Vector Bundles and Moduli Stabilization in Heterotic Theories

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A smooth $E_8 \times E_8$ heterotic model:

- The geometric ingredients include:
  - A Calabi-Yau 3-fold, X
  - A holomorphic vector bundle, V, on X (with structure group $G \subset E_8$)
- Compactifying on X leads to $\mathcal{N} = 1$ SUSY in $4D$, while V breaks $E_8 \to G \times H$, where H is the Low Energy GUT group
  - $G = SU(n)$, $n = 3, 4, 5$ leads to $H = E_6, SO(10), SU(5)$
  - Add Wilson Lines (if $\pi_1(X) \neq 0$) to break $SU(5) \to SU(3) \times SU(2) \times U(1)$
- Matter and Moduli
  - H-charged matter, $H^1(X, V)$, $H^1(X, V^\vee)$, $H^1(X, \wedge^2 V)$, ...
  - $X \Rightarrow h^{1,1}(X)$ - Kähler moduli and $h^{2,1}(X)$ - Complex structure moduli
  - $V \Rightarrow h^1(X, V \times V^\vee)$ Bundle moduli
- Numerous heterotic models known with Standard Model spectrum, but need moduli stabilization

- A standard tool for moduli stabilization in string theory is a two-step (KKLT) scenario: 1) Stabilize CS moduli w/ flux, 2) Add D-terms + non-perturbative effects for everything else...
  - This approach is hard in heterotic. Need to balance $W \sim W_{flux} + Ae^{-aS+bT}$
  - Unlike in Type II theories, have only one type of flux (NS). Very hard to find a minimum.
  - Also, such solutions generically deform the system far away from CY/Kähler geometry. Very little is known about vector bundles on such geometries. Hard to incorporate model building.
- Happily, there is an alternative: A supersymmetric vacuum to the theory must satsify the Hermitian Yang-Mills Equations

$$\delta\chi = 0 \Rightarrow \begin{cases} F_{ab} = F_{\bar{a}\bar{b}} = 0 \\ g^{a\bar{b}} F_{a\bar{b}} = 0 \end{cases}$$

- Solution depends on complex structure, Kähler and bundle moduli. Some regions of moduli space will provide a solution, some not.

Holomorphic Vector bundles

- Recall, a vector bundle is said to be holomorphic if $F_{ab} = F_{\bar{a}\bar{b}} = 0$
- Suppose we begin with a holomorphic bundle w.r.t a fixed complex structure. What happens as we vary the complex structure? Must a bundle stay holomorphic for any variation $\delta\mathfrak{z}^I v_I \in h^{2,1}(X)$? $\Rightarrow$ No.
- Infinitesimally, we must solve:

$$\delta\mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)}_{|c|\bar{b}]} + 2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]} = 0$$

- Rotation of $F^{1,1}$ into $F^{0,2}$ plus change in $F^{0,2}$ due to change in gauge connection.
- Question: For each $\delta\mathfrak{z}^I$ is there a δA which compensates?
- Answer: Not in general (but fluctuation hard to solve directly).
- The Central Idea: Use bundle holomorphy to constrain C.S. Moduli

Deformation Space – $Def(V, X)$: Simultaneous holomorphic deformations of V and X. The tangent space is $H^1(X, \mathcal{Q})$, defined via Atiyah Sequence

$$0 \to V \otimes V^\vee \to \mathcal{Q} \stackrel{\pi}{\to} TX \to 0$$

$\mathcal{Q}$ is defined by the tangent to the projectivized total space of the bundle $\mathbb{P}(V) \to X$. $H^1(X, \mathcal{Q})$ are the real moduli of a heterotic theory!

- The long exact sequence in cohomology gives us

$$0 \to H^1(V \otimes V^\vee) \to H^1(\mathcal{Q}) \stackrel{d\pi}{\to} H^1(TX) \stackrel{\alpha}{\to} H^2(V \otimes V^\vee) \to \dots$$

- We must determine: $H^1(X, \mathcal{Q}) = H^1(X, V \otimes V^*) \oplus Ker(\alpha)$ where

$$\alpha = [F^{1,1}] \in H^1(V \otimes V^\vee \otimes TX^\vee)$$

  is the Atiyah Class

- C.S. moduli allowed $\alpha(\delta_{\mathfrak{z}} v) = 0 \quad (0 \in H^2(V \times V^\vee))$. I.e. in $Ker(\alpha)$,

$$\delta_{\mathfrak{z}}{}^I v^c_{I[\bar{a}]} F^{(0)}_{|c|\bar{b}]} = -2D^{(0)}_{[\bar{a}} \delta A_{\bar{b}]}$$

4D Field Theory

- For the $4d$ Theory: We have the Gukov-Vafa-Witten superpotential $W = \int_X \Omega \wedge H$ where $H = dB - \frac{3\alpha'}{\sqrt{2}}\left(\omega^{\mathrm{3YM}} - \omega^{\mathrm{3L}}\right)$.
  $\omega^{\mathrm{3YM}} = A \wedge dA - \frac{1}{3} A \wedge A \wedge A$.
- In Minkowski vacuum, F-terms: $F_{C_i} = \frac{\partial W}{\partial C_i} = -\frac{3\alpha'}{\sqrt{2}} \int_X \Omega \wedge \frac{\partial \omega^{\mathrm{3YM}}}{\partial C_i}$
- Dimensional Reduction Anzatz: $A_\mu = A_\mu^{(0)} + \delta A_\mu + \bar{\omega}_\mu^i \delta C_i + \omega_\mu^i \delta \bar{C}_i$

$$\delta(F_{C_i}) = \int_X \epsilon^{\bar{a}\bar{c}\bar{b}} \epsilon^{abc} \Omega^{(0)}_{abc} 2\bar{\omega}^{xi}_{\bar{c}} \, \mathrm{tr}(T_x T_y) \left(\delta \mathfrak{z}^I v^c_{I[\bar{a}} F^{(0)y}_{|c|\bar{b}]} + 2 D^{(0)}_{[\bar{a}} \delta A^y_{\bar{b}]}\right)$$

- In general, $\mathfrak{z}$ is stabilized at the compactification scale. To write explicit F-terms F_{C_i}, must find a region of moduli space where $\mathfrak{z}$ is light.
- Stabilize C.S. moduli perturbatively in a SUSY Minkowski vacuum (with $W = 0$),.
- Topologically trivial flux → Still a CY manifold!.

A simple example

- Consider an $SU(2)$ bundle defined by extension:

$$0 \to \mathcal{L} \to V \to \mathcal{L}^\vee \to 0$$

Controlled by the nontrivial "gluing" of two line bundles, $Ext^1(\mathcal{L}^\vee, \mathcal{L})$. In principle, such a bundle can stabilize arbitrarily many moduli.

- For example, consider $\mathcal{L} = \mathcal{O}(-2,-2,1,1)$ on the CY, $X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array}\right]^{4,68}$
- Why this one? Here $Ext^1(\mathcal{L}^\vee, \mathcal{L}) = H^1(X, \mathcal{O}(-4,-4,2,2)) = 0$ generically. Hence cannot define the bundle for general complex structure!
- Happily, cohomology can "jump" at higher co-dimensional loci in Complex Structure moduli space.

- This is a clear example of "structural" C.S. dependence in V.
- Mathematically: Let $\mathcal{A} = \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$. Given, $p \in H^0(\mathcal{A}, \mathcal{O}(2,2,2,2))$, the Koszul sequence for X gives us

$$0 \to \mathcal{O}(-2,-2,-2,-2) \otimes {\mathcal{L}_{\mathcal{A}}}^{\otimes 2} \xrightarrow{p} {\mathcal{L}_{\mathcal{A}}}^{\otimes 2} \to {\mathcal{L}_X}^{\otimes 2} \to 0$$

$$H^1(X, \mathcal{L}^{\otimes 2}) = ker(p)\ , \quad p : H^2(\mathcal{A}, \mathcal{O}(-2,-2,-2,-2) \otimes \mathcal{L}_{\mathcal{A}}) \xrightarrow{p} H^2(\mathcal{A}, \mathcal{L}^{\otimes 2})$$

$$H^2(X, \mathcal{L}^{\otimes 2}) = coker(p)$$

Question: How can we vary $p = p_0 + \delta p$ so that $ker(p) \neq 0$?

In field theory:

- E_7 Singlets: $C_+ \in H^1(\mathcal{L}^{\otimes 2})$, $C_- \in H^1(\mathcal{L}^{\vee \otimes 2})$. ("Jump" together)
- Superpotential: $W = \lambda_{ia}(\mathfrak{z})\, C_+^i C_-^a$
  - Choose Vacuum: $< C_+ > \neq 0$ and $< C_- > = 0$.
  - Non-trivial F-term: $\frac{\partial W}{\partial C_-^a} = \lambda_{ia}(\mathfrak{z}) < C_+^i > = 0$
  - In fluctuation $\delta \left(\frac{\partial W}{\partial C_-^b} \right) = \frac{\partial \lambda_{ib}}{\partial \mathfrak{z}_\perp^I} < C_+^i > \delta \mathfrak{z}_\perp^I$

$\frac{\partial \lambda_{ia}}{\partial \mathfrak{z}^I}$ vanishes along locus with $\lambda = 0$. $\perp$ to locus, $\delta \mathfrak{z}_\perp^I$ gets a mass. Agrees with Atiyah Computation.

Three ways to determine/engineer C.S. Stabilization:

1. Atiyah Class Computation: Directly compute $Im(\alpha)$, $\alpha : H^1(TX) \to H^2(V \times V^\vee)$
2. Field Theory (solve F-terms)
3. Study "Jumping" of key bundle support.

- Intuitively, expect these three to give the same answer, but at first pass the computations look very different.
- However, can rigorously prove that these views are equivalent. No.3 is the easiest.
- Can be done for Extension, Monad, Spectral Cover, Serre and Maruyama Constructions, etc.. For each, simple "structural" failures of holomorphy can be found.

Global questions...

- Everything we have discussed so far involves fluctuation around a point in C.S. moduli space. Big limitiation: You have to know where to start.
- Hard to find isolated solutions (wanted for Pheno) this way.
- New Idea: Represent CS Loci (vacuum solutions to the F-terms) as an algebraic variety
- Mathematically, cohomology can be expressed with polynomial representatives. Changes PDEs to Algebra. From Koszul Sequence:
- $H^1(X, \mathcal{L}^{\otimes 2}) = ker(p)$ forms an ideal in the combined moduli space

$$p(\text{complex structure})s(\text{bundle moduli}) = 0 \quad \text{in cohomology}$$

- Using Groebner Basis methods (Singular + StringVacua)

Form the Ideal ⇒ Primary Decompose ⇒ Elimination to remove s

- Now we can scan over all possible starting points!

In order to describe the ideal on one slide, quotient by freely acting $\mathbb{Z}_2 \times \mathbb{Z}_4$ symmetry. Then we have $ps = 0 \in H^2(\mathcal{A}, \mathcal{L}^{\otimes 2})$, where

$$
\begin{aligned}
p = \quad & c_1 x_{1,0}x_{1,1}x_{2,0}x_{2,1}x_{3,0}x_{3,1}x_{4,0}x_{4,1} + \\
& c_9 \left(x_{1,0}^2 x_{3,0}x_{3,1}x_{4,0}x_{4,1}x_{2,0}^2 + x_{1,1}^2 x_{3,0}x_{3,1}x_{4,0}x_{4,1}x_{2,0}^2 + x_{1,0}^2 x_{2,1}^2 x_{3,0}x_{3,1}x_{4,0}x_{4,1} + x_{1,1}^2 x_{2,1}^2 x_{3,0}x_{3,1}x_{4,0}x_{4,1} \right) + \\
& c_3 \left(x_{1,1}^2 x_{2,0}x_{2,1}x_{4,0}x_{4,1}x_{3,0}^2 + x_{1,0}x_{1,1}x_{2,1}^2 x_{3,1}x_{4,0}^2 x_{3,0} + x_{1,0}x_{1,1}x_{2,0}^2 x_{3,1}x_{4,1}^2 x_{3,0} + x_{1,0}^2 x_{2,0}x_{2,1}x_{3,1}^2 x_{4,0}x_{4,1} \right) + \\
& c_4 \left(x_{1,0}x_{1,1}x_{2,0}^2 x_{4,0}x_{4,1}x_{3,0}^2 + x_{1,1}^2 x_{2,0}x_{2,1}x_{3,1}x_{4,0}^2 x_{3,0} + x_{1,0}^2 x_{2,0}x_{2,1}x_{3,1}x_{4,1}^2 x_{3,0} + x_{1,0}x_{1,1}x_{2,1}^2 x_{3,1}^2 x_{4,0}x_{4,1} \right) + \\
& c_5 \left(x_{1,0}x_{1,1}x_{2,1}^2 x_{4,0}x_{4,1}x_{3,0}^2 + x_{1,0}^2 x_{2,0}x_{2,1}x_{3,1}x_{4,0}^2 x_{3,0} + x_{1,1}^2 x_{2,0}x_{2,1}x_{3,1}x_{4,1}^2 x_{3,0} + x_{1,0}x_{1,1}x_{2,0}^2 x_{3,1}^2 x_{4,0}x_{4,1} \right) + \\
& c_6 \left(x_{1,0}^2 x_{2,0}x_{2,1}x_{4,0}x_{4,1}x_{3,0}^2 + x_{1,0}x_{1,1}x_{2,0}^2 x_{3,1}x_{4,0}^2 x_{3,0} + x_{1,0}x_{1,1}x_{2,1}^2 x_{3,1}x_{4,1}^2 x_{3,0} + x_{1,1}^2 x_{2,0}x_{2,1}x_{3,1}^2 x_{4,0}x_{4,1} \right) + \\
& c_7 \left(x_{1,1}^2 x_{2,1}^2 x_{3,0}^2 x_{4,0}^2 + x_{1,0}^2 x_{2,1}^2 x_{3,1}^2 x_{4,0}^2 + x_{1,1}^2 x_{2,0}^2 x_{3,0}^2 x_{4,1}^2 + x_{1,0}^2 x_{2,0}^2 x_{3,1}^2 x_{4,1}^2 \right) + \\
& c_8 \left(x_{1,0}^2 x_{2,1}^2 x_{3,0}^2 x_{4,0}^2 + x_{1,0}^2 x_{2,0}^2 x_{3,1}^2 x_{4,0}^2 + x_{1,1}^2 x_{2,1}^2 x_{3,0}^2 x_{4,1}^2 + x_{1,1}^2 x_{2,0}^2 x_{3,1}^2 x_{4,1}^2 \right) + \\
& c_2 \left(x_{1,0}x_{1,1}x_{2,0}x_{2,1}x_{3,0}^2 x_{4,0}^2 + x_{1,0}x_{1,1}x_{2,0}x_{2,1}x_{3,1}^2 x_{4,0}^2 + x_{1,0}x_{1,1}x_{2,0}x_{2,1}x_{3,0}^2 x_{4,1}^2 + x_{1,0}x_{1,1}x_{2,0}x_{2,1}x_{3,1}^2 x_{4,1}^2 \right) + \\
& c_{10} \left(x_{1,1}^2 x_{2,0}^2 x_{3,0}^2 x_{4,0}^2 + x_{1,1}^2 x_{2,1}^2 x_{3,1}^2 x_{4,0}^2 + x_{1,0}^2 x_{2,0}^2 x_{3,0}^2 x_{4,1}^2 + x_{1,0}^2 x_{2,1}^2 x_{3,1}^2 x_{4,1}^2 \right) + \\
& c_{11} \left(x_{1,0}^2 x_{2,0}^2 x_{3,0}^2 x_{4,0}^2 + x_{1,1}^2 x_{2,0}^2 x_{3,1}^2 x_{4,0}^2 + x_{1,0}^2 x_{2,1}^2 x_{3,0}^2 x_{4,1}^2 + x_{1,1}^2 x_{2,1}^2 x_{3,1}^2 x_{4,1}^2 \right)
\end{aligned}
$$

$$
\begin{aligned}
s = \; & \frac{b_1}{x_{1,0}^2 x_{1,1}^2 x_{2,0}^2 x_{2,1}^2} + \frac{b_2}{x_{1,0}^3 x_{1,1} x_{2,0}^3 x_{2,1}} + \frac{b_2}{x_{1,0} x_{1,1}^3 x_{2,0}^3 x_{2,1}} + \frac{b_3}{x_{1,0}^4 x_{2,0}^2 x_{2,1}^2} + \frac{b_3}{x_{1,1}^4 x_{2,0}^2 x_{2,1}^2} + \frac{b_2}{x_{1,0}^3 x_{1,1} x_{2,0} x_{2,1}^3} + \\
& \frac{b_2}{x_{1,0} x_{1,1}^3 x_{2,0} x_{2,1}^3} + \frac{b_3}{x_{1,0}^2 x_{1,1}^2 x_{2,0}^4} + \frac{b_4}{x_{1,0}^4 x_{2,0}^4} + \frac{b_4}{x_{1,1}^4 x_{2,0}^4} + \frac{b_3}{x_{1,0}^2 x_{1,1}^2 x_{2,1}^4} + \frac{b_4}{x_{1,0}^4 x_{2,1}^4} + \frac{b_4}{x_{1,1}^4 x_{2,1}^4}
\end{aligned}
$$

$x_{a,i}$-homogeneous coordinates on $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

Disconnected Loci

- Let's perform the analysis for the example... $\mathcal{L} = \mathcal{O}(-4, -4, 2, 2)$ on $X^{4,68}/\mathbb{Z}_2 \times \mathbb{Z}_4$.
- Primary Decomposing and Eliminating gives 27 distinct loci in Complex Structure Moduli Space
- Branches to the solution space range in dimension from 7 to zero (i.e. from 3 to all of the complex structure moduli fixed by V).
- Having found isolated point-like solutions, we might think we can declare victory...
- But for the given values of C.S., we still have to check transversality of the CY: $p = 0 = dp$
- While generic CICY is smooth, once we are fixed to very special points in C.S., singularities are a real concern...

Singularities in the CY

Dimension in CS	7	5	4	3	2	1	0
Dimension of Singularities in X	0	0	smooth	0	0	0	2

- For this example, of the 27 branches to the solution space, all but one force the CY to be singular. The 4-dimensional locus given by

$$c_{10} - c_{11} = 0, c_8 - c_{11} = 0, c_7 - c_{11} = 0, c_4 + c_5 = 0, c_3 + c_6 = 0, c_2 c_9 - c_1 c_{11} = 0$$

  leads to a smooth CY.

- Can we do anything with the singular solutions?
- Locally, for dim(Sing)≤ 1 we can imagine resolving (i.e. blowing-up) the singularities.
- Unfortunately, for the case at hand, all isolated point-like solutions are too badly singular. However, we can consider one of the larger loci...

Blowing up and "Splitting" CICYs

- Consider the 5-dimensional locus with pt-like singularities in CY
- Locally, we can resolve these singular pts. But have to worry about global issues: CY condition? What happens to the bundle? Symmetries?
- Happily, there are some resolutions of singular CYs that we have good control over: Conifold Transitions.
- CY defining poly takes the form $p = f_1 f_3 - f_2 f_4 = 0$. Topologically a cone over $S^3 \times S^2$. Can be resolved by introducing new $\mathbb{P}^1$ direction

$$\begin{pmatrix} f_1 & f_2 \\ f_3 & f_4 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix} = 0$$

- This "resolved" manifold can be described as a new CICY, called a "split" of the initial singular one.
- Can explicitly track divisors, extension bundles, and symmetries through this geometric transition.

- Consider the following split of the Tetraquadric:

$$X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array}\right] \Rightarrow \left[p_{(2,2,2,2)} \to f^1{}_{(2,0,2,0)} f^3{}_{(0,2,0,2)} - f^2{}_{(2,0,2,0)} f^4{}_{(0,2,0,2)} = 0\right] \Rightarrow X_{split} = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^1 & 2 & 0 \\ \mathbb{P}^1 & 0 & 2 \\ \mathbb{P}^1 & 2 & 0 \\ \mathbb{P}^1 & 0 & 2 \end{array}\right]$$

- Quotienting both sides by $\mathbb{Z}_2 \times \mathbb{Z}_4$, this "split" locus intersects the dim 5 CS locus above, leads to 8 singular pts on the CY. Resolving such pts, $X \to X_{split}$.
- To check, can repeat the analysis independently with $\mathcal{L} = \mathcal{O}(0, -2, -2, 1, 1)$ on X_{split}.
- This time, 14 branches ranging from dimension 3 to 0. The resolution X gives a 2 dimensional locus in the CS space of a smooth X_{split}.
- All CICYs connected by such transitions. Reid's Fantasy?
- Not yet dynamical transitions, but provides interesting web of "stabilizing" bundles on CYs...

Conclusions

- The presence of a holomorphic vector bundle constrains C.S. moduli ⇒ can be used as a hidden sector mechanism for moduli stabilization
- The C.S. can be stabilized at the perturbative level without moving away from a CY manifold
  - Avoids problems of naive KKLT scenarios in heterotic
  - Allows us to keep heterotic model-building toolkit
- Stabilized values fully determined for use in physical couplings
- The full moduli-dependent vacuum space can be determined using computational algebraic geometry
- Some singular solutions can be resolved ⇒ Gives insight into connections between possible base manifold CYs.

The End