An Algorithmic Approach to Identifying Swiss Cheese Geometries

Brent D. Nelson

Northeastern University

String Phenomenology 2012

The Collaboration

James Gray Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München

Yang-Hui He Department of Mathematics, City University, London

Vishnu Jejjala Centre for Theoretical Physics, University of the Witwatersrand, Johannesburg, South Africa

Benjamin Jurke & BDN Department of Physics, Northeastern University, Boston

Joan Simon School of Mathematics and Maxwell Institute for Mathematical Sciences, University of Edinburgh

String Theory in the Cloud: An Application

⇒ Research supported by a grant from the NSF Division of Computing and Communication Foundations

- Includes a donation of 500,000 core-hours on Microsoft's cloud computing network using the Azure platform
- We thus have the world's largest hammer...

String Theory in the Cloud: An Application

⇒ Research supported by a grant from the NSF Division of Computing and Communication Foundations

- Includes a donation of 500,000 core-hours on Microsoft's cloud computing network using the Azure platform
- We thus have the world's largest hammer...

⇒ Search the Kreuzer-Skarke database of 500 million Calabi-Yau threefolds (CY_3's), realized as hypersurfaces in toric varieties, for manifolds which are 'Swiss Cheese'

- Heuristically, manifolds with moduli stabilized such that one (or more) modulus controls the overall volume, other moduli control volume of smaller, blow-up cycles
- Separation of cycles allows for realization of the large volume limit (LVL), and thus the LARGE Volume Scenario (LVS)

Balasubramanian, Berglund, Conlon, Quevedo, JHEP **0503** (2005) 007
Conlon, Quevedo, Suruliz, JHEP **0508** (2005) 007
Blumenhagen, Moster, Plauschinn, JHEP **0801** (2008) 105
Cicoli, Conlon, Quevedo, JHEP **0810** (2008) 058

Why Swiss Cheese Calabi-Yau Threefolds?

⇒ Phenomenology

- Allows for separation of scales → supergravity approximation trustworthy
- 'Two-step' stabilization mechanism justified parametrically
- Soft supersymmetry breaking terms give a reasonable phenomenology

  Conlon, Abdussalam, Quevedo, Suruliz, JHEP **0701** (2007) 032

  Conlon, Kom, Suruliz, Allanach, Quevedo JHEP **0708** (2007) 061
- Inflation can be realized in the string moduli sector in this scenario

  Bond, Kofman, Prokushkin, Vaudrevange, PRD **75** (2007) 123511

⇒ Necessary data exists

- Everything we need to know is (in principle) captured by the intersection form and the Kahler cone for the construction
- This information can be obtained from the Kreuzer-Skarke database

  `http://hep.itp.tuwien.ac.at/ kreuzer/CY/`

  Kreuzer & Skarke, Adv.Theor.Math.Phys. **4** (2002) 1209

⇒ Much early work already exists

- General discussion of LVL with examples provided by Cicoli et al.

  Cicoli, Conlon, Quevedo, JHEP **0810** (2008) 058

More on all of these as we proceed...

Moduli Sector in Type IIB

⇒ Type IIB string theory compactified on Calabi-Yau threefold (CY$_3$)

- Internal mathematical consistency provides large number of constraints, but still much freedom in specifying CY$_3$
- Moduli space typically rich and of high dimension

⇒ Geometrical moduli of a CY$_3$ $\mathcal{X}$ determined by the number of embedded 2-and 3-spheres

- 2-cycles: $h^{1,1}(\mathcal{X}) = b_2$ Kähler moduli
- 3-cycles: $h^{1,2}(\mathcal{X}) = (b_3/2) - 1$ complex structure moduli
- Axio-dilaton

⇒ A note (plea!) on conventions...

t^i Parameterize the Kähler form J directly

T^i Complexified volumes which include the RR 4-form charge along the associated 4-cycle D_i

τ^i 4-cycle volumes of divisors D_i, quadratic in t^is; equal to one half the complex scalar fields T_i in the 4D $\mathcal{N} = 1$ chiral multiplets of the effective theory

Supergravity Lagrangian in Type IIB

$\Rightarrow$ Moduli Kähler potential with leading $(\alpha')^3$ correction

$$\begin{aligned} K &= -\ln\left(-\int_{\mathcal{X}} \Omega \wedge \bar{\Omega}\right) - \ln(S - \bar{S}) - 2\ln\left(\hat{\mathcal{V}} + \frac{\xi}{2g_s^{\frac{3}{2}}}\right) \\ &= K_{cs} - 2\ln\left(2/g_s\right) - 2\ln\left(\hat{\mathcal{V}} + \hat{\xi}\right) \; ; \quad \hat{\xi} \equiv \frac{\xi}{2g_s^{\frac{3}{2}}} \end{aligned}$$

Becker, Becker, Haack, Louis, JHEP **0206** (2002) 069

- ξ is a constant determined by the topology of the Calabi-Yau manifold via

$$\xi = -\frac{\chi(\mathcal{X})\zeta(3)}{2(2\pi)^3} \approx -0.002423 \cdot \chi(\mathcal{X})$$

- Last term includes the leading $(\alpha')^3$ correction, but no string loop corrections

$\Rightarrow$ We will need $\xi > 0$ later for moduli stabilization

- Thus we need $\chi(\mathcal{X}) < 0$, so CY_3 must satisfy $h^{2,1}(\mathcal{X}) > h^{1,1}(\mathcal{X}) \geq 2$
- That is, the number of complex structure moduli is strictly larger than the number of Kahler moduli

Supergravity Lagrangian in Type IIB

⇒ Superpotential includes Gukov-Vafa-Witten potential and non-perturbative corrections that depend (via coefficients a_i) on Kähler moduli T_i

$$W = W_{\mathrm{GVW}} + W_{\mathrm{np}} = \int_{\mathcal{X}} G_3 \wedge \Omega + \sum_{i=1}^{h^{1,1}} A_i(S, U_j)^{-a_i T_i}$$

- Sum ranges over all 4-cycles D_i
- Threshold prefactors A_i depend on complex structure moduli U_j and the axio-dilaton S
- The no-scale structure of W_{GVW} is broken due to the explicit dependency of the non-perturbative term W_{ns} on the Kähler moduli T_i
- The a_i arise from non-perturbative corrections: $a_i = 2\pi$ for D-brane instantons and $a_i = 2\pi/N$ for gaugino condensates

Moduli Kahler Metric and Stabilization

⇒ The limit where $\xi \to 0$ (neglect α' correction to K) results in block diagonal Kähler metric for the moduli

- This is the KKLT scenario with two step minimization
- $W_{\rm GVW}$ stabilizes U_j and S at first order, with $\langle W_{\rm GVW}\rangle \to W_0$ for step two
- Non-perturbative superpotential $W_{\rm np}$ then stabilizes Kähler moduli T_i

⇒ Nice outcome – but is it universally justifiable?

- Inclusion of α' correction introduces non-diagonal Kähler metric
- However, if we achieve a minimum with $a_s\tau_s = \ln(\mathcal{V})$ then in the limit of large volume $\mathcal{V}$

Balasubramanian, Berglund, Conlon, Quevedo, JHEP **0503** (2005) 007
Conlon, Quevedo, Suruliz, JHEP **0508** (2005) 007

$$K^{-1}_{U\bar{U}}F_U\bar{F}_{\bar{U}}, K^{-1}_{S\bar{S}}F_S\bar{F}_{\bar{S}} \sim (\frac{1}{\mathcal{V}^0})$$

$$K^{-1}_{S\bar{T}}F_S\bar{F}_{\bar{T}}, K^{-1}_{T\bar{T}}F_T\bar{F}_{\bar{T}} - 3|W|^2 \sim (\frac{1}{\mathcal{V}^1})$$

⇒ The Large Volume Limit (LVL) allows parametrically controlled 'two-step' stabilization

The Large Volume Scenario

⇒ Consider a simple $h^{1,1} = 2$ example where $\mathcal{V} = \tau_L^{3/2} - \tau_s^{3/2}$ which is a special case of the "strong cheese" form

$$\mathcal{V} = \tau_L^{3/2} - \sum_{i=1}^{N_{\rm small}} \tau_{s_i}^{3/2}$$

⇒ In this case we have a limit where $\tau_L \to \infty$ and $a_s\tau_s = \ln(\mathcal{V})$

- The full scalar potential for τ_s in the large volume limit is

$$V \sim \left[\frac{1}{\mathcal{V}} a_s^2 |A|^2 \sqrt{\tau_s} e^{-2a_s\tau_s} - \frac{1}{\mathcal{V}^2} a_s\tau_s |AW| e^{-a_s\tau_s} + \frac{\xi}{\mathcal{V}^3} |W|^2 \right] \sim \mathcal{O}\left(\frac{1}{\mathcal{V}^3} \right)$$

- Cancellation in this case made possible because the inverse Kähler metric for the small cycle obeys the scaling relation

$$K^{\tau_s\tau_s} \sim \sqrt{\tau_s}\,\mathcal{V}$$

- We will call this crucial property of the inverse Kähler metric the *homogeneity condition*

Known Swiss Cheese Manifolds

⇒ A (partial) catalog of known Swiss Cheese (SC) manifolds

Blumenhagen, Moster, Plauschinn, JHEP **0801** (2008) 058
Collinucci, Kreuzer, Mayrhofer, Walliser, JHEP **0709** (2009) 074
Blumenhagen, Braun, Grimm, Weigand, NPB **815** (2009) 1
Cicoli, Kreuzer, Mayrhofer, arXiv:1107.0383
+ Previously cited papers...

- $\mathbb{P}^4_{1,1,1,6,9}[18]$, $h^{1,1} = 2$
- $\mathbb{P}^4_{1,3,3,3,5}[15]$, $h^{1,1} = 3$
- $\mathcal{F}_{11} = \mathrm{CY}^{3,111}/\mathbb{Z}_2$, $h^{1,1} = 3$
- $\mathbb{P}^4_{1,1,3,10,15}[30]$, $h^{1,1} = 4$
- $\mathbb{P}^4_{1,2,2,10,15}[30]$, $h^{1,1} = 4$
- $\mathbb{P}^4_{1,1,2,2,6}[12]/\mathbb{Z}_2$, $h^{1,1} = 4$
- $M_n^{(dP_1)^n}, \; n = 0, \ldots, 8$
- K3-fibered Calabi-Yaus with a del Pezzo divisor, $h^{1,1} = 4$

⇒ Most can be written in 'strong cheese' form, the last two sets by construction

Towards a Universal Description of SC Manifolds

⇒ Let $\tau_1, \ldots, \tau_{N_{\rm small}}$ remain small as $\mathcal{V} \to \infty$ and $\tau_{N_{\rm small}+1}, \ldots, \tau_{h^{1,1}(\mathcal{X})} \to \infty$

The scalar potential V admits a set Ξ of non-supersymmetric AdS minima at exponentially large volume located at $\mathcal{V} \sim e^{a_i \tau_i}$ for all small cycles $i = 1, \ldots, N_{\rm small}$ if and only if:

- $h^{2,1}(\mathcal{X}) > h^{1,1}(\mathcal{X}) \geq 2$, which implies that $\xi > 0$, and
- each small cycle of volume τ_j is actually a blow up mode resolving a point-like singularity

The set Ξ of AdS vacua can then be characterized as follows:

$$\begin{aligned} \Xi &= \text{single point if } h^{1,1}(\mathcal{X}) = N_{\rm small} + 1 \\ &= h^{1,1}(\mathcal{X}) - N_{\rm small} - 1 \ \text{ flat directions if } h^{1,1}(\mathcal{X}) > N_{\rm small} + 1 \end{aligned}$$

Cicoli, Conlon, Quevedo, JHEP **0810** (2008) 058

⇒ Note that the requirement that the small cycle(s) correspond to a blow-up mode guarantees the *homogeneity condition*

The Geometry of Calabi-Yau Threefolds

⇒ Let $D_1, \ldots, D_n$ be a divisor basis via $\mathrm{Div}(\mathcal{X}) \cong H^{1,1}(\mathcal{X};)$, where $D_i \subset \mathcal{X}$ is a 4-cycle divisor of the threefold $\mathcal{X}$

- Let D_i be the Poincaré-dual $(1,1)$-form of the divisor D_i
- The Kähler structure is expressed by the expansion of the symplectic Kähler form $J \in \Omega^{1,1}(\mathcal{X})$ via

$$J = \sum_{i=1}^{n} t^i [D_i]$$

- The topology is encoded in the triple intersection numbers

$$\kappa_{ijk} = \int_{\mathcal{X}} [D_i] \wedge [D_j] \wedge [D_k]$$

⇒ With these definitions, the overall volume $\mathcal{V}$ of $\mathcal{X}$ is given by

$$\mathcal{V} = \int_{\mathcal{X}} \Omega_3 \wedge \bar{\Omega}_3 = \frac{1}{3!} \int_{cX} J \wedge J \wedge J = \frac{1}{6} \kappa_{ijk} t^i t^j t^k$$

- The volumes of the four-cycle divisors D_i are given by

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t_i} = \frac{1}{2!} \int_{\mathcal{X}} [D_i] \wedge J \wedge J = \frac{1}{2} \kappa_{ijk} t^j t^k$$

Interlude on the Kähler Cone

⇒ The space of all viable Kähler forms which give rise to smooth compact Kähler manifolds is called the Kähler cone $\mathcal{K}_{\mathcal{X}}$

- The cone can be represented by an $n_F \times h^{1,1}(\mathcal{X})$ matrix of integers $\mathcal{K} = [\mathcal{K}_i^\kappa]$, where n_F is the number of facets
- In this case the Kähler parameters are constrained by a set of n_F inequalities

$$\sum_{i=1}^{h^{1,1}} \mathcal{K}_i^\kappa t^i \geq 0 \quad \text{for} \quad \kappa = 1, \ldots, n_F.$$

⇒ When $n_F = h^{1,1}(\mathcal{X})$ and $\mathcal{K}_i^\kappa$ is a square matrix, the Kähler cone is said to be *simplicial*

- When, in addition, the matrix $\mathcal{K}_i^\kappa$ is invertible, the manifold $\mathcal{X}$ is called *simple*
- In such cases we can absorb the Kähler cone conditions into a renormalization of the Kähler parameters themselves via

$$\hat{D}_{\hat{\imath}} = \sum_i \hat{D}_i (\mathcal{K}^{-1})^i_{\hat{\imath}} \qquad \hat{t}^{\hat{\imath}} = \sum_i \mathcal{K}^{\hat{\imath}}_i t^i$$

$$\hat{\kappa}_{\hat{\imath}\hat{\jmath}\hat{k}} = \sum_{i,j,k} \kappa_{ijk} (\mathcal{K}^{-1})^i_{\hat{\imath}} (\mathcal{K}^{-1})^j_{\hat{\jmath}} (\mathcal{K}^{-1})^k_{\hat{k}}$$

The Virtues of 'Manifest Cheese'

⇒ The previous interlude ought to suggest that the basis of D_i or τ_i in which the 4D supergravity Lagrangian looks 'simple' may not be the basis you inherit from the construction of the CY$_3$

⇒ Key question: how can we make a distinction between large cycle divisors and small cycle divisors which is robust and fully general?

- Many key properties of SC manifolds (such as the universal definition given above) tend to assume such a separation already exists
- That's fine if you are lucky enough to have the right basis right off the bat, or you are working in $h^{1,1}(\mathcal{X}) = 2$ for which gleaning the 'right' basis is easy

⇒ Example: Let $D_L, D_{s_1}, \ldots, D_{s_n}$ be a 'convenient' divisor basis for $\mathcal{X}$, as is usually assumed in the literature

- Then $D_{s_i} + D_{s_j}$ gives another 'small' divisor
- But $D_L + D_{s_i}$ gives a new (fake) 'large' divisor

⇒ A general basis change will change the number of (apparently) large 4-cycles!

⇒ For a generic intersection form $\hat{\kappa}_{ijk}$ the implied divisor basis $\hat{D}_i$ is *not* in a 'convenient' basis

Towards a General Scanning Algorithm

⇒ Everything we need to know to identify SC manifolds should be in the intersection form (+ Kähler cone conditions)

- Naively, you might expect you could always go to the 'strong' cheese form by simply diagonalizing the intersection form using some non-degenerate matrix $A \in GL(h^{1,1};\mathbb{Z})$

$$\kappa_{ijk} \longrightarrow A_{i\tilde{i}} A_{j\tilde{j}} A_{k\tilde{k}} \kappa_{\tilde{i}\tilde{j}\tilde{k}}$$

- This gives you $\begin{pmatrix} h^{1,1}+2 \\ 3 \end{pmatrix}$ cubic equations in $(h^{1,1})^2 - 1$ variables
- This is not computationally tractable – need a faster base change

Relating Everything to the Kähler Parameters

$\Rightarrow$ Back up a step: define the rotation on the 4-cycles

$$\tau_i = \sum_{\tilde{j}=1}^{h^{1,1}} A_i{}^{\tilde{j}} \tilde{\tau}_{\tilde{j}}$$

$\Rightarrow$ The requirement for the rotation is that the newly defined τ_i four-cycle volumes are such that we obtain the two desired classes:

1. Large cycles τ_{L_A}: τ_I for $I = 1, \ldots, N_{\mathrm{large}}$
2. Small cycles τ_{s_a}: τ_α for $\alpha = N_{\mathrm{large}} + 1, \ldots, h^{1,1}(\mathcal{X})$

$\Rightarrow$ Now let $\vec{t} = (t^1, \ldots, t^n)$ denote a (column) vector of the Kähler parameters

- Consider the intersection form restricted to a particular four-cycle divisor D_i as a symmetric matrix $(\kappa_{(i)})_{jk} := \kappa_{ijk}$
- Then re-express the four-cycle volumes in terms of the Kähler parameters as

$$\tau_i = \frac{1}{2}\kappa_{ijk} t^j t^k = \frac{1}{2}\vec{t}^{\,*} \kappa_{(i)} \vec{t}$$

where $\vec{t}^{\,*}$ refers to the transposed row vector of $\vec{t}$

Separating 'Large' and 'Small' Kähler Parameters

⇒ Now split the Kähler parameters into $\vec{t} = \lambda_A \vec{t}_{\mathrm{L}_A} + \gamma_a \vec{t}_{\mathrm{s}_a}$

- The LVL corresponds to some $\lambda_A \to \infty$ for some set of the $A = 1, \ldots, N_{\mathrm{large}}$
- Translating this split to the four-cycle volumes we obtain

$$\tau_i = \frac{1}{2}\left[\lambda_A \lambda_B \cdot (\vec{t}^*_{\mathrm{L}_A} \kappa_{(i)} \vec{t}_{\mathrm{L}_B}) + 2\lambda_A \gamma_b \cdot (\vec{t}^*_{\mathrm{L}_A} \kappa_{(i)} \vec{t}_{\mathrm{s}_b}) + \gamma_a \gamma_b \cdot (\vec{t}^*_{\mathrm{s}_a} \kappa_{(i)} \vec{t}_{\mathrm{s}_b})\right]$$

⇒ From looking at the λ power counting we see that

- For large cycles τ_I we require $\vec{t}^*_{\mathrm{L}_A} \kappa_{(I)} \vec{t}_{\mathrm{L}_B} \neq 0$ OR $\vec{t}^*_{\mathrm{L}_A} \kappa_{(I)} \vec{t}_{\mathrm{s}_b} \neq 0$

- For small cycles τ_α we require $\kappa_{(\alpha)} \vec{t}_{\mathrm{L}_A} = 0$

⇒ This last condition also ensures the homogeneity condition

$$\begin{aligned}
K^{-1}_{ij} &= -\frac{2}{9}\left(2\mathcal{V} + \hat{\xi}\right) \kappa_{ijk} t^k + \frac{4\mathcal{V} - \hat{\xi}}{\mathcal{V} - \hat{\xi}} \tau_i \tau_j \\
\frac{K^{-1}_{\alpha\alpha}}{\mathcal{V}} &= -\frac{4}{9} \kappa_{\alpha\alpha i} t^i + \frac{4(\tau_\alpha)^2}{\mathcal{V}} + \mathcal{O}(\mathcal{V}^{-2}) \\
&= -\frac{4}{9} (\kappa_{(\alpha)} \vec{t})_\alpha \sim \sqrt{\tau_\alpha}
\end{aligned}$$

Final System to Check

⇒ The full system of equations will include the rotation matrices $A_i{}^{\tilde{\jmath}}$

1. Small cycles: $A_\alpha{}^{\tilde{\jmath}}(\kappa_{(\tilde{\jmath})}\vec{t}_{\mathrm{L}_A}) = 0$
2. Basis change: $\det[A_i{}^{\tilde{\jmath}}] \neq 0$
3. $K^{-1}_{\alpha\alpha}$ homogeneity: $A^{\tilde{\imath}}_\alpha A^{\tilde{\jmath}}_\alpha(\kappa_{(\tilde{\imath})}\vec{t}_{\mathrm{s}_a})_{\tilde{\jmath}} \neq 0$
4. Non-triviality: $\det\left(\vec{t}_{\mathrm{L}_1}, \ldots, \vec{t}_{\mathrm{s}_1}, \ldots\right) \neq 0$
5. Kähler cone: $(\vec{t}_{\mathrm{L}_A})^{\tilde{\imath}} \geq 0$

⇒ We must solve these for $A_i{}^{\tilde{\jmath}}$, $\vec{t}_{\mathrm{L}_A}$, and $\vec{t}_{\mathrm{s}_a}$ over the rational numbers

⇒ Conversion to a form suitable for `Singular` + `Mathematica`

⇒ Hang ups

- Non-simplicial Kahler cones
- Primary decomposition sometimes makes the problem worse
- Speed – redundancy fixing

⇒ Most of this slide covered by B. Jurke in his talk tomorrow!!

Our Results

⇒ **No** Swiss Cheese among complete-intersection Calabi-Yau (CICY) manifolds with $h^{1,1} \leq 4$

- Not surprising – CICY's with $h^{1,1} \leq 4$ are all favorable manifolds
- No blow-up modes at all
- First non-favorable manifold occurs at $h^{1,1} = 5$ for the CICYs

⇒ Hypersurfaces in Toric Varieties

	$h^{1,1} = 2$	$h^{1,1} = 3$	$h^{1,1} = 4$
Number of Polytopes	36	244	1197
Number of Geometries	39	306	5930
Simplicial Kähler Cones	39	266	3513
Number of Swiss Cheeses	22	94	302

⇒ Number of 'strong' form Swiss Cheese's

- $h^{1,1} = 2$: 22 out of 22
- $h^{1,1} = 3$: 50 out of 94
- $h^{1,1} = 4$: 106 out of 302

Example 1

⇒ A new Swiss Cheese with $(h^{1,1}, h^{1,2}) = (2, 38)$ and intersection form

$$\kappa_{111} = 0 \,, \qquad \kappa_{112} = \kappa_{121} = \kappa_{211} = 1 \,,$$
$$\kappa_{122} = \kappa_{212} = \kappa_{221} = 3 \,, \qquad \kappa_{222} = 9 \,.$$

⇒ From this, we immediately read off the overall volume $\mathcal{V}$ as well as the volumes of the two four-cycles

$$\mathcal{V} = \frac{1}{2}t_1^2 t_2 + \frac{3}{2}t_1 t_2^2 + \frac{3}{2}t_2^3$$
$$\tilde{\tau}_1 = t_1 t_2 + \frac{3}{2}t_2^2, \quad \tilde{\tau}_2 = \frac{1}{2}t_1^2 + 3t_1 t_2 + \frac{9}{2}t_2^2$$

⇒ The $\tilde{\tau}$ variables are in an arbitrary basis. The rotation to a basis of small and large cycles is given by

$$\begin{pmatrix} \tau_{\rm s} \\ \tau_{\rm L} \end{pmatrix} = \begin{pmatrix} -3 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{\tau}_1 \\ \tilde{\tau}_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t_1^2 \\ \frac{1}{2}(t_1 + 3t_2)^2 \end{pmatrix}$$

⇒ In this basis the volume appears in strong form

$$\mathcal{V} = \frac{\sqrt{2}}{9}\left(\tau_{\rm L}^{\frac{3}{2}} - \tau_{\rm s}^{\frac{3}{2}}\right)$$

Example 1

⇒ Diagonal components of inverse Kähler metric in this basis are

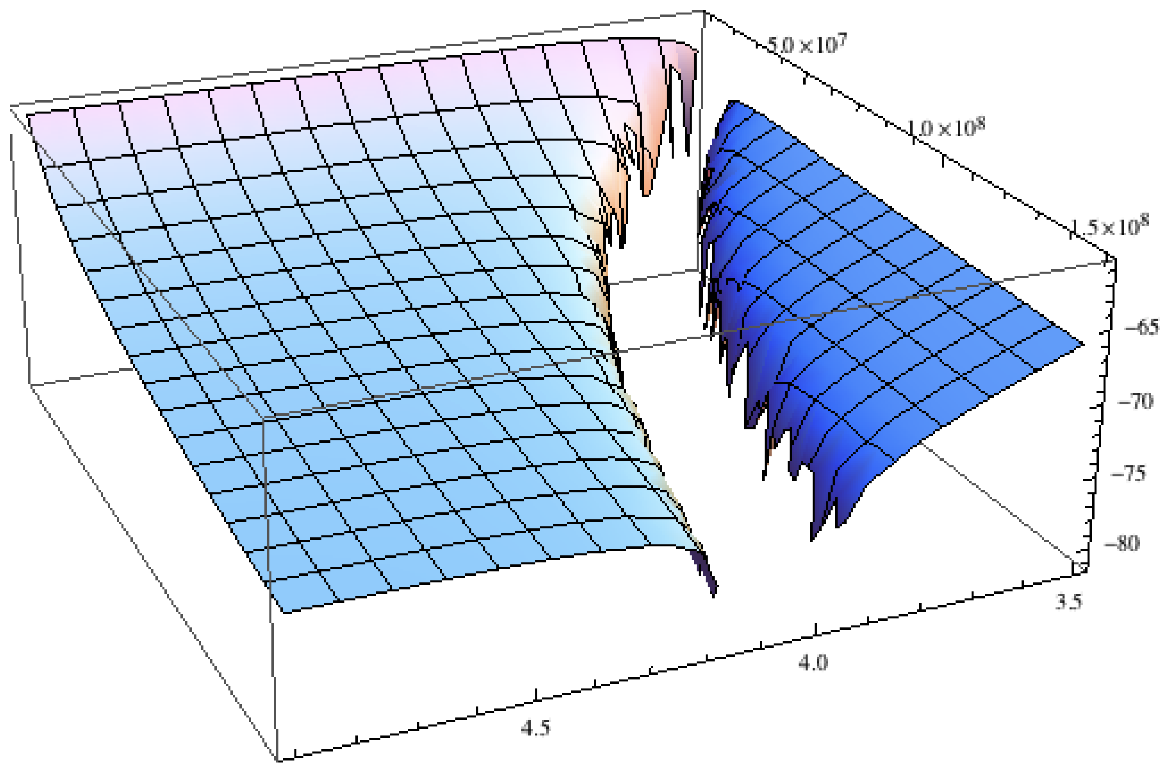
$$\frac{K_{\rm ss}^{-1}}{\mathcal{V}} = \frac{4}{3}t_1 + (t_2^{-2}), \quad \frac{K_{\rm LL}^{-1}}{\mathcal{V}} = 50t_2 + \frac{50}{3}t_1 + (t_2^{-2}) \; .$$

and since $t_1 \sim \sqrt{\tau_{\rm s}}$ we have homogeneity $K_{\rm ss}^{-1}/\mathcal{V} \sim \sqrt{\tau_{\rm s}}$

⇒ We therefore expect to find an LVL minimum for the potental

$$V = \frac{1}{\mathcal{V}} a_1^2 |A_1|^2 \sqrt{\tau_s} e^{-2a_1\tau_s} - \frac{1}{\mathcal{V}^2} a_1 \tau_s |A_1 W| e^{-a_1\tau_s} + \frac{\hat{\xi}|W|^2}{\mathcal{V}^3}$$

⇒ Take $a_1 = 2\pi$, $A_1 = 1$, $W = 20$, $\hat{\xi} = 1.31$



Example 2

$\Rightarrow$ A new (strong) Swiss Cheese with $h^{1,1} = 3$ and intersection form, normalized by Kahler cone

$$\kappa_{ijk} = \begin{pmatrix} \begin{pmatrix} 135 \\ 21 \\ 72 \end{pmatrix} & \begin{pmatrix} 21 \\ 3 \\ 12 \end{pmatrix} & \begin{pmatrix} 72 \\ 12 \\ 36 \end{pmatrix} \\ \begin{pmatrix} 21 \\ 3 \\ 12 \end{pmatrix} & \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix} & \begin{pmatrix} 12 \\ 2 \\ 6 \end{pmatrix} \\ \begin{pmatrix} 72 \\ 12 \\ 36 \end{pmatrix} & \begin{pmatrix} 12 \\ 2 \\ 6 \end{pmatrix} & \begin{pmatrix} 36 \\ 6 \\ 18 \end{pmatrix} \end{pmatrix}$$

$\Rightarrow$ This gives rise to a volume $\mathcal{V}(t)$

$$\mathcal{V} = 135t_1^3 + 63t_1^2t_2 + 9t_1t_2^2 + 216t_1^2t_3 + 72t_1t_2t_3 + 6t_2^2t_3 + 108t_1t_3^2 + 18t_2t_3^2 + 18t_3^3$$

$\Rightarrow$ Connection of four cycles to the Kähler parameters

$$\tau_1 = (6t_1 + t_2 + 3t_3)^2, \; \tau_2 = \frac{1}{2}(3t_1 + t_2)^2, \; \tau_3 = \frac{t_2^2}{6}$$

$\Rightarrow$ This leads to a 'strong form' version of the volume

$$\mathcal{V} = \frac{2}{3}\tau_1^{3/2} - \frac{\sqrt{8}}{3}\tau_2^{3/2} - 2\sqrt{6}\tau_3^{3/2}$$

Conclusion and Future Directions

⇒ Summary: We provide an efficient algorithm for identifying Swiss Cheese geometries independent of construction. 418 manifolds which admit a LVL are identified with $h^{1,1} \leq 4$. The method is (in principle) scalable to arbitrarily large numbers of Kähler moduli

⇒ The papers

- First paper: the algorithm
- Second paper: comprehensive list of models plus analytic results

⇒ Future directions

- Deal with non-simplicial Kähler cones
- Attack higher $h^{1,1}$ values
- Consider phenomenology and cosmology of this class of models