Non-geometric fluxes in higher dimensions: II

based on 1202.3060 and 1204.1979 in collaboration with

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$\Delta p \cdot \Delta q \geq \frac{1}{2}\hbar$

Overview

- **Rewriting** of the Double Field Theory (DFT) action to make non-geometric fluxes appear
- Introduction of new tensor calculus for the winding derivatives to reveal **geometric interpretation**
- Connection to 10-dimensional **supergravity** by solving the strong constraint

Strategy

- Perform a **field redefinition** on the DFT action $S_{\mathrm{DFT}}(\mathcal{E}, d)$ of [Hohm, Hull, Zwiebach: 2010a]
- **Covariantise** the resulting action with respect to half of the DFT gauge transformations
  - Introduce a covariant winding derivative $\tilde{\nabla}^i$
  - Introduce a winding Riemann tensor $\check{\mathcal{R}}^{ij}{}_k{}^l$

Field redefinition 1

- **Step 1**: Relation from the supergravity context (D. Andriot's talk)

$$\tilde{\mathcal{E}}(X) = \left(\tilde{g}^{-1} + \beta\right)(x, \tilde{x}) = (g + b)^{-1}(x, \tilde{x}) = \mathcal{E}^{-1}(X)$$

Problematic inverse!

- **Step 2**: T-duality in Double Field Theory

$$\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}\ , \quad X' = hX$$

with

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(D, D) \quad \text{and} \quad X = (x, \tilde{x})$$

Field redefinition 2

- Special case: T-duality in **all directions**

$$h = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \Rightarrow \mathcal{E}'(\tilde{x}, x) = \mathcal{E}^{-1}(x, \tilde{x})$$

- **Step 3**: Compare

$$\left(\tilde{g}^{-1} + \beta\right)(x, \tilde{x}) = \tilde{\mathcal{E}}(x, \tilde{x}) = \mathcal{E}'(\tilde{x}, x) = \left(g' + b'\right)(\tilde{x}, x)$$

Rules for the field redefinition

$$g' \to \tilde{g}^{-1} \ , \quad b' \to \beta \ , \quad \partial_i \leftrightarrow \tilde{\partial}^i$$

- Each term of $S_{\mathrm{DFT}}(\mathcal{E}, d)$ is **invariant separately**
- Dilaton density e^{-2d} remains unchanged

Field redefinition 3

Effectively: **Simply invert all indices!**

$$e^{-2d}\Big[-\frac{1}{4}\,g^{ik}g^{jl}\,g^{pq}\left(\mathcal{D}_p\mathcal{E}_{kl}\,\mathcal{D}_q\mathcal{E}_{ij}-\mathcal{D}_i\mathcal{E}_{lp}\mathcal{D}_j\mathcal{E}_{kq}-\bar{\mathcal{D}}_i\mathcal{E}_{pl}\,\bar{\mathcal{D}}_j\mathcal{E}_{qk}\right)$$
$$+\,g^{ik}g^{jl}\big(\mathcal{D}_i d\;\bar{\mathcal{D}}_j\mathcal{E}_{kl}+\bar{\mathcal{D}}_i d\;\mathcal{D}_j\mathcal{E}_{lk}\big)+4g^{ij}\mathcal{D}_i d\,\mathcal{D}_j d\;\Big]$$

$$\downarrow$$

$$e^{-2d}\Big[-\frac{1}{4}\,\tilde{g}_{ik}\tilde{g}_{jl}\tilde{g}_{pq}\left(\tilde{\mathcal{D}}^p\tilde{\mathcal{E}}^{kl}\,\tilde{\mathcal{D}}^q\tilde{\mathcal{E}}^{ij}-\tilde{\mathcal{D}}^i\tilde{\mathcal{E}}^{lp}\tilde{\mathcal{D}}^j\tilde{\mathcal{E}}^{kq}-\bar{\tilde{\mathcal{D}}}^i\tilde{\mathcal{E}}^{pl}\,\bar{\tilde{\mathcal{D}}}^j\tilde{\mathcal{E}}^{qk}\right)$$
$$+\,\tilde{g}_{ik}\tilde{g}_{jl}\big(\tilde{\mathcal{D}}^i d\;\bar{\tilde{\mathcal{D}}}^j\tilde{\mathcal{E}}^{kl}+\bar{\tilde{\mathcal{D}}}^i d\;\tilde{\mathcal{D}}^j\tilde{\mathcal{E}}^{lk}\big)+4\tilde{g}_{ij}\tilde{\mathcal{D}}^i d\,\tilde{\mathcal{D}}^j d\;\Big]$$

Covariantisation of winding derivatives

- **Gauge symmetry** of DFT is parametrised by $\xi^M = (\tilde{\xi}_i, \xi^i)$
- Now covariantise action with respect to the ξ^i gauge transformations
- The new objects transform as [Hohm, Hull, Zwiebach: 2010b]

$$\begin{aligned} \delta_\xi \tilde{g}_{ij} &= \mathcal{L}_\xi \tilde{g}_{ij} \\ \delta_\xi \beta^{ij} &= \mathcal{L}_\xi \beta^{ij} + \Delta_\xi \beta^{ij} \end{aligned}$$

- "**Covariant**" means

$$\delta_\xi A = \mathcal{L}_\xi A$$

- Strategy: rewrite the action in terms of covariant objects

New objects 1: Scalars

- A scalar is covariant, but its winding derivative is not

$$\delta_\xi \left(\tilde{\partial}^i \phi\right) \neq \mathcal{L}_\xi \left(\tilde{\partial}^i \phi\right)$$

- Introduce a **new derivative**

$$\tilde{D}^i := \tilde{\partial}^i - \beta^{ij}\partial_j$$

- The object $\tilde{D}^i \phi$ is covariant
- Moreover, the non-geometric fluxes already appear

$$\left[\tilde{D}^i, \tilde{D}^j\right] = -R^{ijk}\partial_k - Q_k{}^{ij}\tilde{D}^k, \quad R^{ijk} := \tilde{D}^{[i}\beta^{jk]}$$

New objects 2: Tensors

- Define **covariant derivative** for vectors

$$\tilde{\nabla}^i V^j := \tilde{D}^i V^j - \check{\Gamma}_k{}^{ij} V^k$$

Introducing a new connection $\check{\Gamma}_k{}^{ij}$

- Demand two **constraints**
  a) $\tilde{\nabla} g = 0$
  b) $\left[\tilde{\nabla}^i, \tilde{\nabla}^j\right] \phi = -R^{ijk} \partial_k \phi$

$$\Rightarrow \quad \check{\Gamma}_k{}^{ij} = \tilde{\Gamma}_k{}^{ij} + g_{kl} g^{p(i} Q_p{}^{j)l} - \frac{1}{2} Q_k{}^{ij}$$

New objects 3

- Define new **Riemann tensor**

$$\big[\tilde{\nabla}^i, \tilde{\nabla}^j\big] V_k \;=\; -R^{ijp}\nabla_p V_k + \check{\mathcal{R}}^{ij}{}_k{}^l V_l$$

and find:

$$\check{\mathcal{R}}^{ij}{}_k{}^l \;=\; \tilde{D}^i \check{\Gamma}_k{}^{jl} - \tilde{D}^j \check{\Gamma}_k{}^{il} + \check{\Gamma}_k{}^{iq}\,\check{\Gamma}_q{}^{jl} - \check{\Gamma}_k{}^{jq}\,\check{\Gamma}_q{}^{il} + Q_q{}^{ij}\,\check{\Gamma}_k{}^{ql} - R^{ijq}\,\Gamma^l{}_{qk}$$

- **Scalar curvature**

$$\check{\mathcal{R}} := g_{ij}\check{\mathcal{R}}^{ij}\ , \quad \check{\mathcal{R}}^{ij} := \check{\mathcal{R}}^{ki}{}_k{}^j$$

New DFT action 1

- Rewritten action now contains

$$
\begin{aligned}
\mathcal{R} &= \text{Scalar curvature based on } \partial_i \\
\check{\mathcal{R}} &= \text{New scalar curvature based on } \tilde{D}^i \\
R^2 &= \text{R-flux tensor contribution} \\
(\partial\phi)^2 &= \text{Dilaton term based on } \partial_i \\
(\tilde{D}^i\phi)^2 &= \text{Dilaton term based on } \tilde{D}^i
\end{aligned}
$$

$$
\begin{aligned}
S_{\mathrm{DFT}} \; = \; \int dx d\tilde{x} \sqrt{|\tilde{g}|}\, e^{-2\tilde{\phi}} \Big[& \mathcal{R} + \check{\mathcal{R}} - \frac{1}{12} R^{ijk} R_{ijk} \\
& + 4(\partial\tilde{\phi})^2 + 4\big(\tilde{D}^i\tilde{\phi} + \mathcal{T}^i\big)^2 \Big]
\end{aligned}
$$

New DFT action 2

- **Each term** covariant under ξ gauge transformations
  $\rightarrow$ half of DFT gauge transformations manifest
- Action still invariant under $\tilde{\xi}$ transformations but in a nontrivial manner
- An additional tensor appears

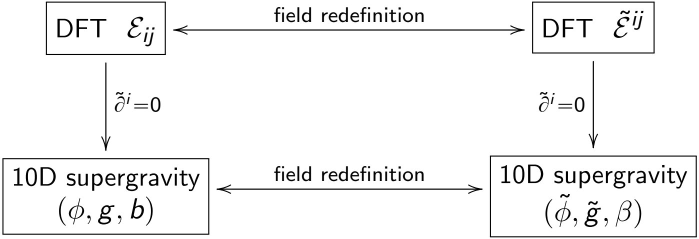
$$\mathcal{T}^i := \check{\Gamma}_k{}^{ki}$$

Important:

- **Q-flux** is hidden in $\check{\mathcal{R}}$ as part of the connection (similar to geometric flux f)
- **R-flux** appears as tensor like H-flux did before

Connection to supergravity

- Solve the strong constraint to reduce to 10D supergravity



- Same result as in 10D but easier computation

Connection to the literature

Our formalism nicely matches with recent results, for example:

- R-flux expression in Scherk-Schwarz reductions of Double Field Theory [Aldazabal et al.: 2011][Geissbühler: 2011]

$$R^{ijk} = 3\left(\tilde{\partial}^{[i}\beta^{jk]} + \beta^{p[i}\partial_p\beta^{jk]}\right)$$

- Bianchi identity [Blumenhagen et al.: 2012]

$$\tilde{\nabla}^{[i}R^{jkl]} = 0$$

Summary

Double Field Theory allows for
a **geometric** interpretation of **non-geometric fluxes**.