(NEW) BORDERS FOR QUANTUM COSMOLOGY

Standpoint and (some) SUSY

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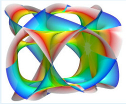
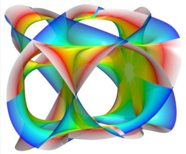
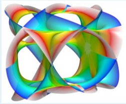
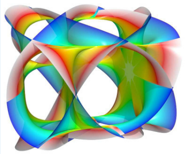
 

Abstract

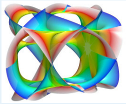
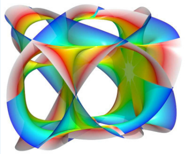
Quantum Cosmology tackles the quantum description of the early universe. It is aimed as an accessible primer that covers the basics, critically discussing ideas and concepts that comprise our current knowledge. The scope for analyzing quantum cosmological models within a supersymmetric framework is *pointed*.

As much as possible, it summarizes *what we know, what we think we know and what we think we do not know* on an equal footing. It is focused for ‘young’, inquisitive minds eager to embark on in-depth research in this field. It is hoped to suggest the tools researchers will need to go on their own, pushing them to ask the right questions rather than seek definitive answers.





N=2 SUSY

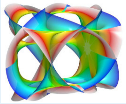
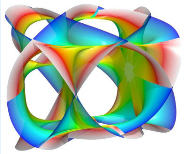
QC SUSY part-I

- Starting from
- $\rightsquigarrow$ bosonic configurations
- $\rightsquigarrow$ General Relativity **or** bosonic sector (strings)
- Setting:

$$ds^2 = -\mathcal{N}^2(t)dt^2 + h_{ij}(t)\omega^i\omega^j$$

$$h_{ij}(t) = \frac{1}{6\pi}e^{2\alpha(t)}\left[e^{2\beta}(t)\right]_{ij} = \frac{1}{6\pi}e^{2\beta^i}\delta_{ij},$$

$$\beta_{ij}(t) \equiv \mathrm{diag}(\beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+)$$

QC SUSY part-I

- Hamiltonian

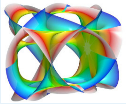
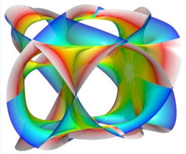
$$\begin{aligned}
\mathcal{H} &= \frac{1}{2}(-p_\alpha^2 + p_+^2 + p_-^2) + U^{(0)}(\alpha, \beta_+, \beta_-) \\
&= \frac{3}{2}\left[(p_1)^2 + (p_2)^2 + (p_3)^2 - 2p_1p_2 - 2p_1p_3 - 2p_2p_3\right] \\
&+ U^{(0)}(\beta^1, \beta^2, \beta^3), \qquad (1)
\end{aligned}$$

- $U^{(0)} \equiv -12\pi^2\, h^{(3)}R$

$$\text{type I}: \qquad U_I^{(0)} \equiv 0 \qquad (2)$$

$$\text{type II}: \qquad U_{II}^{(0)} \equiv \frac{1}{6}e^{4\alpha}e^{-8\beta_+} = \frac{1}{6}e^{4\beta^3}, \qquad (3)$$

$$\begin{aligned}
\text{type IX}: \qquad U_{IX}^{(0)} &\equiv \frac{1}{6}e^{4\alpha}\Big[2e^{4\beta_+}(\cosh(4\sqrt{3}\beta_-) - 1) + e^{-8\beta_+} \\
&-4e^{-2\beta_+}\cosh(2\sqrt{3}\beta_-)\Big] \\
&= \frac{1}{6}\Big[e^{4\beta^1} + e^{4\beta^2} + e^{4\beta^3} - 2e^{2\beta^1+2\beta^2}
\end{aligned}$$

QC SUSY part-I

- **Hidden** supersymmetry (SUSY)
- *Retrieved when* potential $U(q)$ is derivable from a *superpotential* $\mathsf{W}(q)$
- $$U(q) = \frac{1}{2}\mathcal{G}^{XY}(q)\frac{\partial \mathsf{W}(q)}{\partial q^X}\frac{\partial \mathsf{W}(q)}{\partial q^Y},$$
- $\rightsquigarrow$ Hamilton-Jacobi equation
- $\rightsquigarrow$ Euclidean time
- Euclidean action

QC SUSY part-I

- Superpotentials W
  - Bianchi type I:

$$\mathrm{W}_I \equiv 0;$$

  - Bianchi type II:

$$\mathrm{W}_{II} \equiv \frac{1}{6} e^{2\alpha - 4\beta_+} = \frac{1}{6} e^{2\beta^3};$$

  - Bianchi type IX:

$$\begin{aligned} \mathrm{W}_{IX}^{(0)} &\equiv \frac{1}{6} e^{2\alpha} \left[2e^{2\beta_+} \cosh 2\sqrt{3}\beta_- + e^{-4\beta_+} \right] \\ &= \frac{1}{6} \left(e^{2\beta^1} + e^{2\beta^2} + e^{2\beta^3} \right); \qquad (1) \end{aligned}$$

QC SUSY part-I

- Classical Hamiltonian

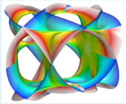
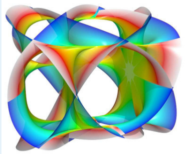
$$\mathcal{H}_c = \frac{1}{2}\mathcal{G}^{XY}(q)\left(p_X p_Y + \frac{\partial \mathrm{W}}{\partial q^X}\frac{\partial \mathrm{W}}{\partial q^Y}\right).$$

- *Quantum* Hamiltonian $\mathcal{H}$,

$$2\mathcal{H} = \tilde{\mathcal{S}}\mathcal{S} + \mathcal{S}\tilde{\mathcal{S}}$$

- where $\mathcal{S}$, $\tilde{\mathcal{S}}$ are linear operators satisfying

$$\mathcal{S}^2 = 0 = \tilde{\mathcal{S}}^2.$$

QC SUSY part-I

- The operators $\mathcal{S}$ and $\tilde{\mathcal{S}}$ have the explicit form

$$\mathcal{S} \equiv \psi^x {e_x}^Y(q)\left(\pi_Y + i\frac{\partial \mathrm{W}}{\partial q^Y}\right) \quad (1)$$

$$\tilde{\mathcal{S}} \equiv \bar{\psi}_x e^{xY}(q)\left(\pi_Y - i\frac{\partial \mathrm{W}}{\partial q^Y}\right), \quad (2)$$

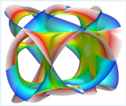
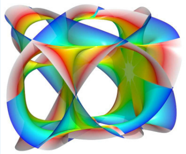
- ${e_x}^Y(q) \equiv e^{-\widetilde{\Omega}}\delta_x^Y$ is the minisuperspace *vielbein* associated with $\mathcal{G}^{XY}(q)$
- They satisfy ${e_x}^Y(q){e_y}^X(q)\eta^{xy} \equiv \mathcal{G}^{XY}(q)$,

QC SUSY part-I

- η^{xy} is the *local* 'Lorentz' metric at the minisuperspace *tangent section*.
- The ψ^x and their adjoint $\bar{\psi}_x$ are fermionic operators,
- $\rightsquigarrow$ constituting Grassmannian (odd) partners of the q^X,
- **Therefore**: introducing fermionic (!) minisuperspace degrees of freedom and enlarging the (minisuperspace) configuration space,
- satisfying

$$\{\psi^x, \psi^y\} = 0 = \{\bar{\psi}_x, \bar{\psi}_y\} \quad (1)$$

$$\{\psi^x, \bar{\psi}_y\} = \delta^x_y . \quad (2)$$

QC SUSY part-I

- Hamiltonian

$$\mathcal{H} = -\frac{\hbar^2}{2}\mathcal{G}^{XY}\frac{\partial}{\partial q^Y}\frac{\partial}{\partial q^X} + \frac{1}{2}\mathcal{G}^{YX}\frac{\partial \mathsf{W}}{\partial q^Y}\frac{\partial \mathsf{W}}{\partial q^X} + \frac{\hbar}{2}{e_x}^Y {e_y}^X \frac{\partial^2 \mathsf{W}}{\partial q^Y \partial q^X}[\bar{\psi}^x, \psi^y] \; ;$$

- Fermion number

$$\mathcal{F} \equiv \bar{\psi}_x \psi^x$$

- Conserved:
-

$$[\mathcal{H}, \mathcal{F}] = 0, [\mathcal{S}, \mathcal{F}] = \mathcal{S}, [\tilde{\mathcal{S}}, \mathcal{F}] = -\tilde{\mathcal{S}}.$$

SQC and SUGRA

SQC SUGRA

- Important "section"...
- ... reduction of 4-dimensional canonical quantum $N = 1$ SUGRA
- E.g., **FRW models:**
- $\rightsquigarrow$ simplest
- $\rightsquigarrow$ 'bottom-up'
- *(prudent) approach on* (i) how *SQC research should proceeded,* (ii) *methods that are to be applied and* (iii) *the results retrieved: general? Useful?*
- FRW Minisuperspace Reduction *Ansätse*: Consistent with
- $\rightsquigarrow$ supersymmetry,
- $\rightsquigarrow$ Lorentz
- $\rightsquigarrow$ general coordinate transformations,
- Purpose: minisuperspace that will inherit invariance under local time translations, supersymmetry and Lorentz transformation.

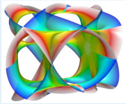
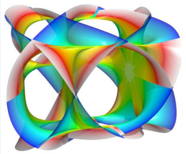
SQC SUGRA

- **Tetrad and Gravitino Ansätze**:
- Closed ($k = +1$) FRW universes have S^3 spatial sections.
- Tetrad $e_\mu^{AA'} = e_\mu^a \sigma_a^{AA'}$ (in 2-spinor notation)

$$e_{a\mu} = \begin{bmatrix} \mathcal{N}(\tau) & 0 \\ 0 & a(\tau) e_{\hat{a}i} \end{bmatrix} , \; e^{a\mu} = \begin{bmatrix} \mathcal{N}(\tau)^{-1} & 0 \\ 0 & a(\tau)^{-1} e^{\hat{a}i} \end{bmatrix} ,$$

- For $\psi^A{}_\mu$ and $\bar{\psi}^{A'}{}_\mu$
- $\rightsquigarrow$ $\psi^A{}_0$ and $\bar{\psi}^{A'}{}_0$ to be functions of time only.
- Furthermore

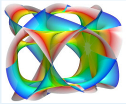
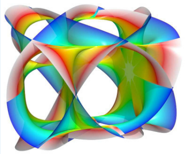
$$\psi^A{}_i = e^{AA'}{}_i \bar{\psi}_{A'} \; , \; \bar{\psi}^{A'}{}_i = e^{AA'}{}_i \psi_A \; ,$$

SQC SUGRA

- Action:
- $\rightsquigarrow$ integration over the spatial hypersurfaces
- $\rightsquigarrow$ minisuperspace with $N = 4$ *local* supersymmetry and time-invariance reparametrization
- (obvious!) *Elimination* of (the presence) of *spatial indices by contraction*.
-

$$
\begin{aligned}
L_{free} \quad = \quad & -\frac{3a}{\mathcal{N}}\left[\dot{a} - \frac{i}{4}\left(\bar{\psi}_{0A'}\bar{\psi}^{A'} - \psi^{A}\psi_{0A}\right)\right]^2 \\
& -i\frac{3}{2}a^3 n_{AA'}\psi^{A}\bar{\psi}^{A} - i\frac{3}{2}a^3 n_{AA'}\bar{\psi}^{A}\dot{\psi}^{A} + 3\mathcal{N}a \\
& +\frac{3}{2}a^2\left(\bar{\psi}_{0A'}\bar{\psi}^{A'} + \psi^{A}\psi_{0A}\right) - \frac{3}{2}\mathcal{N}a^2 n^{AA'}\psi_{A}\bar{\psi}_{A'} \\
& +\frac{3}{16}a^3 n^{AA'}\left(\bar{\psi}_{0A'}\psi_{A}\bar{\psi}^{B'}\bar{\psi}_{B'} + \psi_{0A}\bar{\psi}_{A'}\psi^{B}\psi_{B}\right) \quad (2)
\end{aligned}
$$

SQC SUGRA

- Hamiltonian

$$H = \mathcal{N}\mathcal{H} + \psi_0^A \mathcal{S}_A + \bar{\mathcal{S}}_{A'} \bar{\psi}_0^{A'} + \mathcal{M}^{AB} \mathcal{J}_{AB} + \bar{\mathcal{M}}^{A'B'} \bar{\mathcal{J}}_{A'B'},$$

- $\psi_0^A, \bar{\psi}_0^{A'}$ together with $\mathcal{M}^{AB}, \bar{\mathcal{M}}^{A'B'}$ are Lagrange multipliers.
- $\mathcal{H}_\perp \equiv \mathcal{H}$ Hamiltonian constraint, $\mathcal{S}_A, \bar{\mathcal{S}}_{A'}$ and $\mathcal{J}_{AB}, \bar{\mathcal{J}}_{A'B'}$ denote, respectively, the supersymmetry and Lorentz constraints
- Full set of constraints takes a rather *simple* form

$$\mathcal{S}_A = \psi_A \pi_a - 6ia\psi_a, \qquad (2)$$
$$\mathcal{H} = -a^{-1}(\pi_a^2 + 36a^2), \qquad (3)$$
$$\bar{\mathcal{S}}_{A'} = \bar{\psi}_{A'} \pi_a + 6ia\bar{\psi}_{A'}, \qquad (4)$$
$$\mathcal{J}_{AB} = \psi_{(A} \bar{\psi}^{B'} n_{B)B'} \ . \qquad (5)$$

SQC SUGRA

- E.g., Quantum States for the Vacuum Case
- Momentum variables are represented as

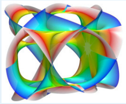
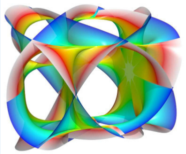
$$\bar{\psi}_A \to \frac{\partial}{\partial \psi^A}, \quad \pi_a \to -i\frac{\partial}{\partial a}.$$

- SUSY constraints:

$$\begin{aligned} \mathcal{S}_A &= -\frac{1}{2\sqrt{3}} a\psi_A \frac{\partial}{\partial a} - \sqrt{3}a^2\psi_A, \\ \bar{\mathcal{S}}_A &= \frac{1}{2\sqrt{3}} a \frac{\partial}{\partial a}\frac{\partial}{\partial \psi^A} - \sqrt{3}a^2 \frac{\partial}{\partial \psi^A}. \end{aligned} \tag{2}$$

- Lorentz:

$$\mathcal{J}_{AB} = \psi_{(A}\frac{\partial}{\partial \psi^{B)}}.$$

SQC SUGRA

- Wave function:

$$\Psi^{FRW}_{SUSY} = \mathsf{A}(a) + \mathsf{B}(a)\psi^F\psi_F. \qquad (2)$$

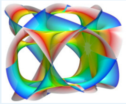
- Equations to solve: $\mathcal{S}_A\Psi = 0$, $\bar{\mathcal{S}}_A\Psi = 0$

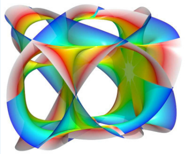
$$\frac{a}{\sqrt{3}}\partial_a \mathsf{B} - 2\sqrt{3}a^2\mathsf{B} = 0, \; \frac{a}{\sqrt{3}}\partial_a \mathsf{A} + 2\sqrt{3}a^2\mathsf{A} = 0.$$

- Solutions are

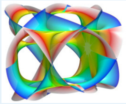
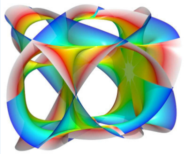
$$\Psi = \mathsf{A}_0 \exp[-3a^2/\hbar] + \mathsf{B}_0 \exp[3a^2/\hbar]\psi_A\psi^A \;,$$

- A_0 and B_0 are independent of a and ψ^A.
- Semi-classical interpretation as $\exp(-I/\hbar)$
- I.e., we get a Hartle–Hawking solution for $\mathsf{B} = 0$



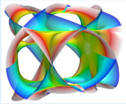


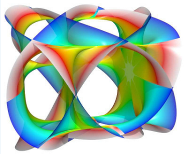
Can it be real?!

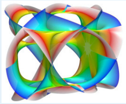
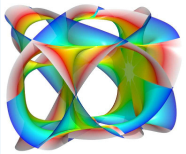
Semiclassical Expansion

- ... *if* (and *how*) our physical (observed) universe can be retrieved from SQC physical states





For Research...

Outlook yet again...

- So,
  1. How can the ‘hidden’ $N = 2$ SUSY be retrieved?
  2. And what about when (super)matter includes a scalar field or a Yang–Mills sector?
  3. *How* precisely can we implement a FRW SQC (minisuperspace) from $N = 1$ SUGRA?
  4. *How* can scalar (super)matter be included?
  5. Why is the case of Bianchi models with a cosmological constant still an issue?
  6. How does the DeWitt supermetric relates to the usual DeWitt metric?
  7. What are the main features of SUGRA (quantum) corrections into the Schrödinger equation?
  8. Can SQC become ‘observational’?