Accelerating Universes in String Theory via Field Redefinition

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Contents

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1 Introduction — Models of inflation and no-go theorem

- **Check of superstring**

Circumstance where quantum effects of gravity are manifest

$\Rightarrow$ **Black hole (singularity)**
Early universe (singularity)

It is extremely urgent and important to study if these problems could be resolved within superstring and if there is any possibility that it gives realistic models

We focus on the early universe ———————————

- **Why inflation is necessary?**
  - **Horizon problem: homogeneity beyond causally connected region**
  - **flatness problem: the present universe is very flat, why?**

Inflation resolves these problems, producing scale invariant density perturbation, in agreement with observation

Moreover the expansion of the present universe is found to be accelerating! (late-time acceleration)

The correct theory of gravity must explain not only the early inflation but also the present accelerating expansion.

- **The first model of inflation:**
  - **positive cosmological constant (A. Guth, K. Sato, 1981)**
    scale factor in the FLRW universe expands exponentially
  - **higher order corrections such as R^2 (A.A. Starobinsky, 1980)**
    these also give similar expansion

Without introducing artificial potential, we would like to have this behavior as a prediction of the fundamental theory, the superstring.

However we find no-go theorem (Gibbons, ... and more)
If $D(>2)$-dimensional sugra is compactified on smooth manifold without boundary and

1. **gravitational interactions do not contain higher derivative terms than ordinary Einstein theory**
2. **all massless fields have positive kinetic term (not ghost)**
3. **d-dimensional Newton constant is finite,**

then we cannot obtain accelerating expansion.

How to avoid no-go theorem

- **additional dof such as D-branes ⇒ Dvali-Tye, KKLT, KKLMMT**
- **time-dependent internal space ⇒ S-brane**

  Alas, problem: obtained e-folding is around 2-3!!

  If we use hyperbolic space for our space, there is accelerating ever-expanding solution.

  Chen-Ho-Neupane-Ohta-Wang, JHEP 0310(2003) 058, hep-th/0306291, JHEP 0611(2006) 044, hep-th/0609043.

  It was found for $m \geq 6 \Rightarrow$ M-theory or string theory! $[(4+m)$ dims.]

$$a(\tau) = \tau + A\tau^{-\sqrt{(m-6)/(m+2)}}$$

- **higher order corrections existing in superstring/M-theory (like Starobinsky) ⇒ Maeda-Ohta '04 – '06**

K. Maeda and N. Ohta, PLB 597 (2004) 400, hep-th/0405205; PRD 71 (2005) 063520, hep-th/0411093; K. Akune, K. Maeda and N. Ohta, PRD 73 (2006) 103506, hep-th/0602242.

We find that there is a case in which we can obtain large e-folding.

Feature of general sols: size of internal space is larger than the Planck scale To obtain 60 e-folding, numerical analysis shows that we have

$$R_0 \sim 4000 m_{11}^{-1}$$

This is rather big (large extra dimensions).

$$m_4^2 = R_0^7 m_{11}^9 \quad \Rightarrow \quad m_{11} \sim 2 \times 10^{-13} m_4 \sim 600 \ \textbf{TeV}$$

If true, we may achieve the energy scale of quantum gravity near future. But it may depend on the initial condition.

- **scalar fields with negative kinetic terms ⇒ Phantom cosmology**
- **non-compact and/or with boundary ⇒ brane world**

Here we report other possibility of inflation due to higher order corrections.

2 Gauss-Bonnet Theory with dilaton (heterotic string) and density perturbation

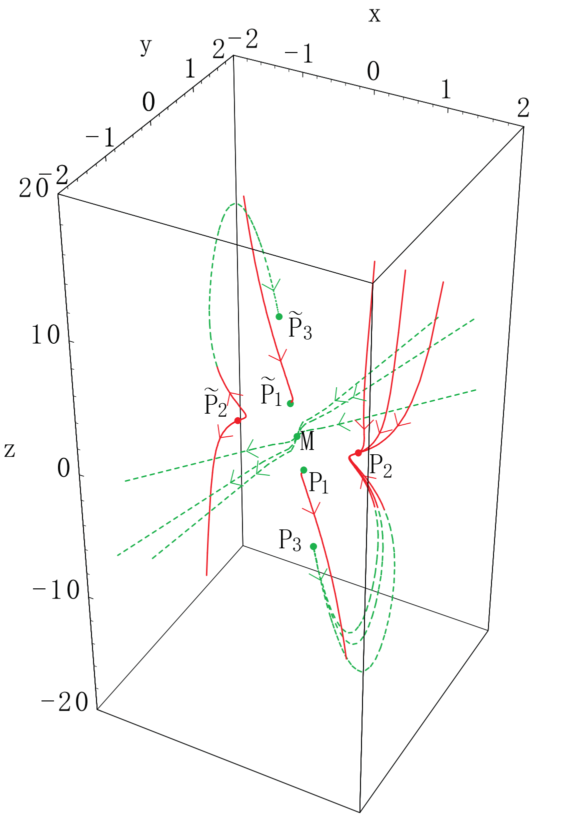
2.1 Solution space in a dilaton coupled GB theory

Similar solution in Gauss-Bonnet theory with dilaton

K. Bamba, Z.-K. Guo and N. Ohta, PTP 118 (2007) 879, arXiv:0707.4334 [hep-th].

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g}\, e^{-2\phi} \left[R + 4(\partial_\mu \phi)^2 + \alpha_2 R_{\rm GB}^2 \right],$$

Field equations gives autonomous system. Fixed points in time-evolution $\Rightarrow$ General solutions are those starting from one to another fixed points.



metric

$$ds_{10}^2 = -e^{2u_0(t)}dt^2 + e^{2u_1(t)}ds_3^2 + e^{2u_2(t)}ds_6^2,$$

Solution space with dilaton. solid (red) line for $d^2a/d\tau^2 > 0$, dotted (green) line for $d^2a/d\tau^2 < 0$
7 fixed points

$$(x, y, z) = \mathbf{M}(0, 0, 0), \quad \mathbf{P}_1(\mp 0.292373, \pm 0.36066, \pm 0.954846),$$
$$\mathbf{P}_2(\pm 0.91822, \mp 0.080285, \pm 0.585906),$$
$$\mathbf{P}_3(\pm 0.161307, \pm 0.161307, \mp 9.30437),$$

Only $\mathbf{P}_2$ gives accelerated expansion
But there is no inflationary expansion

2.2 Field redefinition ambiguity

K. -i. Maeda, N. Ohta and R. Wakebe, Eur. Phys. J. C 72 (2012) 1949 [arXiv:1111.3251 [hep-th]].

Quantum corrections in heterotic string **R.R. Metsaev and A.A. Tseytlin '87**

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-\tilde{g}}\, e^{-2\tilde{\phi}}\left[\tilde{R} + 4(\partial_\mu\tilde{\phi})^2 + \alpha_2\tilde{R}^2_{ABCD}\right],$$

$\Downarrow$ **ghosts**

$R^2_{\rm GB}$ **B. Zwiebach '85**

Ambiguity in the effective action due to field redefinition

$$g_{AB} \to g_{AB} + \delta g_{AB}, \qquad \phi \to \phi + \delta\phi$$

$$\begin{aligned}\delta g_{AB} &= \alpha_2[b_1 R_{AB} + b_2\nabla_A\phi\nabla_B\phi + g_{AB}\{b_3 R + b_4(\nabla\phi)^2 + b_5\nabla^2\phi\}] \\ \delta\phi &= \alpha_2[c_1 R + c_2(\nabla\phi)^2 + c_3\nabla^2\phi]\end{aligned}$$

$$\begin{aligned}S &= \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\, e^{-2\phi}\Big[R + 4(\partial_\mu\phi)^2 + \alpha_2\Big[R^2_{ABCD} + b_1 R^2_{AB} + (b_2+4b_1)R_{AB}\nabla^A\phi\nabla^B\phi \\ &+\left(2c_1 - \frac{1}{2}b_1 - \frac{D-2}{2}b_3\right)R^2 + \left(2c_2 - 8c_1 - \frac{b_2}{2} + 2Db_3 - \frac{D-2}{2}b_4\right)R(\nabla\phi)^2 \\ &+\left(2c_3 + 8c_1 - b_1 - 2(D-1)b_3 - \frac{D-2}{2}b_5\right)R(\nabla^1\phi) - 2(4c_2 - 2b_2 - Db_4)(\nabla\phi)^4 \\ &+(8c_2 - 8c_3 - 3b_2 - 2(D-1)b_4 + 2Db_5)\Box\phi(\nabla\phi)^2 + [8c_3 - 2(D-1)b_5](\Box\phi)^2\Big],\end{aligned}$$

Restrict the generalized effective action to the Galilean type

Second order derivatives in the eom (no ghost)

$$b_1 = -4, \quad b_5 = 4b_3, \quad c_1 = \frac{D-2}{4}b_3 - \frac{1}{2}, \quad c_2 = -2b_3 + \frac{D-2}{4}b_4 + 2, \quad c_3 = (D-1)b_3.$$

$$S = \frac{1}{2\kappa_D^2}\int d^D x\sqrt{-g}\, e^{-2\phi}\Big\{R + 4(\partial_\mu\phi)^2 + \alpha_2\Big[R_{\rm GB}^2 + \lambda(\nabla\phi)^4 + \mu\Big(R^{AB} - \frac{1}{2}Rg^{AB}\Big)\nabla_A\phi\nabla_B\phi + \nu\Box\phi(\nabla\phi)^2\Big]\Big\},$$

$$\lambda + 2(\mu+\nu) + 16 = 0$$

Two free parameters (μ and ν) from the freedom of field redefinition

With field redefinition ambiguity, the result changes much and de Sitter solutions possible

Certainly macroscopic objects (BH, the Universe) should not depend on field redefinition

It would be true if we know the complete action.

But we only know effective action only up to order α'

Our parametrization should cover the exact effective action, if any. So we should search for solutions allowing the ambiguity.

2.3 Results

$$ds_{10}^2 = -dt^2 + e^{2at} d\boldsymbol{x}_3^2 + e^{2bt} d\boldsymbol{y}_6^2, \qquad \phi = ct$$

$$\textbf{Einstein frame:} \qquad ds_{10}^2 = e^{-(6bt-2\phi)} ds_E^2 + e^{2bt} d\boldsymbol{y}_6^2.$$

$$ds_E^2 = e^{6bt-2\phi}(-dt^2 + e^{2at} d\boldsymbol{x}^2)$$

We find that there are one-parameter families of de Sitter solutions($b = 3c$).

case	fixed point (a,b,c)	$H = a$	ν
1. $[a = b]$	$(a, a, 3a)$	$\pm\sqrt{\dfrac{2}{9\mu + 160}}$	$-(3\mu + 32)$
2. $[2(1 + 4a^2 - 8ab$	$(a, -2.94771a, -8.84313a)$	$\pm\dfrac{0.159922}{\sqrt{\mu + 17.0724}}$	$-3.86891\mu - 45.4052$
$-80b^2) = 9b^2\mu)]$	$(a, 0.583777a, 1.75133a)$	$\pm\dfrac{0.807509}{\sqrt{\mu + 18.2148}}$	$-3.40790\mu - 39.2874$

There are other solutions of accelerating expansion.

2.4 Density perturbation

Inflation by higher order corrections can produce scale invariant one ?

(Z.-K. Guo, N. Ohta and S. Tsujikawa, PRD 75 (2007) 023520, astro-ph/0610336.)

Toy model: GB and dilaton higher derivative kinetic term

$$\mathcal{L}_c = -\frac{1}{2}\alpha' \xi(\phi)\left[c_1 R_{\rm GB}^2 + c_2 (\nabla\phi)^4\right], \quad \xi(\phi) = \lambda e^{\mu\phi} \ (\mu = \pm 1 \text{ at tree level.})$$

Typically $\lambda = -\frac{1}{4}, c_1 = 1, c_2 = -1$.

Our approach: no terms other than those from superstrings

Result: If GB is dominant, NG. If higher kinetic term is dominant, OK.

3 Conclusion

No-go theorem can be overcome.
Various models are possible, but fine tuning may be necessary.
In most models, quantum corrections seems to be important.

In particular, higher order corrections are important in the early universe, and they can give generalized de Sitter solutions if field redefinition ambiguity is taken into account. It can produce density perturbation.

Outlook:

- **graceful exit**
- **Reheating ⇒ coupling to matter**

KKLT need complicated and ad hoc settings. Does nature like that?
⇒ String Landscape, anthropic principle ?

How to derive 4-dim
We have 4-dim. as a solution, but it is not clear if it is natural. ⇒ 4-form in M-theory important?
We then need higher order corrections in 4-form?

We hope that our solutions have useful applications!!

Thank you for your attention!