Bundles over nearly-Kähler homogeneous spaces in heterotic string theory with flux

Michael Klaput (University of Oxford)

based on work togehter with Andre Lukas, Cyril Matti and Eirik Svanes

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Why Calabi-Yau?

Usual approach: look for a solution of the form

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- phenomenologically very attractive, recently several hundred standard models (spectrum and more) discovered, see [Anderson, Gray, Lukas, Palti; 1106.4804]

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$$\mathcal{M}_{10} = \underbrace{\mathcal{M}_3}_{\text{maximally symmetric}} \times \mathcal{M}_7 = \mathcal{M}_4 \oslash \mathcal{M}_6$$

now: $\frac{1}{2}$ BPS domain wall $\Rightarrow$ $\mathcal{M}_7$ has G_2 holonomy

$\Leftrightarrow$ $\mathcal{M}_6$ is half-flat

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+ differential calculations become group theoretical problems

+ additional group structure provides guideline $\rightarrow$ G-invariance

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- one-dimensional representations $\rho \Rightarrow$ line bundles

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Subtlety: SUSY + Bianchi does not automatically satisfy EOM.
In fact: EOM not satisfied, but non-vanishing terms are $\mathcal{O}(\alpha'^2/R^4)$

$\longrightarrow$ large radius limit

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$\rightarrow$ all Kähler moduli get fixed through torsion and H-flux
$\rightarrow$ dilaton can be fixed with non-perturbative effects
$\rightarrow$ consistency conditions work against each other, however, consistent parameter regions exist

(see Eirik Svanes' talk for more on moduli stabilization)

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- framework is very accessible to computations

$\rightarrow$ for example calculating thresholds

Thank you very much for your attention!