One-Loop Calculation of the *S* Parameter in Higgsless EW Models

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A. Pich, I. Rosell and JJ SC [arXiv:1206.3454 [hep-ph]]

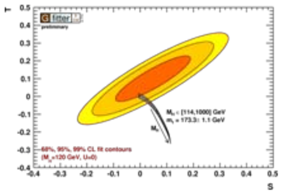
OUTLINE

1) Motivation: EWSB and S-parameter
2) Effective Lagrangian, $\Pi_{30}(s)$ and computation of S
3) High-energy Constraints
4) Phenomenology

Motivation:
EWSB and S-parameter

What?

One-loop calculation of the oblique S parameter within Higgsless models of EWSB*



Why?

What if there is no Higgs?

$M_H \in$ [117.5,118.5] GeV U [122.5,127.5] GeV (?)**

✓ Alternative ways of mass generation?
strongly-coupled higgsless models

How?

Effective approach

a) EWSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$: similar to ChSB in QCD → ChPT ***
b) Strongly-coupled Higgsless models: Resonances like in QCD → RChT (+)
c) Lagrangian with at most two derivatives + short-distance information (+)

+

Dispersive representation from Peskin and Takeuchi'92

* Gfitter
* LEP EWWG
* Zfitter

** Prelimiminary CMS, ATLAS, CDF and D0 Collaborations.

*** Weinberg '79
*** Gasser & Leutwyler '84 '85
*** Bijnens et al. '99 '00

(+) Ecker et al. '89
(+) Cirigliano et al. '06
(+) Pich, Rosell, SC '08.

Higgsless EW resonance models at String Phenomenology '12?

i) Many String Theory constructions contain a **Tower of V and A resonances** *

Strongly-coupled models: → ***Higgsless***

→ ***non-perturbative EWSB***

→ ***Compound resonances instead of a fundamental Higgs.***

Many possibilities in the market:

Technicolor (Original/Walking/Conformal/Holographic…), Extra Dimensions, etc.

ii) EWSB similar to Chiral Symmetry Breaking (ChSB) in QCD: $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$

→ At low energies, same EFT pion Lagrangian, Chiral Perturbation Theory (ChPT)**

E.g. in the SM,

$$\mathcal{L}(\Phi) = \frac{1}{2}\langle (D^\mu\Sigma)^\dagger D_\mu\Sigma\rangle - \frac{\lambda}{16}\left(\langle\Sigma^\dagger\Sigma\rangle - v^2\right)^2 \longrightarrow \mathcal{L}(\Phi) = \frac{v^2}{4}\langle D_\mu U^\dagger D^\mu U\rangle + \mathcal{O}(H/v)$$

* Csaki et al. '04
* Cacciapaglia et al. '04
* Hirn,Sanz '06 …

** Weinberg '79
** Gasser & Leutwyler '84 '85
** Bijnens et al. '99 '00

iii) EFT for the Goldstones (SM gauge symmetry non-linearly realized)

+

Lighest resonances from strongly-coupled Higgsless models:

✓ Similar to Resonance Chiral Theory (RChT) in QCD *

Incidentally, the most naive rescaling	f_π=0.090 GeV	→	v=0.246 TeV
from QCD to EW scale yields:	M_ρ=0.770 GeV	→	M_{V1}=2.1 TeV
	M_{a1}=1.260 GeV	→	M_{A1}=3.4 TeV

iv) Estimation of the S parameter in strongly-coupled EW models:

Equivalent to the determination of L_{10} in ChPT/RChT** → **S-parameter** (EWPO)

*Ecker et al. '89
* Cirigliano et al. '06

** Pich,, Rosell, SC '08

Oblique Electroweak Observables

✓ Universal oblique corrections via the EW boson self-energies (transverse in the Landau gauge)

$$\mathcal{L}_{\rm v.p.} \doteq -\frac{1}{2} W^3_\mu\, \Pi^{\mu\nu}_{33}(q^2) W^3_\nu - \frac{1}{2} B_\mu\, \Pi^{\mu\nu}_{00}(q^2) B_\nu - W^3_\mu\, \Pi^{\mu\nu}_{30}(q^2) B_\nu - W^+_\mu\, \Pi^{\mu\nu}_{WW}(q^2) W^-_\nu$$

✓ **S parameter** → New physics in the difference between the Z self-energies at $Q^2=M_Z^2$ and $Q^2=0$.

$$\Pi_{30}(q^2) \,=\, q^2\,\widetilde{\Pi}_{30}(q^2) \,+\, \frac{g^2\tan\theta_W}{4}\, v^2\,, \;\longrightarrow\; e_3 \,=\, \frac{g}{g'}\,\widetilde{\Pi}_{30}(0)\,, \;\longrightarrow\; S = \frac{16\pi}{g^2}\left(e_3 - e_3^{\rm SM}\right).$$

✓ In this work, dispersive representation introduced by Peskin and Takeuchi*.

$$S \;=\; \frac{16}{g^2\tan\theta_W}\int_0^\infty \frac{{\rm d}s}{s}\left({\rm Im}\widetilde{\Pi}_{30}(s) - {\rm Im}\widetilde{\Pi}_{30}^{\rm SM}(s)\right) \;=$$

$$=\; \int_0^\infty \frac{{\rm d}s}{s}\left(\frac{16}{g^2\tan\theta_W}\,{\rm Im}\widetilde{\Pi}_{30}(s) - \frac{1}{12\pi}\left[1-\left(1-\frac{M_H^2}{s}\right)^3\theta\left(s-M_H^2\right)\right]\right)$$

→ The convergence of the integral needs a vanishing ${\rm Im}\widetilde{\Pi}_{30}(s)$ at short distances.

→ S-parameter defined for an arbitrary reference value M_H

* Peskin and Takeuchi '92.

Effective Lagrangian,

$\Pi_{30}(s)$,

and the calculation of S

The Effective Lagrangian

- Effective theory with at most two derivatives + short-distance info:

SM gauge bosons + EW Goldstones + vector and axial-vector resonances

$$\mathcal{L} = \mathcal{L}_{\rm EW}^{(2)} + \mathcal{L}_{\rm GF} + \mathcal{L}_V + \mathcal{L}_A + \mathcal{L}_{VV}^{\rm kin} + \mathcal{L}_{AA}^{\rm kin} + \mathcal{L}_{VA}$$

$$\mathcal{L}_{\rm EW}^{(2)} = -\frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle + \frac{v^2}{4}\langle u_\mu u^\mu\rangle\,, \qquad \mathcal{L}_{\rm GF} = -\frac{1}{2\xi}(\partial^\mu \vec{W}_\mu)^2\,,$$

$$\mathcal{L}_V + \mathcal{L}_A = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle + \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle\,,$$

$$\mathcal{L}_{RR}^{\rm kin} = -\frac{1}{2}\langle \nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{M_R^2}{2} R_{\mu\nu}R^{\mu\nu}\rangle\,, \qquad (R = V, A)$$

$$\begin{aligned}\mathcal{L}_{VA} &= i\,\lambda_2^{VA}\langle [V^{\mu\nu}, A_{\nu\alpha}] h_\mu^\alpha\rangle + i\,\lambda_3^{VA}\langle [\nabla^\mu V_{\mu\nu}, A^{\nu\alpha}] u_\alpha\rangle \\ &\quad + i\,\lambda_4^{VA}\langle [\nabla_\alpha V_{\mu\nu}, A^{\alpha\nu}] u^\mu\rangle + i\,\lambda_5^{VA}\langle [\nabla_\alpha V_{\mu\nu}, A^{\mu\nu}] u^\alpha\rangle\,,\end{aligned}$$

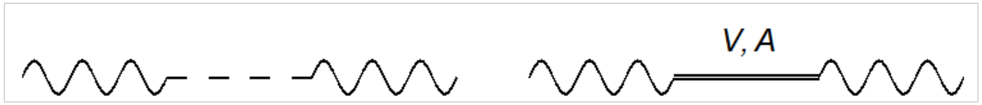
$$\begin{aligned}\kappa &= -2\lambda_2^{\rm VA} + \lambda_3^{\rm VA}\,, \\ \sigma &= 2\lambda_2^{\rm VA} - 2\lambda_3^{\rm VA} + \lambda_4^{\rm VA} + 2\lambda_5^{\rm VA}\,.\end{aligned}$$

We have 7 resonance parameters:

F_V, G_V, F_A, κ, σ, M_V and M_A

→ The high-energy constraints will be crucial

i) At leading-order (LO)*

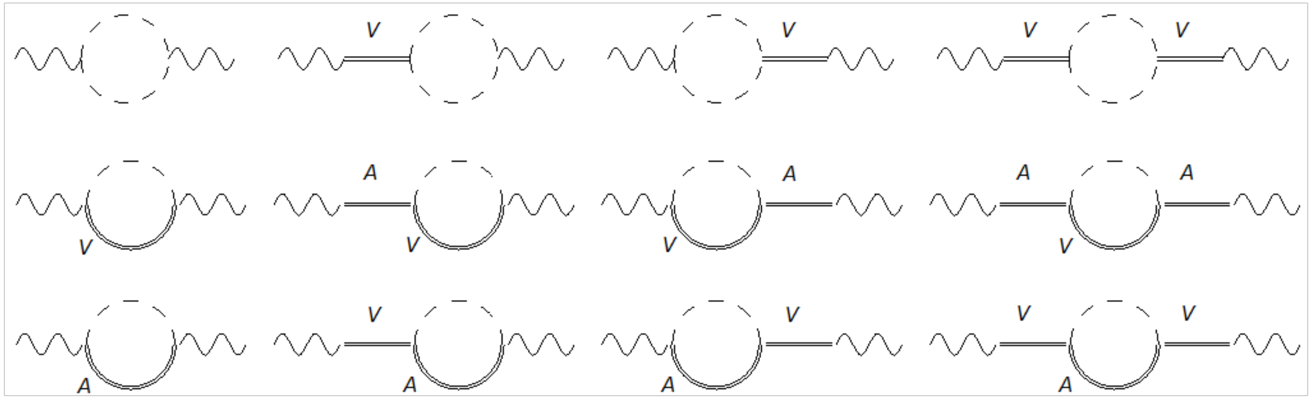


$$\Pi_{30}(s)|_{\rm LO} = \frac{g^2\tan\theta_W}{4}\, s\left(\frac{v^2}{s} + \frac{F_V^2}{M_V^2 - s} - \frac{F_A^2}{M_A^2 - s}\right)$$

$$S_{\rm LO} = 4\pi\left(\frac{F_V^2}{M_V^2} - \frac{F_A^2}{M_A^2}\right)$$

* Peskin and Takeuchi '92.

ii) At next-to-leading order (NLO)*



- ✓ Dispersive relation from tree+1-loop spectral function: $\Pi_{30}(s) = \Pi_{30}(0) + \frac{s}{\pi}\int_0^\infty \frac{\mathrm{d}t}{t\,(t-s)} \mathrm{Im}\Pi_{30}(t)$

  $\pi\pi$, $V\pi$, $A\pi$… (higher cuts dropped)

- ✓ F_R^r and M_R^r are *renormalized* couplings which define the resonance poles at the one-loop level.

$$\Pi_{30}(s)|_{\mathrm{NLO}} = \frac{g^2 \tan\theta_W}{4}\, s \left(\frac{v^2}{s} + \frac{F_V^{r\,2}}{M_V^{r\,2}-s} - \frac{F_A^{r\,2}}{M_A^{r\,2}-s} + \overline{\Pi}(s) \right)$$

⬇

$$S_{\mathrm{NLO}} = 4\pi \left(\frac{F_V^{r\,2}}{M_V^{r\,2}} - \frac{F_A^{r\,2}}{M_A^{r\,2}} \right) + \overline{S}$$

* Barbieri et al.'08
* Cata and Kamenik '10
* Orgogozo and Rynchov '11

High-energy constraints

- ✓ We have 7 resonance parameters: F_V, G_V, F_A, κ, σ, M_V and M_A.
- ✓ The number of unknown couplings can be reduced by using short-distance information.
- ✓ In contrast with the QCD case, we ignore the underlying dynamical theory.

i) Weinberg Sum Rules (WSR)*

$$\Pi_{30}(s) = \frac{g^2 \tan\theta_W}{4}\, s\, [\Pi_{VV}(s) - \Pi_{AA}(s)]$$

$$\left\{ \begin{array}{rcl} \frac{1}{\pi}\int_0^\infty dt\, [\mathrm{Im}\Pi_{VV}(t) - \mathrm{Im}\Pi_{AA}(t)] &=& v^2 \\ \frac{1}{\pi}\int_0^\infty dt\, t\, [\mathrm{Im}\Pi_{VV}(t) - \mathrm{Im}\Pi_{AA}(t)] &=& 0 \end{array} \right.$$

i.i) LO

$$\begin{array}{rcl} F_V^2 - F_A^2 &=& v^2 \\ F_V^2 M_V^2 - F_A^2 M_A^2 &=& 0 \end{array}$$

(1 / 2 constraints)

i.ii) Imaginary NLO

$$\mathrm{Im}\Pi_{V-A}(s) \sim \mathcal{O}\left(\frac{1}{s^{\Delta/2}}\right)$$

(3 / 4 constraints)

i.iii) Real NLO: fixing of $F_{V,A}^r$ or lower bounds**

$$F_V^{r\,2} - F_A^{r\,2} = v^2\,(1 + \delta^{(1)}_{\mathrm{NLO}})$$

$$F_V^{r\,2} M_V^{r\,2} - F_A^{r\,2} M_A^{r\,2} = v^2 M_V^{r\,2} \delta^{(2)}_{\mathrm{NLO}}$$

(constraints on $F_{V,A}^r$)

* Weinberg'67
* Bernard et al.'75.
** Pich, Rosell, SC '08

ii) Additional short-distance constraints

ii.i) $W_LW_L \rightarrow W_LW_L$ scattering* $\quad G_V = \frac{v}{\sqrt{3}}$

ii.ii) $\pi\pi$ Vector Form Factor** $\quad F_V G_V = v^2$

ii.iii) $\pi\gamma$ Axial Form Factor*** $\quad F_V - 2G_V = F_A\,(2\kappa + \sigma)$

3 additional constraints!

- ✓ We have up to 9 (7) constraints with 2 (1) WSR and 7 resonance parameters: we cannot consider all the constraints at the same time, some approximately

- ✓ We consider different combinations of constraints → consistency check

* Bagger et al.'94
* Barbieri et al.'08
** Ecker et al.'89
*** Pich, Rosell, SC '08

Phenomenology

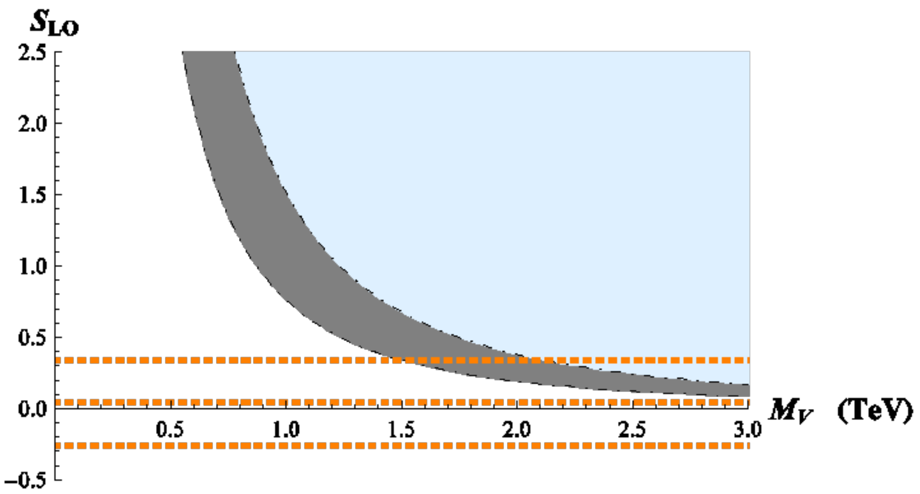
S = 0.04 ± 0.10 * (M_H=0.120 TeV)

i) LO results

i.i) 1st and 2nd WSRs

$$S_{\rm LO} \; = \; \frac{4\pi v^2}{M_V^2}\left(1+\frac{M_V^2}{M_A^2}\right)$$

$$\frac{4\pi v^2}{M_V^2} < S_{\rm LO} < \frac{8\pi v^2}{M_V^2}$$



i.ii) Only 1st WSR (lower bound for M_A>M_V)

$$S_{\rm LO} \; = \; 4\pi\left\{\frac{v^2}{M_V^2}+F_A^2\left(\frac{1}{M_V^2}-\frac{1}{M_A^2}\right)\right\}$$

$$S_{\rm LO} > \frac{4\pi v^2}{M_V^2}$$

At LO M_V > 1.5 TeV at 3σ

* Gfitter
* LEP EWWG
* Zfitter

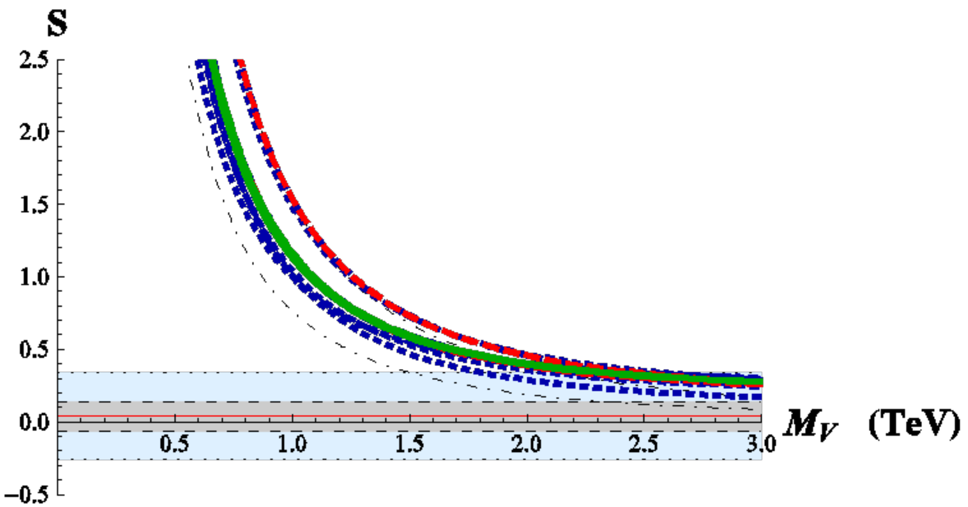
ii) NLO results: 1st and 2nd WSRs

($\pi\pi + V\pi + A\pi$ required)

- ✓ 1st and 2nd WSRs at LO and NLO:
  - → 6 constraints:
  - → M_V the only free parameter

- ✓ 8 solutions

- ✓ Only 2 approximately compatible with VFF, AFF and scattering constraints (green).



At NLO with the 1st and 2nd WSRs
$M_V > 1.8$ TeV at 3σ

- ✓ If alternatively we consider
  1st and 2nd WSR only from NLO
  + VFF and AFF constraints
  (6 constraints),

  then heavier result $M_V > 2.4$ TeV at 3σ

iii) NLO results: <u>only 1st WSR</u>

(only $\pi\pi$; lower bound for $M_A > M_V$)

- ✓ 1st WSR at NLO equivalent to the VFF constraints:

  → The only free parameter is M_V

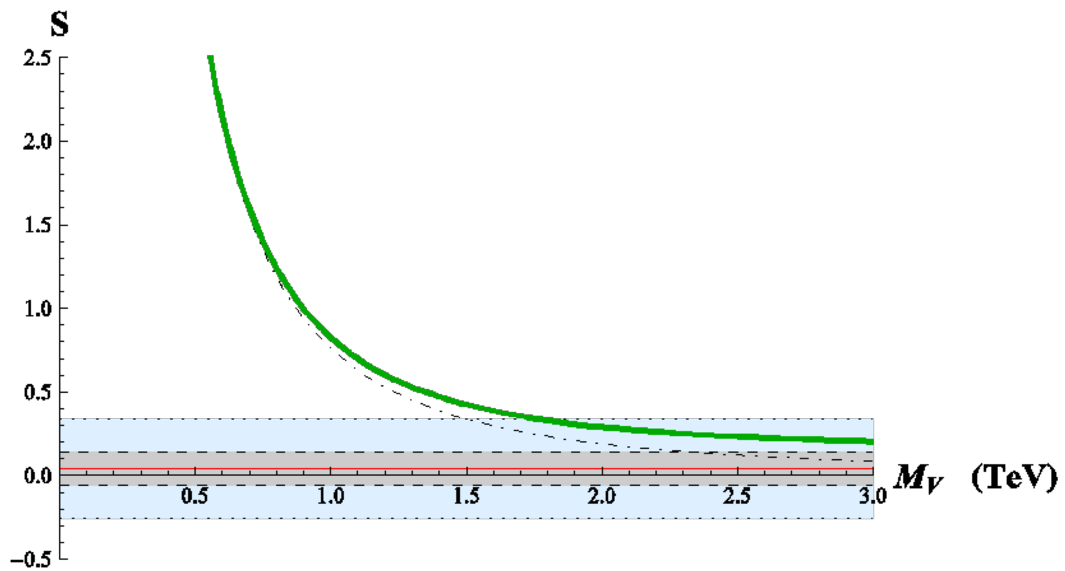
- ✓ Without the 2nd WSR, only lower bounds on S

- ✓ Simple form:

$$S_{\rm NLO} > \frac{4\pi v^2}{M_V^{r\,2}} \left(1 + \delta_{\rm NLO}^{(1)}\right) + \overline{S}$$

$$\delta_{\rm NLO}^{(1)} = \frac{M_V^2}{48\pi^2 v^2}$$

$$\bar{S} = \frac{1}{12\pi}\left[\ln\left(\frac{M_V^2}{M_H^2}\right) - \frac{17}{6}\right]$$



At NLO with only the 1st WSR
M_V > 1.8 TeV at 3σ

(only $\pi\pi$; lower bound for $M_A > M_V$)

iv) NLO results: only 1st WSR

($\pi\pi + V\pi + A\pi$; lower bound for $M_A > M_V$)

- ✓ 1st WSR at NLO + VFF and AFF constraints (5 constraints):

  the only free parameters, M_V and $r = M_A/M_V$

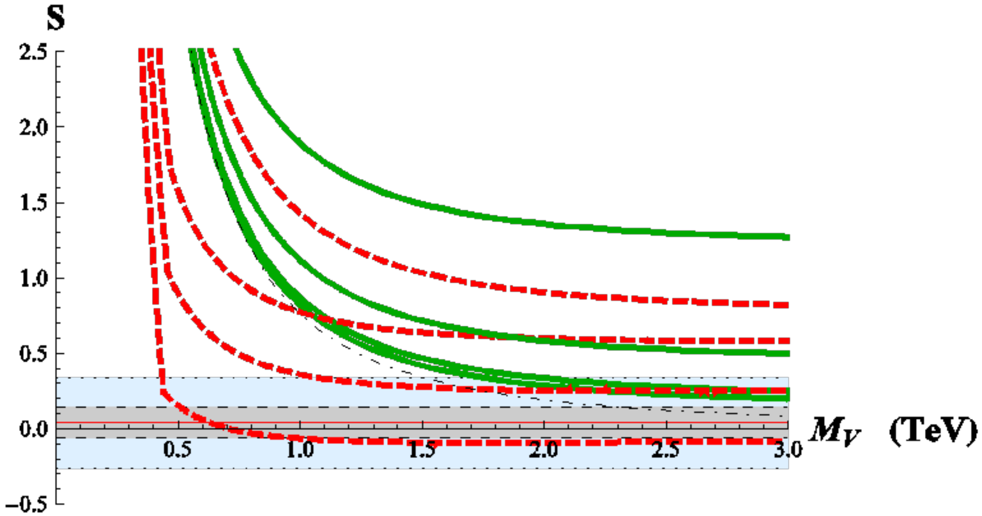
- ✓ Imposing that $F_V^2 - F_A^2 > 0$

  → only 2 solutions (**green** and **red**)

- ✓ The red solution is clearly disfavoured:

  hugeviolation of 2nd WSR at LO and NLO

  + large corrections in the 1st WSR at NLO
  + large $M_V \Leftrightarrow M_A$ splitting
  + In any case $M_A > 1.2$ TeV at 3σ

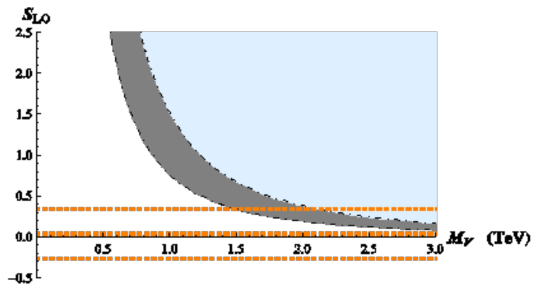


At NLO with only the 1st WSR
$M_V > 1.8$ TeV at 3σ

($\pi\pi + V\pi + A\pi$; lower bound for $M_A > M_V$)

Conclusions

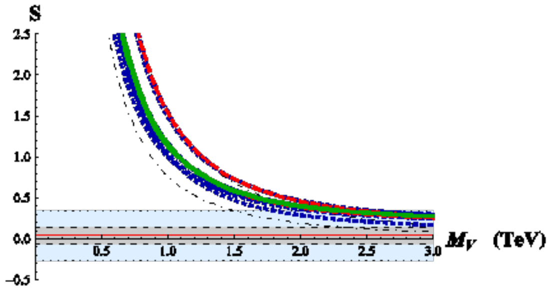
- Improvements over previous NLO calculation:
  - ✓ Dispersive calculation: no unphysical cut-offs Λ_{UV}.
  - ✓ A more general Lagrangian.
  - ✓ Short-distance information → crucial ingredient

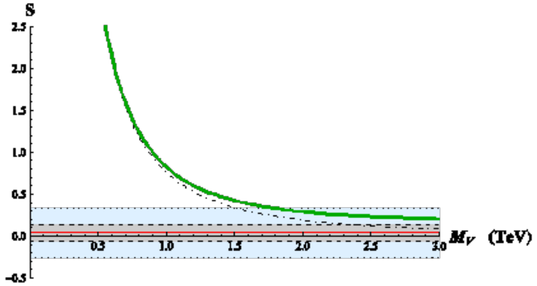


- We have considered different possibilites:
  - ✓ LO
  - ✓ NLO: with the 1st and 2nd WSR

    with only the 1st WSR *(only $\pi\pi$; $M_A > M_V$)*

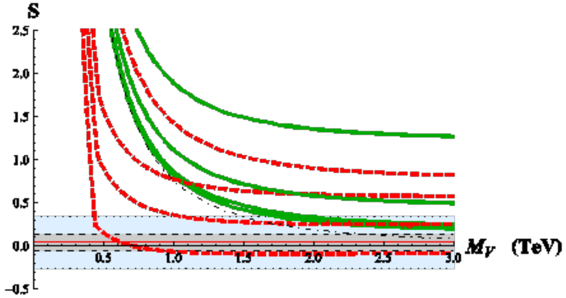
    with only the 1st WSR *($\pi\pi + V\pi + A\pi$; $M_A > M_V$)*





- **RESULTS:**
  - ✓ At LO M_V > 1.5 TeV at 3σ
  - ✓ At NLO M_V > 1.8 TeV at 3σ

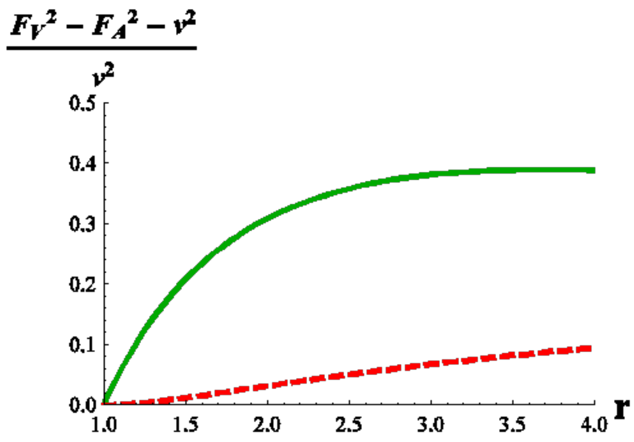
→ **High resonance mass scale, beyond 1 TeV**

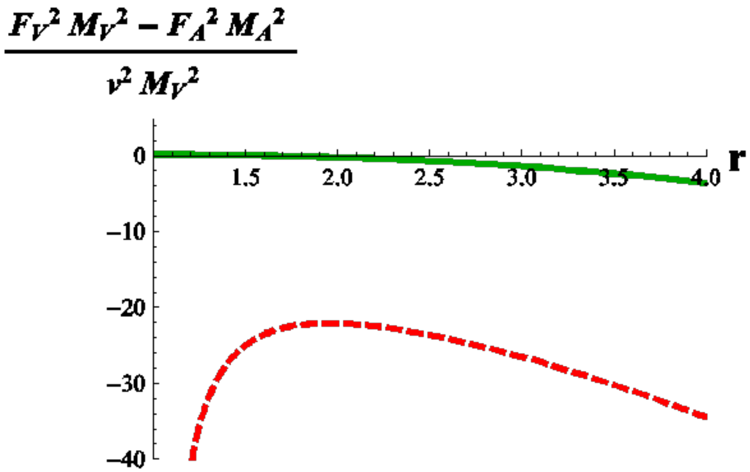


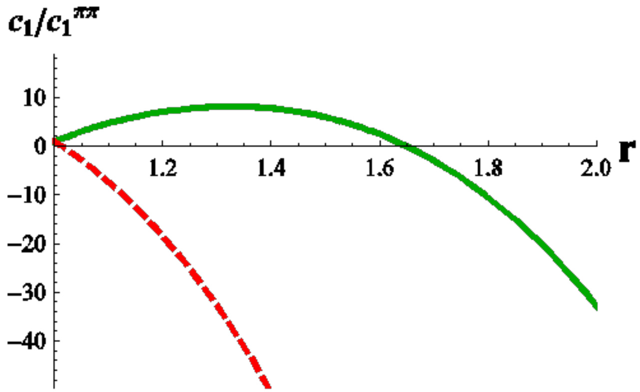
Red solution, clearly disfavoured ***(only 1st WSR → lower bound)***:
large corrections in the 1st WSR at NLO
\+ large $M_V \Leftrightarrow M_A$ splitting
\+ In any case M_A>1.2 TeV at 3σ

Future work

- ✓ Oblique T parameter.
- ✓ Dispersive representation?


$\frac{F_V^2 - F_A^2 - v^2}{v^2}$
0.5
0.4
0.3
0.2
0.1
0.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
r


$\frac{F_V^2 M_V^2 - F_A^2 M_A^2}{v^2 M_V^2}$
0
−10
−20
−30
−40
1.5
2.0
2.5
3.0
3.5
4.0
r


$c_1/c_1^{\pi\pi}$
10
0
−10
−20
−30
−40
1.2
1.4
1.6
1.8
2.0
r