Strings in Non-geometric Backgrounds

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(Bhg, Deser, Plauschinn, Rennecke , arXiv:1106.0316)

(Bhg, Deser, Lüst, Plauschinn, Rennecke , arXiv:1205.1522)

Introduction

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String theory is described by 2D non-linear sigma model

$$\mathcal{S} = \frac{1}{2\pi\alpha'} \int_\Sigma d^2z \,(\,G_{ab} + B_{ab})\, \partial X^a \,\overline{\partial} X^b + \ldots \; ,$$

where conformal invariance provides the string equations of motion

$$\begin{aligned} 0 &= \beta^G_{ab} = \alpha'\Big(R_{ab} - \frac{1}{4}\, H_a{}^{cd}\, H_{bcd} + 2\nabla_a\nabla_b\Phi\Big) + O(\alpha'^2) \\ 0 &= \beta^B_{ab} = \alpha'\Big(-\frac{1}{2}\nabla_c H^c{}_{ab} + \alpha' H_{ab}{}^c\nabla_c\Phi\Big) + O(\alpha'^2) \\ 0 &= \beta^\Phi_{ab} = \frac{1}{4}(d - d_{\text{crit}}) + \alpha'\Big((\nabla\Phi)^2 - \frac{1}{2}\nabla^2\Phi - \frac{1}{24}H^2\Big) + O(\alpha'^2)\,. \end{aligned}$$

Leading order: Einstein gravity with a two-form B_{ab} and a scalar Φ.

Introduction

Introduction

FAQ: Does there exist a purely geometric description?

At leading order: H can be described as torsion of the Riemannian geometry. Aspects of higher order generalization are analyzed in (Bhg, Deser, Plauschinn, Rennecke, *Palatini-Lovelock-Cartan Gravity*, arXiv:1202.4934; see talk of Plauschinn)

There exist conformal field theories which cannot be identified with such simple large radius geometries. These are left-right asymmetric like asymmetric orbifolds. Applying T-duality leads to the chain of fluxes (Shelton, Taylor, Wecht, hep-th/0508133)

$$H_{abc} \leftrightarrow f_{ab}{}^{c} \leftrightarrow Q_a{}^{bc} \overset{?}{\leftrightarrow} R^{abc} ,$$

Q and R are non-geometric fluxes. What is the nature of R-flux?

Classical non-associativity

Classical non-associativity

Generalized geometry resp. double field theory suggests to make a change of field variables (Grana,Minasian, Petrini, Waldram, arXiv:0807.4527)

$$\tilde{g}_{ij} = g_{ij} - B_{im}\, g^{mn}\, B_{nj}\ , \qquad \beta^{ij} = -g^{im}\, B_{mn}\, \tilde{g}^{nj}\ .$$

$\beta = \beta^{ij}\partial_i \wedge \partial_j$ is an anti-symmetric bi-vector.

This defines a quasi-Poisson structure as follows

$$\{f,g\} = \beta^{ij}\,(\partial_i f)\,(\partial_j g)\ ,$$

where $f, g \in \mathcal{C}^\infty(M)$. In general, this bracket does not satisfy the Jacobi identity but one finds

$$\{\{f,g\},h\} + \text{cycl.} = R^{ijk}\,(\partial_i f)\,(\partial_j g)\,(\partial_k h)\ .$$

Classical non-associativity

Classical non-associativity

Here, the non-geometric R-flux is given by

$$R^{ijk} = 3\,\beta^{[\underline{i}m}\,\partial_m\beta^{\underline{jk}]}$$

The non-geometric flux Q-flux is defined as ${Q_k}^{ij} = \partial_k\beta^{ij}$, which is not a tensor.

Given a VEV to the fluxes, the quantum version of the classical Poisson relations for the coordinates becomes

$$[x^i, x^j] = \oint_{C_x} {Q_k}^{ij} dy^k \;, \qquad\qquad [x^i, x^j, x^k] = R^{ijk} \;.$$

Non-commutativity: Wilson line of Q-flux

Non-associativity: local R-flux

(see work of (Bouwknegt, Hannabuss, Mathai))

Quantum NCA-structure

Quantum NCA-structure

Can one find evidence for such a structure from a genuine CFT computation?

- Compute cyclic double commutator

$$\big[X^\mu, X^\nu, X^\rho\big] := \lim_{\sigma_i \to \sigma} \big[[X^\mu(\sigma_1,\tau), X^\nu(\sigma_2,\tau)], X^\rho(\sigma_3,\tau)\big] + \text{cyclic}$$

  for WZW-model (Bhg, Plauschinn, arXiv:1010.1263)

- Compute commutators by direct quantization of closed strings in linear B-field (Lüst, arXiv:1010.1361)(Condeescu, Florakis, Lüst,arXiv:1202.6366

- Conformal perturbation theory around flat geometry with constant H-flux + CFT T-duality (Bhg, Deser, Lüst, Plauschinn, Rennecke, arXiv:1106.0316)

Quantum NCA-structure

Quantum NCA-structure

Consider

$$ds^2 = \sum_{a=1}^{N} \left(dX^a\right)^2, \qquad H = \frac{2}{\alpha'^2}\,\theta_{abc}\, dX^a \wedge dX^b \wedge dX^c\,.$$

From the string EOM, this background should correspond to a CFT up to linear order in H.

Conformal perturbation theory in a gauge $B_{ab} = \frac{1}{3} H_{abc}\, X^c$, i.e.

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 \qquad \text{with} \qquad \mathcal{S}_1 = \frac{1}{2\pi\alpha'}\,\frac{H_{abc}}{3} \int_\Sigma d^2z\, X^a \partial X^b \,\overline{\partial} X^c\,.$$

We expect $\mathcal{S}_1$ to be a marginal operator (only) up to linear order in H.

Quantum NCA-structure

Quantum NCA-structure

One finds

- At linear order in H, redefinition of holomorphic currents

$$\mathcal{J}^a(z,\overline{z}) = J^a(z) - \tfrac{1}{2} H^a{}_{bc}\, J^b(z)\, X_R^c(\overline{z}) \; .$$

- Correction to the two-point function of two coordinates

$$\delta_2 \big\langle X^a(z_1,\overline{z}_1)\, X^b(z_2,\overline{z}_2) \big\rangle \sim H^a{}_{pq}\, H^{bpq}\; \log |z_1 - z_2|^2 \;\log \epsilon \; ,$$

  where ϵ is a cut-off $\rightarrow$ RG-running.

- Three-point coupling

$$\big\langle \mathcal{X}^a(z_1,\overline{z}_1)\, \mathcal{X}^b(z_2,\overline{z}_2)\, \mathcal{X}^c(z_3,\overline{z}_3) \big\rangle \sim H^{abc} \Big[\mathcal{L}\Big(\frac{z_{12}}{z_{13}}\Big) - \mathcal{L}\Big(\frac{\overline{z}_{12}}{\overline{z}_{13}}\Big) \Big]$$

  with $\mathcal{L}(z) = L(z) + L\left(1 - \frac{1}{x}\right) + L\left(\frac{1}{1-x}\right)$ and $L(z)$ the Rogers dilogarithm.

CFT T-duality

CFT T-duality

Define T-duality as usual in CFT

$$\begin{array}{ccc} \mathcal{X}^a_L(z) & \xrightarrow{\text{T-duality}} & +\mathcal{X}^a_L(z)\,, \\ \mathcal{X}^a_R(\overline{z}) & & -\mathcal{X}^a_R(\overline{z})\,. \end{array}$$

Under a T-duality in all three directions: momentum modes in the R-flux background correspond to winding modes in the H-flux background.

Therefore, the three-point function in the R-flux background should read

$$\left\langle \mathcal{X}^a(z_1,\overline{z}_1)\,\mathcal{X}^b(z_2,\overline{z}_2)\,\mathcal{X}^c(z_3,\overline{z}_3)\right\rangle^R \sim R^{abc}\left[\mathcal{L}\left(\frac{z_{12}}{z_{13}}\right)+\mathcal{L}\left(\frac{\overline{z}_{12}}{\overline{z}_{13}}\right)\right].$$

Tachyon correlator

Tachyon correlator

Define tachyon vertex operator as

$$\mathcal{V}(z,\overline{z}) = :\exp\big(ip\cdot\mathcal{X}\big): \ .$$

- It is primary and has conformal dimension $(h,\overline{h}) = (\frac{\alpha'}{4}p^2, \frac{\alpha'}{4}p^2) = (1,1)$. i.e. it is a physical quantum state of the deformed theory.
- The integrated 3-point scattering amplitude becomes

$$\langle\, \mathcal{T}_1\, \mathcal{T}_2\, \mathcal{T}_3\, \rangle^{H/R} = \int \prod_{i=1}^{3} d^2 z_i\, \delta^{(2)}(z_i - z_i^0)\, \delta(p_1+p_2+p_3)$$
$$\exp\Big[-i\theta^{abc}\, p_{1,a}p_{2,b}p_{3,c}\big[\mathcal{L}\big(\tfrac{z_{12}}{z_{13}}\big) \mp \mathcal{L}\big(\tfrac{\overline{z}_{12}}{\overline{z}_{13}}\big)\big]\Big]_\theta .$$

(at linear order in $\theta = H/R$).

Tachyon correlator

Tachyon correlator

Under a permutation σ of the vertex operators, the $\exp(\ldots)$-factor in the integrand picks up a phase

$$\left\langle \mathcal{V}_{\sigma(1)} \mathcal{V}_{\sigma(2)} \mathcal{V}_{\sigma(3)} \right\rangle^{H/R} = \exp\Big[i \left(\tfrac{1+\epsilon}{2}\right) \mathrm{sign}(\sigma)\, \pi^2\, \theta^{abc}\, p_{1,a}\, p_{2,b}\, p_{3,c} \Big] \left\langle \mathcal{V}_1\, \mathcal{V}_2\, \mathcal{V}_3 \right\rangle^{H/R}$$

where we used $L(z) + L(1-z) = L(1)$ and $\epsilon = \mp 1$ for H/R-flux.

- Phase is independent of world-sheet coordinates
- For H-flux the phase is trivial
- For R-flux it is non-trivial
- CFT amplitude is crossing symmetric, after $\delta(p_1 + p_2 + p_3)$

Three-product

Three-product

The phase can be described by a non-trivial three-product

$$f_1(x) \vartriangle f_2(x) \vartriangle f_3(x) \stackrel{\text{def}}{=}$$

$$\exp\left(\frac{\pi^2}{2}\, \theta^{abc}\, \partial_a^{x_1}\, \partial_b^{x_2}\, \partial_c^{x_3}\right) f_1(x_1)\, f_2(x_2)\, f_3(x_3)\Big|_x ,$$

where we used the notation $(\ )|_x = (\ )|_{x_1=x_2=x_3=x}$.

A three-bracket for the coordinates x^a can then be defined as

$$\left[x^a, x^b, x^c\right] = \sum_{\sigma \in P^3} \mathrm{sign}(\sigma)\, x^{\sigma(a)} \vartriangle x^{\sigma(b)} \vartriangle x^{\sigma(c)} = 3\pi^2\, \theta^{abc},$$

consistent with the WZW result in (Bhg, Plauschinn, arXiv:1010.1263).

Math-structure: Anchor map

Math-structure: Anchor map

In addition to the quasi-Poisson structure, the bi-vector β also induces a natural map $\beta^\sharp : T^*M \to TM$ (Grana,Minasian, Petrini, Waldram, arXiv:0807.4527)

$$\beta^\sharp(\eta)(\xi) = \beta(\eta, \xi) \qquad\qquad \text{for all} \quad \xi \in T^*M \,,$$

which is called an anchor. In components, this becomes

$$e^i_\sharp = \beta^\sharp(e^i) = \beta^{ij}\partial_j \ .$$

Two kinds of derivative operators:

- $e_i = \partial_i$
- $e^i_\sharp = \beta^{ij}\partial_j$

.

Pre-Roytenberg algebra

Pre-Roytenberg algebra

Now, $\{e_i, e_\sharp^i\} \in TM$ satisfy the commutation relations

$$[e_i, e_j] = 0 \,,$$

$$[e_i, e_\sharp^j] = Q_i{}^{jk}\, e_k \,,$$

$$[e_\sharp^i, e_\sharp^j] = R^{ijk}\, e_k + Q_k{}^{ij}\, e_\sharp^k \,,$$

where as before $R^{ijk} = 3\,\beta^{[\underline{i}m}\,\partial_m \beta^{\underline{jk}]}$ and $Q_k{}^{ij} = \partial_k \beta^{ij}$.
However, the fluxes H and f do not yet appear.
Choosing a non-coordinate frame e_a and redefine

$$\mathcal{H}_{abc} = H_{abc} \,,$$

$$\mathcal{F}_{ab}{}^c = f_{ab}{}^c - H_{abm}\,\beta^{mc} \,,$$

$$\mathcal{Q}_a{}^{bc} = Q_a{}^{bc} + H_{amn}\,\beta^{mb}\,\beta^{nc} \,,$$

$$\mathcal{R}^{abc} = R^{abc} - H_{mnp}\,\beta^{ma}\,\beta^{nb}\,\beta^{pc} \,,$$

Pre-Roytenberg algebra

Pre-Roytenberg algebra

one finds

$$[e_a, e_b] = \mathcal{F}_{ab}{}^c\, e_c + \mathcal{H}_{abc}\, e^c_\sharp$$
$$[e_a, e^b_\sharp] = \mathcal{Q}_a{}^{bc}\, e_c - \mathcal{F}_{ac}{}^b\, e^c_\sharp$$
$$[e^a_\sharp, e^b_\sharp] = \mathcal{R}^{abc}\, e_c + \mathcal{Q}_c{}^{ab}\, e^c_\sharp,$$

what we call a pre-Roytenberg algebra (Roytenberg, math/9910078).

Bianchi identities

Bianchi identities

Deduce the Bianchi identities for the various fluxes from the Jacobi identities of the pre-Roytenberg algebra (known before only for constant fluxes).

Bianchi identity for the H-flux

$$\mathrm{I} \;:\; 0 = \nabla_{[\underline{a}}\,\mathcal{H}_{\underline{bcd}]} = \partial_{[\underline{a}}\,\mathcal{H}_{\underline{bcd}]} - \tfrac{3}{2}\,\mathcal{F}_{[\underline{ab}}{}^{m}\,\mathcal{H}_{m\underline{cd}]}\ .$$

Next, we can evaluate the following four equations

$$\mathrm{II} \;:\; 0 = \big[[e_a, e_b], e_c\big] + \text{cycl.} \qquad \mathrm{III} \;:\; 0 = \big[[e_a, e_b], e^c_\sharp\big] + \text{cycl.}$$

$$\mathrm{IV} \;:\; 0 = \big[[e_a, e^b_\sharp], e^c_\sharp\big] + \text{cycl.} \qquad \mathrm{V} \;:\; 0 = \big[[e^a_\sharp, e^b_\sharp], e^c_\sharp\big] + \text{cycl.},$$

Bianchi identities

Bianchi identities

which lead to the four Bianchi identities

$$\mathrm{II}:\quad 0 = \Big(\partial_{[\underline{c}}\,\mathcal{F}_{\underline{ab}]}{}^{d} + \mathcal{F}_{[\underline{ab}}{}^{m}\,\mathcal{F}_{\underline{c}]m}{}^{d} + \mathcal{H}_{[\underline{ab}\,m}\,\mathcal{Q}_{\underline{c}]}{}^{md}\Big) + \Big(\partial_{[\underline{c}}\,\mathcal{H}_{\underline{ab}]n} - 2\mathcal{F}_{[\underline{ab}}{}^{m}\,\mathcal{H}_{\underline{cn}]m}\Big)\beta^{nd}\;,$$

$$\mathrm{III}:\quad 0 = \Big(\beta^{cm}\partial_m\mathcal{F}_{ab}{}^{d} + 2\partial_{[\underline{a}}\,\mathcal{Q}_{\underline{b}]}{}^{cd} - \mathcal{H}_{mab}\mathcal{R}^{mcd} - \mathcal{F}_{ab}{}^{m}\mathcal{Q}_{m}{}^{cd} + 4\mathcal{Q}_{[\underline{a}}{}^{[\underline{c}m}\mathcal{F}_{m\underline{b}]}{}^{\underline{d}]}\Big) + \Big(\beta^{cm}\partial_m\mathcal{H}_{abn} - 2\partial_{[\underline{a}}\mathcal{F}_{\underline{b}]n}{}^{c} - 3\mathcal{H}_{m[\underline{ab}}\mathcal{Q}_{\underline{n}]}{}^{mc} + 3\mathcal{F}_{[\underline{ab}}{}^{m}\mathcal{F}_{m\underline{n}]}{}^{c}\Big)\beta^{nd}\;,$$

Bianchi identities

Bianchi identities

$$\mathrm{IV}:\quad 0=\Big(-\partial_a\mathcal{R}^{bcd}-2\beta^{[\underline{c}m}\partial_m\mathcal{Q}_a{}^{\underline{b}]d}+3\mathcal{Q}_a{}^{[\underline{b}m}\mathcal{Q}_m{}^{\underline{cd}]}-3\mathcal{F}_{am}{}^{[\underline{b}}\mathcal{R}^{\underline{cd}]m}\Big)+\Big(2\beta^{[\underline{c}m}\partial_m\mathcal{F}_{an}{}^{\underline{b}]}-\partial_a\mathcal{Q}_n{}^{bc}+\mathcal{Q}_m{}^{bc}\mathcal{F}_{an}{}^{m}+\mathcal{R}^{bcm}\mathcal{H}_{man}-4\mathcal{Q}_{[\underline{a}}{}^{[\underline{b}m}\mathcal{F}_{m\underline{n}]}{}^{\underline{c}]}\Big)\beta^{nd}$$

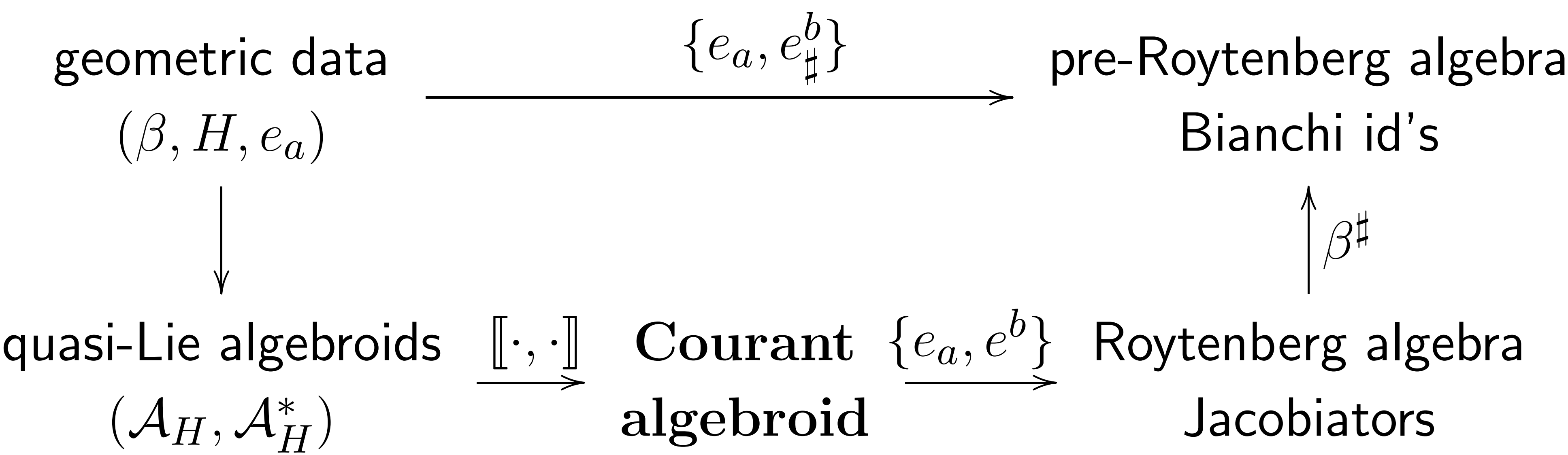
$$\mathrm{V}:\quad 0=\Big(\beta^{[\underline{c}m}\,\partial_m\mathcal{R}^{\underline{ab}]d}-2\mathcal{R}^{[\underline{ab}m}\,\mathcal{Q}_m{}^{\underline{cd}]}\Big)+\Big(\beta^{[\underline{c}m}\,\partial_m\mathcal{Q}_n{}^{\underline{ab}]}+\mathcal{R}^{[\underline{ab}m}\,\mathcal{F}_{mn}{}^{\underline{c}]}+\mathcal{Q}_m{}^{[\underline{ab}}\,\mathcal{Q}_n{}^{\underline{c}]m}\Big)\beta^{nd}$$

Lie and Courant Algebroids

Lie and Courant Algebroids

There is a mathematically intriguing structure behind all this:
Lie-algebroids and Courant algebroids

(see also (Halmagyi, arXiv:0805.4571 & 0906.2891))



(for more details see talk of A.Deser)

Conclusions

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Research program on non-geometric string theory, motivated by

- Asymmetric CFTs
- T-duality of geometric configurations

Taken some initial steps

- CFT analysis of R-flux backgrounds $\rightarrow$ NCA structure
- Fluxes as torsion for higher curvature corrections
- Mathematical framework: Lie and Courant algebroids (Bianchi-identities)

Future

- Differential Geometry of algebroids $\rightarrow$ contact to DFT
  (talks by Lüst, Andriot, Patalong)
- Deformation quantization of quasi-Poisson structures.