Moduli Stabilisation of Heterotic String Theory with Flux

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Work done in collaboration with:
Michael Klaput, Andre Lukas, Cyril Matti,
arXiv:1005.5302, arXiv:1107.3573, upcoming

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- Can write down explicitly g_{uv}, J and Ω.
- No complex structure moduli: $H^3(X) = 0$.

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- The Kähler form, and the complex three-form may be expanded in terms of these

$$J = v^i \omega_i, \quad \Omega = \mathcal{Z}\alpha_0 - \mathcal{G}_{\mathcal{Z}}\beta^0,$$

Similar to the Calabi-Yau setting. Here $\mathcal{G}$ is a second order homogeneous polynomial in $\mathcal{Z}$ (the prepotential in the Calabi-Yau setting), and $\mathcal{G}_{\mathcal{Z}} = \partial_{\mathcal{Z}}\mathcal{G}$.

- In Michael Klaput's talk, we heard that this domain wall solution solves the supersymmetry equations and the Bianchi identity exactly, and the equations of motion are solved to $\mathcal{O}(\frac{\alpha'^2}{R^4})$. We now want to match this to a 4d supergravity.

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- We also expand the NS-NS two-form and three forms in terms of the internal geometry:

$$B = b^i \omega_i ,$$

$$H = \mathrm{d}B + \frac{\alpha'}{4}(\omega_{YM} - \omega_L) = b^i e_i \beta - \mu\alpha ,$$

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- We have a Kahler potential, and the Gukov-Vafa-Witten superpotential:

$$W \sim \int_X \Omega \wedge (H + i\mathrm{d}J) = \sqrt{8}(e_i T^i - \mu \frac{\mathcal{G}_{\mathcal{Z}}}{\mathcal{Z}}).$$

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- Note that W is independent of ϕ, and so ϕ is a flat direction. Solved by adding non-perturbative effects, e.g. a gauging condensate $W_g = ke^{-cS}$. Here k is a constant of $\mathcal{O}(1)$ (in units of α'), while c is a number depending on the condensing gauge group (its Coxeter number).

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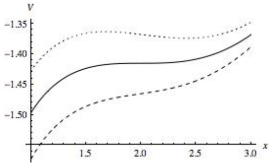

Figure: The supergravity potential as a function of $x = cs$. We have set $v^1 = v^2 = 0$, and v^3 to the value corresponding to $\partial_{v^3} V = 0$. The dashed line corresponds to $k < k_{min}$, the full line corresponds to $k = k_{min}$, and the dotted line corresponds to $k > k_{min}$.

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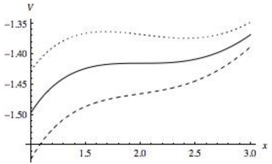

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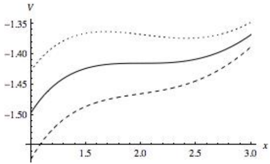

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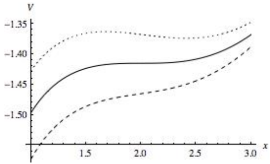

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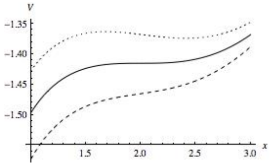

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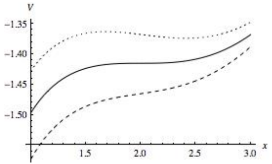

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- Worst case, we can still learn something about the physics of non-Kähler compactifications (e.g. Strominger system).

- Thank you for your attention!