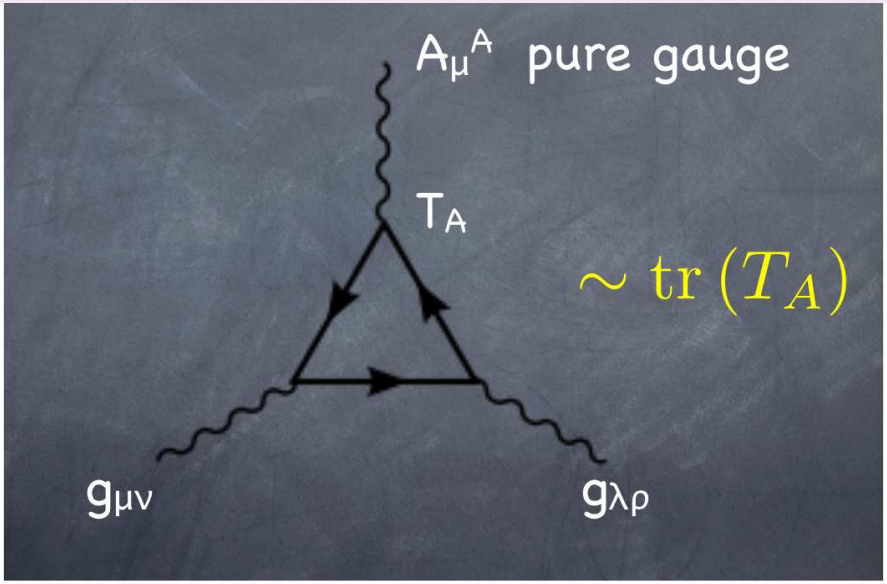
- Chiral Vortical Conductivity: defined from the retarded correlation function of $J_A^i(x)$ and $T^{0j}(x')$.



Chiral Vortical Conductivity

- 1 loop calculation **[Landsteiner, Megías, Pena-Benitez, PRL107 '11]**:

$$(\sigma^{\mathcal{V}})_A = \underbrace{\frac{1}{8\pi^2}\sum_{B,C} d_{ABC}\,\mu^B\,\mu^C}_{\text{Chiral Anomaly}} + \underbrace{\frac{T^2}{24}\mathrm{tr}\,(T_A)}_{\text{Gauge-Gravitational Anomaly!!!}}$$



Computation in Holography: See Pena-Benitez's talk.