$SO(10)$ Resolutions in F-theory

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$SU(5)$ GUTs in F-theory

- Compactify on a Calabi-Yau 4-fold Y_4 which is an elliptic fibration over a base B_3 with various singularities
- Codimension 1 A_4 singularity gives GUT surface with $SU(5)$ gauge group
- Matter comes from codimension 2 singularity enhancements, $\bar{\mathbf{5}}$ from A_5 ($SU(6)$) enhancement, $\mathbf{10}$ from D_5 ($SO(10)$) enhancement
- Yukawa interactions at codimension 3 enhancements.
  $\mathbf{10} \times \bar{\mathbf{5}} \times \bar{\mathbf{5}}$ Yukawa from D_6 ($SO(12)$) enhancement.
  $\mathbf{10} \times \mathbf{10} \times \mathbf{5}$ Yukawa from E_6 enhancement

Problem?

- `1107.0733` - Esole and Yau
- If we do enhancement and resolution of SU(5) explicitly, we don't get an E_6 enhancement point where we expect to, we instead get a singularity which corresponds to the Dynkin diagram of E_6 with a node missing

Solution

- `1108.1794` - Marsano and Schäfer-Nameki
- Enhancements do not give a higher gauge group, but give new states in $SU(5)$
- So think in terms of $SU(5)$ root lattice instead of a larger one
- Get structure of the $\mathbf{10} \times \mathbf{10} \times \mathbf{5}$ Yukawa, despite having one less component

$SO(10)$ GUTs in F-theory

- D_5 singularity in codimension 1
- **16** from E_6 enhancement
- **10** from D_6 enhancement
- $\mathbf{16} \times \mathbf{16} \times \mathbf{10}$ Yukawa from E_7 enhancement

Initial resolution

- Start with Tate form for elliptic fibration Y_4 over base B_3 with an $SO(10)$ singularity at $z = 0$:

$$wy^2 + b_1 wxyz + b_3 w^2 yz^2 = x^3 + b_2 wx^2 z + b_4 w^2 xz^3 + b_6 w^3 z^5$$

$$(w, x, y) \in \mathbb{P}^2, \qquad b_i, z \text{ functions on } B_3$$

- We then resolve the singularity by performing blow ups

- $z = 0$ splits into six components, intersecting these with each other gives the negative Cartan matrix of $SO(10)$ (extended)

Initial resolution continued

- $z = 0$ splits into six components (Cartan divisors), intersecting these with each other gives the negative Cartan matrix of $SO(10)$ (extended):

$$\begin{pmatrix} -2 & 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 \\ 1 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2 \end{pmatrix}$$

Matter

- Get matter in the **16** at $z = b_2 = 0$, here one of the Cartan divisors becomes reducible:

$$(0,1,-2,1,1) \to (-2,1,0,0,0) + (1,0,-1,1,0) + (1,0,-1,0,1)$$

$(1,0,-1,1,0)$ and $(1,0,-1,0,1)$ are both weight vectors of the **16**.

- Matter in the **10** we get at $z = b_3 = 0$, here we have another splitting of a Cartan divisor:

$$(0,0,1,-2,0) \to (0,0,1,0,-2) + (0,0,0,-1,1) + (0,0,0,-1,1)$$

where $(0,0,0,-1,1)$ is a weight vector of the **10**

Yukawa

- The $\mathbf{16} \times \mathbf{16} \times \mathbf{10}$ Yukawa interaction is located at $z = b_2 = b_3 = 0$
- If we start with from the **16** matter curve $z = b_2 = 0$ and then set $b_3 = 0$, we see the component with weight vector
$$(1, 0, -1, 0, 1)$$
becomes reducible, and splits into two components
$$(1, 0, -1, 1, 0) + (0, 0, 0, -1, 1)$$
- These last two weight vectors correspond to **16** and **10** respectively
- Only 7 components altogether, E_7 is expected to have 8

G-flux

- Quantization condition:

$$G + \frac{1}{2} c_2 \left(\tilde{Y}_4 \right) \in H^4(\tilde{Y}_4, \mathbb{Z})$$

- This condition cannot be satified without imposing additional constraints on the base B_3 or the GUT surface S_2

- `1205.5688` - Küntzler and Schäfer-Nameki, demonstrated why this is the case by considering local flux

Conclusions and Outlook

- $SO(10)$ has similar to $SU(5)$ - node missing from Dynkin diagram of singularity at Yukawa enhancement
- G-flux quantization condition requires extra constraints
- Would be interesting to see if any of these additional constraints could be derived rather than imposed