Non-geometric fluxes in higher dimensions: I

David ANDRIOT

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arXiv:1106.4015 by D. A., M. Larfors, D. Lüst, P. Patalong
arXiv:1202.3060 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong
arXiv:1204.1979 by D. A., O. Hohm, M. Larfors, D. Lüst, P. Patalong
and work in progress...

Relations to many recent papers...

26/06/2012, String Phenomenology 2012,
Isaac Newton Institute, Cambridge, UK

Introduction

Non-geometry in 4d and 10d SUGRA

Restrict to NSNS sector: $g_{mn},\ b_{mn},\ \phi$.

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Generated by integers ${Q_k}^{mn},\ R^{kmn}$: "non-geometric fluxes".
($\Leftrightarrow$ specific gaugings in gauged SUGRA).

hep-th/0508133 by J. Shelton, W. Taylor, B. Wecht

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A 10d non-geometric config. of fields:
typically looks ill-defined: not single-valued, global issues

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- Progress in relating 4d/10d non-geometry:
  - Field redefinition on NSNS fields $\Rightarrow$ Q, R in 10d Lag.
  - New fields globally defined (in some examples)
    $\Rightarrow$ compactify, get scalar potential ✓

Field redefinition

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Key object: β: antisymmetric bivector β^{mn}.

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Arguments in GCG: β related to non-geometry / to ${Q_k}^{mn}$, R^{kmn}

hep-th/0609084, arXiv:0708.2392 by P. Grange, S. Schäfer-Nameki

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β appears via a reparametrization of the gen. metric $\mathcal{H}$:

$$\mathcal{H} = \begin{pmatrix} g - bg^{-1}b & bg^{-1} \\ -g^{-1}b & g^{-1} \end{pmatrix} = \begin{pmatrix} \tilde{g} & -\tilde{g}\beta \\ \beta\tilde{g} & \tilde{g}^{-1} - \beta\tilde{g}\beta \end{pmatrix}, \ \tilde{g}: \text{ new metric}$$

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$$\Leftrightarrow \quad \begin{array}{l} g = (\tilde{g}^{-1} + \beta)^{-1} \tilde{g}^{-1} (\tilde{g}^{-1} - \beta)^{-1} \\ b = -(\tilde{g}^{-1} + \beta)^{-1} \beta (\tilde{g}^{-1} - \beta)^{-1} \end{array} \quad \Leftrightarrow \quad (g + b) = (\tilde{g}^{-1} + \beta)^{-1}$$

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Apply it on NSNS Lagrangian?
β could be related to non-geo. fluxes $\Rightarrow$ would they appear?

Rewriting of the NSNS Lagrangian

$$\mathcal{L} = e^{-2\phi}\sqrt{|g|}\left(\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{2.3!} H_{kmn} H_{pqr} \ g^{kp} g^{mq} g^{nr}\right)$$

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(assumption: $\beta^{km}\partial_m \cdot = 0$)

$$\begin{aligned}
\mathcal{R} = \widetilde{\mathcal{R}} &- \partial_k \tilde{g}_{su} \partial_m \tilde{g}_{pq} \left(2\tilde{g}^{km}\tilde{g}^{uq}\tilde{g}^{ps} + 2\tilde{g}^{pq}\tilde{g}^{ks}\tilde{g}^{mu} + \frac{1}{2}\tilde{g}^{uq}\tilde{g}^{sm}\tilde{g}^{kp}\right) \\
&- \tilde{g}_{pq}\partial_k\beta^{pk}\partial_m\beta^{qm} - \frac{1}{2}\tilde{g}_{pq}\partial_k\beta^{qm}\partial_m\beta^{pk} \\
&+ 2\tilde{g}^{km}\tilde{g}^{pq}\partial_k\partial_m\tilde{g}_{pq} + 2\tilde{g}^{km}(G^{-1})_{pq}\partial_k\partial_m G^{qp} \\
&+ \partial_m G^{vl}\Big(- 2\tilde{g}^{mr}\tilde{g}^{ks}(G^{-1})_{lv}\partial_k\tilde{g}_{rs} - \tilde{g}^{rs}\tilde{g}^{km}(G^{-1})_{lv}\partial_k\tilde{g}_{rs} \\
&\quad + \tilde{g}^{ms}\tilde{g}^{ru}(G^{-1})_{lu}\partial_v\tilde{g}_{rs} - \tilde{g}^{km}\tilde{g}^{rs}(G^{-1})_{ls}\partial_k\tilde{g}_{vr}\Big) \\
&+ \partial_m G^{vl}\left((G^{-1})_{lq}\partial_v G^{qm} + \frac{1}{2}g_{lq}\partial_v G^{mq}\right) \\
&- \partial_m G^{vl}\partial_k G^{ps}\frac{1}{2}\tilde{g}^{km}\left(2(G^{-1})_{lv}(G^{-1})_{sp} + 5(G^{-1})_{sv}(G^{-1})_{lp} + g_{sl}\tilde{g}_{pv}\right)
\end{aligned}$$

where $G = \tilde{g}^{-1} + \beta$.

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$$= e^{-2\tilde{\phi}}\sqrt{|\tilde{g}|}\left(\widetilde{\mathcal{R}} + 4(\partial\tilde{\phi})^2 - \frac{1}{2}|Q|^2\right) + \partial(\dots) = \tilde{\mathcal{L}} + \partial(\dots)$$

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where $Q_k{}^{mn} = \partial_k \beta^{mn}$, $|Q|^2 = \frac{1}{2!} Q_k{}^{mn} Q_p{}^{qr}\ \tilde{g}^{kp} \tilde{g}_{mq} \tilde{g}_{nr}$

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$\Rightarrow$ also get $R^{mnp} = 3\ \beta^{k[m}\partial_k\beta^{np]}$, $|R|^2 = \frac{1}{3!}R^{kmn}R^{pqr}\ \tilde{g}_{kp}\tilde{g}_{mq}\tilde{g}_{nr}$

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Q-, R-fluxes appear in 10d NSNS via field redefinition
Relation to 4d Q-, R-fluxes/non-geo. terms?
$\Rightarrow$ dimensional reduction

The dimensional reduction

The 4d scalar potential

Split 10d $\Rightarrow$ 4d max. sym. space-time $\times$ 6d compact $\mathcal{M}$
Compactification ansatz: $\mathrm{d}s_{10}^2 = \mathrm{d}s_4^2 + \mathrm{d}s_6^2$ (no warp factor), b, β purely internal

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Only two 4d scalar fields: volume ρ and dilaton σ:

$$g_{6ij} = \rho \ g_{6ij}^{(0)} \ , \quad e^{-\phi} = e^{-\phi^{(0)}} \ \sigma\rho^{-\frac{3}{2}} \ , \ e^{\phi^{(0)}} = g_s$$

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$$S_E = M_4^2 \int \mathrm{d}^4 x \ \sqrt{|g^E|} \left(\mathcal{R}_4^E + \mathrm{kin} - \frac{1}{M_4^2} V(\rho, \sigma) \right)$$

arXiv:0712.1196 by E. Silverstein

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where $V(\rho,\sigma) = \sigma^{-2}\left(\rho^{-3}\ V_H^0 + \rho^{-1}\ V_{\mathcal{R}}^0\right)$

$$V_H^0 = \frac{M_4^2}{v_0}\int \mathrm{d}^6x\ \sqrt{|g_6^{(0)}|}\ \frac{1}{12}\, H_{ijk}^{(0)}\, H_{lmn}^{(0)}\ g_6^{il(0)} g_6^{jm(0)} g_6^{kn(0)}$$

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Non-geometric terms? Most general (NSNS) potential:

$$V(\rho,\sigma) = \sigma^{-2} \left(\rho^{-3} \ V_H^0 + \rho^{-1} \ V_{\mathcal{R}}^0 + \rho \ V_Q^0 + \rho^3 \ V_R^0 \right)$$

arXiv:0711.2512 by M. P. Hertzberg, S. Kachru, W. Taylor, M. Tegmark

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The 4d scalar potential

Split 10d $\Rightarrow$ 4d max. sym. space-time $\times$ 6d compact $\mathcal{M}$
Compactification ansatz: $\mathrm{d}s_{10}^2 = \mathrm{d}s_4^2 + \mathrm{d}s_6^2$ (no warp factor), b, β purely internal

Only two 4d scalar fields: volume ρ and dilaton σ:

$$g_{6ij} = \rho \ g_{6ij}^{(0)} \ , \quad e^{-\phi} = e^{-\phi^{(0)}} \ \sigma \rho^{-\frac{3}{2}} \ , \ e^{\phi^{(0)}} = g_s$$

$\hookrightarrow$ in $\mathcal{S}_{\rm NSNS} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}x \ \mathcal{L}$, integrate 6d, go to 4d Einstein fr.:

$$S_E = M_4^2 \int \mathrm{d}^4x \ \sqrt{|g^E|} \left(\mathcal{R}_4^E + \mathrm{kin} - \frac{1}{M_4^2} V(\rho, \sigma) \right)$$

arXiv:0712.1196 by E. Silverstein

where $V(\rho,\sigma) = \sigma^{-2} \left(\rho^{-3} \ V_H^0 + \rho^{-1} \ V_{\mathcal{R}}^0 \right)$

$$V_H^0 = \frac{M_4^2}{v_0} \int \mathrm{d}^6x \ \sqrt{|g_6^{(0)}|} \ \frac{1}{12} H_{ijk}^{(0)} H_{lmn}^{(0)} \ g_6^{il(0)} g_6^{jm(0)} g_6^{kn(0)}$$

$$V_{\mathcal{R}}^0 = -\frac{M_4^2}{v_0} \int \mathrm{d}^6x \ \sqrt{|g_6^{(0)}|} \ \mathcal{R}_6^{(0)}$$

Non-geometric terms? Most general (NSNS) potential:

$$V(\rho,\sigma) = \sigma^{-2} \left(\rho^{-3} \ V_H^0 + \rho^{-1} \ V_{\mathcal{R}}^0 + \rho \ V_Q^0 + \rho^3 \ V_R^0 \right)$$

arXiv:0711.2512 by M. P. Hertzberg, S. Kachru, W. Taylor, M. Tegmark

With $\tilde{\mathcal{L}}$ instead of $\mathcal{L}$, we get the ✓ 4d potential

We get $V(\rho,\sigma) = \sigma^{-2}\left(\rho^{-1}\ V^0_{\mathcal{R}} + \rho\ V^0_Q + \rho^3\ V^0_R\right)$ where

$$V^0_{\mathcal{R}} = -\frac{M_4^2}{v_0}\int \mathrm{d}^6x\ \sqrt{|\tilde{g}_6^{(0)}|}\ \widetilde{\mathcal{R}}_6^{(0)}$$

$$V^0_R = \frac{M_4^2}{v_0}\int \mathrm{d}^6x\ \sqrt{|\tilde{g}_6^{(0)}|}\ \frac{1}{12} R^{ijk(0)} R^{lmn(0)}\ \tilde{g}^{(0)}_{6il}\tilde{g}^{(0)}_{6jm}\tilde{g}^{(0)}_{6kn}$$

$$\begin{aligned} V^0_Q = -\frac{M_4^2}{v_0}\int \mathrm{d}^6x\ \sqrt{|\tilde{g}_6^{(0)}|}\ \Bigg(& -\frac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}^{rs}\ Q_r{}^{kl}\ Q_s{}^{ij} + \frac{1}{2}\tilde{g}_{pq} Q_k{}^{lp} Q_l{}^{kq} \\ & + \tilde{g}_{jl}\tilde{g}_{pq}\beta^{jm}\left(Q_k{}^{lp}\partial_m\tilde{g}^{kq} + \partial_k\tilde{g}^{lp}\ Q_m{}^{kq}\right) \\ & - \frac{1}{4}\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}_{pq}\left(\beta^{pr}\beta^{qs}\partial_r\tilde{g}^{kl}\partial_s\tilde{g}^{ij} - 2\beta^{ir}\beta^{js}\partial_r\tilde{g}^{lp}\partial_s\tilde{g}^{kq}\right) \\ & + \frac{1}{2\sqrt{|\tilde{g}|}}\ \tilde{g}^{pq}\partial_k\tilde{g}_{pq}\ \partial_m\left(\sqrt{|\tilde{g}|}\tilde{g}_{ij}\beta^{ik}\beta^{jm}\right)\Bigg)_6^{(0)} \end{aligned}$$

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4d Q is not clearly identified...
↪ DFT brings an interpretation for the Q-terms, the role of Q
(see talk of Peter Patalong)

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Relation between 4d/10d non-geometry:
for a 10d non-geometric configuration,
field redefinition + dimensional reduction
$\Rightarrow$ generates 4d non-geometric terms

Conclusion and outlook

- GCG ⇒ field redefinition $(g, b, \phi) \leftrightarrow (\tilde{g}, \beta, \tilde{\phi})$
  Rewriting NSNS Lag.: $\mathcal{L}(g, b, \phi) = \tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi}) + \partial(\dots)$
  10d ${Q_k}^{mn} = \partial_k \beta^{mn}$ (for $\beta^{km}\partial_m \cdot = 0$), $R^{mnp} = 3\ \beta^{k[m}\partial_k \beta^{np]}$
- 10d NSNS non-geometry ⇒ $\tilde{\mathcal{L}}(\tilde{g}, \beta, \tilde{\phi})$ has no global issue...
  $\hookrightarrow$ dim. reduction of $\tilde{\mathcal{L}}$ ⇒ 4d non-geometric potential ✓
  Relation between 4d/10d non-geometry
- Extend to RR sector (S-duality)
  and D-brane/O-plane sources (new objects?)
  $\hookrightarrow$ new interesting backgrounds, 10d de Sitter solutions...
- Relations between non-commutativity and non-geometry