# Anomalies and Remnant Symmetries in Heterotic Constructions

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PRD **85** [arXiv:1203.5789] and work in progress

# Motivation

- MSSM superpotential contains (potentially) bad terms:

$$W_{\text{bad}} \supset \mu H_u H_d + QLd^c + u^c d^c d^c + LLe^c + QQQL + u^c u^c d^c e^c + \cdots$$

- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, $\mathbb{Z}_4^R$,...) [many people here]

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- Forbid/constrain these operators by (discrete) symmetries, (matter parity, proton hexality, $\mathbb{Z}_4^R$,...) [many people here]
- Anomaly constraints on such symmetries (bottom-up) from consistency and grand unification?
- Appearance of discrete symmetries as remnants of gauge or Lorentz symmetries in string constructions?
- Simple example: Consider heterotic $\mathbb{Z}_3$ orbifold, its blowup and GLSM realisation

# Contents

# Green–Schwarz Mechanism

Requires [Green, Schwarz '84]

a) factorisation of anomaly polynomial, $I_6 = X_4 Y_2$, i.e. $Y_2 = F_2 = \mathrm{d}A_1$ ("anomalous $U(1)$")
b) axion field $a$ with shift gauge transformation

$$\delta a = -\lambda \qquad \delta A_1 = \mathrm{d}\lambda$$

c) axion coupling to $X_4$

$$S_{\mathrm{GS}} = \int \frac{1}{2} \left|\mathrm{d}a + A_1\right|^2 + a\, X_4 \quad \Longrightarrow \quad \delta S_{\mathrm{GS}} = -\int I_4^{(1)} = -\mathcal{A}$$

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Shift & coupling ⇒ Anomaly cancellation

Sum of factorised anomalies $I_6 = \sum_a Y^a X^a$: Cancelled by set of axions

# Green–Schwarz Mechanism: Axion or 2-Form

- Dual description

$$\begin{array}{rcl} \text{axion } a & \leftrightarrow & \text{2-form } C_2 \\ \text{shift } \delta a \sim Y_2 & \leftrightarrow & \text{coupling } C_2 \wedge Y_2 \\ \text{coupling } a\, X_4 & \leftrightarrow & \text{shift } \delta C_2 \sim X_4 \end{array}$$

- In particular: Kalb–Ramond $B_2$ dual to "universal axion" $a_0$
  $\rightsquigarrow$ always present in heterotic compactifications

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- In particular: Kalb–Ramond $B_2$ dual to "universal axion" $a_0$
$\rightsquigarrow$ always present in heterotic compactifications
- $X_4$ contains field strengths of other gauge group factors $G_i$, weighted with (arbitrary) anomaly coefficients:

$$X_4 = A_{\text{grav}-U(1)} \operatorname{tr} R^2 + \sum_i A_{G_i^2 - U(1)_A} \operatorname{tr} F_i^2$$

- Universal axion $a_0$ couples universally via $X_4^{\text{uni}}$ (fixed by transformation of $B_2$) – this is a special case!

# Anomaly Universality from Underlying $SU(5)$?

Take $SU(5) \times U(1)_X$, flavour-blind matter charges, $R$ or non-$R$ symmetry

| Matter | gauginos | | Higgses | |
|---|---|---|---|---|
| $\bar{\mathbf{5}}, \mathbf{10}$ | SM | $X, Y$ | $\mathbf{3}$'s | $\mathbf{2}$'s |
| $A_5 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$ | $+5R$ | | $+C_H$ | |
| $A_3 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$ | $+3R$ | $+2R$ | $+C_H$ | |
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| $A_1 = \frac{3}{2}\left(3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} - 4R\right)$ | | $+5R$ | $+\frac{2}{5}C_H$ | $+\frac{3}{5}C_H$ |

$\left(C_H = \frac{1}{2}\left(q_{H_u} + q_{H_d} - 2R\right)\right)$

- Breaking by (discrete) orbifold twists of fluxes changes zero mode spectrum

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- doublet-triplet splitting $\rightsquigarrow$ generic non-universality

# Implications for $G_{\mathsf{SM}} \times \mathbb{Z}_N$

- $\mathbb{Z}_N^2$ or $\mathbb{Z}_N^3$ anomalies not meaningful

  [Ibanez, Ross '91; Banks, Dine '92; Araki et al. '08]
- No need for $\mathbb{Z}_N - G^2$ anomalies to be universal
- Less restrictions on phenomenologically interesting symmetries
- E.g. non-$R$ $\mathbb{Z}_6^X$ with

  $$q_{\mathbf{10}} = 1\,, \qquad q_{\bar{\mathbf{5}}} = 5\,, \qquad q_{H_u} = 4\,, \qquad q_{H_d} = 0$$

  allows Yukawas and Weinberg operator, forbids $\mu$ term and $\not{B}$, $\not{L}$ operators (up to dimension five)
- In string models, possibly remnants of universality exist

# $T^6/\mathbb{Z}_3$ Orbifold and its Blowup

For illustration, consider simple $T^6/\mathbb{Z}_3$ orbifold model:
$V = \frac{1}{3}\left(1,1,-2,0^5\right)\left(0^8\right)$, no Wilson lines

| | |
|---|---|
| Gauge group | $E_6 \times SU(3)\,[\times E_8]$ |
| spectrum | $3\left(\mathbf{27},\overline{\mathbf{3}}\right) + 27\left[(\mathbf{27},\mathbf{1}) + 3\,(\mathbf{1},\mathbf{3})\right]$ |

$\Rightarrow$ No anomalous $U(1)$, i.e. universal axion does not shift, no FI term

Blowup: Smooth out geometry

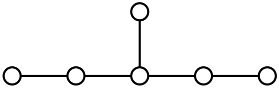
- Heuristically, VEVs for twisted states ("blow-up modes") $\leftrightarrow$ volumes of resolution cycles
- All volumes large $\rightsquigarrow$ description in supergravity regime, i.e. compactification on smooth CY with gauge bundle
- Gauge bundle data $\leftrightarrow$ representations of blow-up modes

# Blowup as CY with Bundle

[Groot Nibbelink et al. 07-09]

Toric resolution, replace singularities by exceptional divisors ($\mathbb{P}^2$s) $E_r$
Line bundles, i.e. Abelian fluxes given by bundle vectors:

$$\mathcal{F} = V_r^I H_I E_r$$



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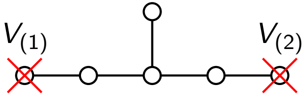


$\Rightarrow$ gauge group $SO(8) \times U(1)_A \times U(1)_B \times SU(3)$

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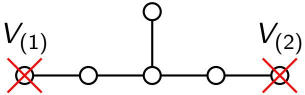


$\Rightarrow$ gauge group $SO(8) \times U(1)_A \times U(1)_B \times SU(3)$

Massless spectrum depends on distribution of flux among exceptional divisors (given by $(k, p, q)$ with $k + p + q = 27$)

Flux gauges shift symmetries of new axions $\Rightarrow$ both $U(1)$'s are massive, even though flux is Abelian

# Blowup Anomalies

$\Rightarrow$ anomaly polynomial $I_6 = \int_X I_{12}$ with backgrounds inserted, or from triangle diagrams

$$\Rightarrow I_6 \sim {F_A}^3 \cdot \left(\frac{k-6}{12}\right) + F_A F_B^2 \cdot \left(\frac{k-18}{4}\right)$$
$$+ F_A \left[\operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{k-9}{2}\right)$$
$$+ F_B \left[\frac{1}{8} F_B^2 + \frac{1}{48} F_A^2 + \operatorname{tr} F_{SU(3)}^2 + \operatorname{tr} F_{SO(8)}^2 + \frac{7}{48} \operatorname{tr} R^2\right] \cdot \left(\frac{p-q}{2}\right)$$

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- For $p = q$, $U(1)_B$ is omalous, while $U(1)_A$ is always anomalous

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- For $p = q$, $U(1)_B$ is omalous, while $U(1)_A$ is always anomalous
- Remnant universality among non-Abelian groups from one $E_8$

# Remnant Gauge Symmetry

Remnant non-$R$ symmetries: discrete subgroup of $U(1)_A \times U(1)_B$ which leaves VEVs invariant
Blow-up modes:

$$\mathbf{1}_{4,0}\,, \quad \mathbf{1}_{-2,-2}\,, \quad \mathbf{1}_{-2,2}$$

$\Rightarrow$ discrete remnant $\mathbb{Z}_4 \times \mathbb{Z}_4$, generated by

$$T_\pm : \phi_{(q_A,q_B)} \longrightarrow \exp\left\{\frac{2\pi \mathrm{i}}{4}\left(q_A \pm q_B\right)\right\}\phi_{(q_A,q_B)}$$

However: Charges of all massless fields are even under both $\mathbb{Z}_4$s
$\rightsquigarrow$ only $\mathbb{Z}_2 \times \mathbb{Z}_2$ realised on massless spectrum

Both $\mathbb{Z}_2$ factors are omalous

# $R$ Symmetries from Orbifolds

$R$ transformations from sublattice rotations act as

$$\mathcal{R} : \Phi \longrightarrow e^{2\pi \mathrm{i} v R} \Phi$$

where $v = \left( \underline{\frac{1}{3}, 0, 0} \right)$, and our blow-up modes have
$R = q_{\text{sh}} - \Delta N = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$

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Symmetry conventions somewhat tricky – upshot: $\mathbb{Z}_{18}$ symmetry with charges for

$$(\text{bosons}, \text{fermions}, \theta) = \frac{1}{18}\,(2n, 2n-3, 3)$$

Remnant $R$ symmetries: unbroken combinations of the three sublattice rotations and $U(1)_{A,B}$

$\Rightarrow$ only a trivial $\mathbb{Z}_2$ $R$ symmetry survives in blow-up

# Model: GLSM Description

[Witten '93;Groot Nibbelink '10; Blaszczyk et al. '11]

[See talk by Fabian Ruehle on Thursday]

Algebraically, describe the orbifold by ($\mathbb{P}^2[3]$ is a $T^2$)

$$\frac{\mathbb{P}^2[3] \times \mathbb{P}^2[3] \times \mathbb{P}^2[3]}{\mathbb{Z}_3}$$

Blowup (crepant resolution) in $(0,2)$ GLSM description:

- 2D SUSY field theory with $U(1)$ symmetries, fields $\sim$ coordinates – flows to worldsheet CFT in IR
- Geometry cut out by $F$ and $D$ term equations, GLSM FI terms become CY Kähler parameters
- To resolve singularities, introduce extra coordinates (exceptional divisors) and $U(1)$s
- Gauge bundle given by "chiral-Fermi" superfields $\Lambda_I$ related to bundle vectors, i.e. fluxes

# $R$ Symmetries in the GLSM

Set of $F$ and $D$ terms fixes geometry.
$\exists$ discrete transformations of the fields which leave $F$ and $D$ terms invariant
$\rightsquigarrow$ $R$ symmetries if holomorphic three-form $\Omega$ transforms [Witten 85]

$$\Omega \sim \eta^T \Gamma_{ijk} \eta \quad \Rightarrow \quad Q_R(\Omega) = Q_R(W) = 2$$

Different types of $R$ symmetries:

- Phases $z \to e^{2\pi \mathrm{i}/3} z$: always possible (but see next slide)
  $\rightsquigarrow$ $\mathbb{Z}_6$ $R$ symmetries
- Permutations of fields: Only possible for special values of Kähler parameters – corresponds to groupwise exchange of exceptional divisors

# $R$ Symmetry breaking in GLSM

$\mathbb{P}^2$ coordinates $z_{i\alpha}$ only appear as $z_{i\alpha}^3$ or $|z_{i\alpha}|^2$
$\Rightarrow$ unbreakable $\mathbb{Z}_3$ rotations?

(Presumably) broken by marginal deformations of Kähler potential terms in presence of gauge bundles ($\rightsquigarrow$ massless charged 4D matter)

[Groot Nibbelink '10]

$$\int \mathrm{d}^2\theta^+ \phi_{4\mathrm{d}}(x^\mu) N(z) \qquad \underbrace{\Lambda\bar{\Lambda}}_{\text{gauge bundle fields}}$$

Fits with orbifold: Bundle corresponds to blowup
Presence of deformations depends on the geometric phase – controlled by Kähler (FI) parameters which may force certain fields to vanish

$\Rightarrow$ Generically, no $R$ symmetry in blow-up (all FI terms large), but enhanced at certain loci of parameter space

# Conclusions

- Anomalies are generically not universal: Not required for anomaly cancellation, not generic from unification
- Orbifold anomalous $U(1)$ is the exception because it is cancelled purely by the universal axion
- For generic string compactifications, many axions possible
- Non-$R$ symmetries can forbid the $\mu$ term

# Conclusions

- Anomalies are generically not universal: Not required for anomaly cancellation, not generic from unification
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- For generic string compactifications, many axions possible
- Non-$R$ symmetries can forbid the $\mu$ term
- Line bundles do reduce the rank via the axion shift – also omalous $U(1)$s can become massive
- Blow-ups can leave gauged discrete subgroups unbroken – important for phenomenology
- On orbifold, $R$ symmetries exist but are broken by the blow-up

# Outlook

- "Geometry part" of GLSM generically has many "unbreakable" $R$-like symmetries – seem to be broken by the gauge bundle, but better understanding of their breaking required
- Linked to determination of charged massless spectrum
- Non-generic type of $R$ symmetries: Exchange symmetries, appearing for certain loci in Kähler moduli space, e.g. exchange of exceptional divisors if their volumes are equal
- Study these symmetries for more realistic models, including Wilson lines etc.
- Orbifold blowups are only a small part of CY landscape...

# Backup: Discrete Symmetries and Higher Anomalies

[Ibanez, Ross; Banks, Dine; Araki; Araki et al.]

Discrete (finite) symmetries have some subtleties (as compared to $U(1)$s):

- Charges defined $\mod N$
- (Majorana) mass terms possible for charged states, so anomaly could be influenced by heavy states
- Path integral Jacobian involves field strengths

$\Rightarrow$ only linear $\mathbb{Z}_N$ anomalies well-defined

$\Rightarrow$ for non-Abelian discrete groups, only $\mathbb{Z}_N$ subgroups relevant

# Backup: Table of Operators and Charges

| Operator | | | | |
|---|---|---|---|---|
| $(LH_u)^2$ | | | | |
| $H_u H_d$ | | | | |
| $LH_u$ | | | | |
| $\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}$ | | | | |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\overline{\mathbf{5}}$ | | | | |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10} H_d$ | | | | |
| $LH_u H_d H_u$ | | | | |

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| Operator | SM Yukawas | Weinberg Op. | | |
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| $(LH_u)^2$ | $4R - 4q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$ | $2R$ | | |
| $H_u H_d$ | $4R - 3q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$ | $5R - 5q_{\mathbf{10}} + k\frac{N}{2}$ | | |
| $LH_u$ | $2R - 2q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$ | $R + k\frac{N}{2}$ | | |
| $\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$ | $q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$ | $-2R + 5q_{\mathbf{10}}$ | | |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$ | $3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$ | $-R + 5q_{\mathbf{10}} + k\frac{N}{2}$ | | |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$ | $4R + 2q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$ | $3R + k\frac{N}{2}$ | | |
| $LH_u H_d H_u$ | $6R - 5q_{\mathbf{10}}$ | $6R - 5q_{\mathbf{10}}$ | | |

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| $LH_u$ | $2R - 2q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$ | $R + k\frac{N}{2}$ | 3 |
| $\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$ | $q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$ | $-2R + 5q_{\mathbf{10}}$ | 5 |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$ | $3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$ | $-R + 5q_{\mathbf{10}} + k\frac{N}{2}$ | 2 |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$ | $4R + 2q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$ | $3R + k\frac{N}{2}$ | 3 |
| $LH_u H_d H_u$ | $6R - 5q_{\mathbf{10}}$ | $6R - 5q_{\mathbf{10}}$ | 1 |

$\rightsquigarrow$ forbid all bad terms e.g. by non-$R$ $\mathbb{Z}_6^X$ with

$$q_{\mathbf{10}} = 1\,, \qquad q_{\bar{\mathbf{5}}} = 5\,, \qquad q_{H_u} = 4\,, \qquad q_{H_d} = 0$$

$\mathbb{Z}_2$ matter parity as subgroup

# Backup: Table of Operators and Charges

| Operator | SM Yukawas | Weinberg Op. | $\mathbb{Z}_6^X$ | $SO(10)$ |
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| $H_u H_d$ | $4R - 3q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$ | $5R - 5q_{\mathbf{10}} + k\frac{N}{2}$ | 4 | 0 |
| $LH_u$ | $2R - 2q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$ | $R + k\frac{N}{2}$ | 3 | $R$ |
| $\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}$ | $q_{\mathbf{10}} + 2q_{\bar{\mathbf{5}}}$ | $-2R + 5q_{\mathbf{10}}$ | 5 | $3R$ |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$ | $3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}}$ | $-R + 5q_{\mathbf{10}} + k\frac{N}{2}$ | 2 | $4R$ |
| $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,H_d$ | $4R + 2q_{\mathbf{10}} - q_{\bar{\mathbf{5}}}$ | $3R + k\frac{N}{2}$ | 3 | $3R$ |
| $LH_u H_d H_u$ | $6R - 5q_{\mathbf{10}}$ | $6R - 5q_{\mathbf{10}}$ | 1 | $R$ |

$\rightsquigarrow$ forbid all bad terms e.g. by non-$R$ $\mathbb{Z}_6^X$ with

$$q_{\mathbf{10}} = 1\,, \qquad q_{\bar{\mathbf{5}}} = 5\,, \qquad q_{H_u} = 4\,, \qquad q_{H_d} = 0$$

$\mathbb{Z}_2$ matter parity as subgroup

Requiring $SO(10)$ relations for matter, we need $R = 1$

# Backup: Group Theory Issues in Blowup

[Groot Nibbelink et al.; Blaszczyk et al.]

- Orbifold picture: Switch on VEVs of twisted states – Higgs-type breaking of gauge symmetries, including rank reduction
- Smooth CY: Abelian gauge bundle does not reduce rank
- Solution: Flux creates axion shifts which break $U(1)$s Stueckelberg-style – axions related to phases of VEVved states ("twisted axions")
- Matching of states involves field redefinitions – relates VEVs to anomalies

- Orbifold blowups rare among CYs $\rightsquigarrow$ generically, one cannot "redefine away" anomalies into orbifold $\oplus$ VEV
- Blowup CYs: largish $h^{1,1} \sim \mathcal{O}(20-50)$, small $h^{2,1} \sim$ (none – a few)