Systematic Investigations of the Heterotic String Landscape

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Heterotic String Landscape Research Group

- Current Ph.D. students: Jared Greenwald, Douglas Moore, Yanbin Deng
- Former Postdoc: Dr. Tibra Ali (Perimeter)
- Ph.D. graduates: Matthew Robinson, Richard Obousy, Tim Renner
- M.S. graduate Kristen Pechan
- Summer undergraduate students/ REU: Cameron Buescher, Michael Janas, Gunner Miller, Scott Ruhnau, William Hicks, Leslie Vestal
- Dr. Markus Hunziker (Baylor Math Dept.)

Investigations of Free Fermionic [1-3] Heterotic String Landscape (FFHSL) :

I. Systematic study of the gauge group parameter space of the FFHSL with

(a) increasing order Z_N, N>2 twisted sectors, (Up to N= 32) &

(b) increasing layer L (number of independent gauge sectors)

II. Initial findings of systematic study of range of phenomenology for NAHE [4] (Nanopoulos, Antoniadis, Hagelin, Ellis)-based models, especially for gauge groups

-- $SO(6)xSO(4) \sim SU(4)xSU(2)_L xSU(2)_R$ [5]

-- Flipped SU(5)

-- (Near) MSSM's

III. Initial findings of new variation of NAHE-based models

with SO(10) x U(1) → E_6: [SO(10) x $SO(6)^3$ x E8] → [E_6 x $U(1)^5$ x SO(22)]

and mirror extension (not possible from NAHE) E_6 x $U(1)^{10}$ x E_6 mirror model

Our research has been developed or reported in:

Initial Systematic Investigations of the Landscape of Low Layer NAHE Variation Extensions. Timothy Renner, Jared Greenwald, Douglas Moore, Gerald Cleaver, arXiv:1111.1917 [hep-ph]

Initial Systematic Investigations of the Landscape of Low Layer NAHE Extensions. Timothy Renner, Jared Greenwald, Douglas Moore, Gerald Cleaver, arXiv:1111.1263 [hep-ph]

Systematic Investigations of the Free Fermionic Heterotic String Gauge Group Statistics: Layer 1 Results. D. Moore, J. Greenwald, T. Renner, M. Robinson, C. Buescher, M. Janas, G. Miller, S. Ruhnau, G. Cleaver. Mod.Phys.Lett. A26 (2011) 2411-2426; arXiv:1107.5758 [hep-ph]

Redundancies in Explicitly Constructed Ten Dimensional Heterotic String Models.Timothy Renner, Jared Greenwald, Douglas Moore, Gerald Cleaver, Int.J.Mod.Phys. A26 (2011) 4451-4473; arXiv:1107.3138 [hep-ph]

Note on a NAHE Variation. Jared Greenwald, Kristen Pechan, Tim Renner, Tibra Ali, Gerald Cleaver Nucl.Phys. B850 (2011) 445-462; arXiv:0912.5207 [hep-ph]

A Non-Standard String Embedding of E(8). Richard K. Obousy, Matthew B. Robinson, Gerald B. Cleaver, Mod.Phys.Lett. A24 (2009) 1577-1582; arXiv:0810.1038 [hep-ph]

Free Fermionic Heterotic Model Building and Root Systems. M. Robinson, G. Cleaver, Matthew B. Hunziker, Mod.Phys.Lett. A24 (2009) 2703-2715; arXiv:0809.5094 [hep-th]

Free Fermionic Heterotic Models

Real fermion phases (boundary conditions)

$\Psi \rightarrow -\exp[i\pi\alpha]\,\Psi$, for v = 0 (antiperiodic) or 1 (periodic)

Complex fermion phases

$\Psi \rightarrow -\exp[i\pi\,\alpha]\,\Psi$, for $-1 < \alpha = 2m/N \bmod 2 \leq 1$

with m = 0, …, N-1 in Z_N twisted sector (Order N)

carrying U(1) charge $Q = \frac{1}{2}\alpha + F$, with F= 0,1,-1

Each (up to 22) complex right-mover ~ U(1) gauge generator

→ maximum rank 22 gauge group

Each set (maximum of six) of two left-right real fermions reduces rank by 1

→ minimum rank for KM level-1 is 16.

Free Fermionic Heterotic Models

In D large spacetime dimensions, model defined by a set of L basis vectors v_I, I=1 to L (L referred to as the Layer), specifying different phase (i.e., charge combinations for each of the (D-2) + 3 (10-D) left-moving fermions and 32 + 2(10-2) right-moving fermions (in a real basis)

$$v_1 = [v_{1,1},\ v_{1,2}, \ldots , v_{1,20} \,\|\, \overline{v_{1,1}}, \overline{v_{1,2}}, \ldots, \overline{v_{1,22}}\ ]$$

$$v_2 = [v_{2,1},\ v_{2,2}, \ldots , v_{2,20} \,\|\, \overline{v_{2,1}}, \overline{v_{2,2}}, \ldots, \overline{v_{2,22}}\ ]$$

.

.

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$$v_L = [v_{L,1},\ v_{L,2}, \ldots , v_{L,20} \,\|\, \overline{v_{L,1}}, \overline{v_{L,2}}, \ldots, \overline{v_{L,22}}\ ]$$

Free Fermionic Heterotic Models

GSO projection k matrix (with range of components of k_{IJ} as for v_J)

$$k_{IJ} + k_{JI} = \tfrac{1}{2}\, v_I \cdot v_J \qquad \text{(mod 2)}$$

$$N_I k_{IJ} = 0 \qquad \text{(mod 2)}$$

$$k_{II} + k_{I1} = -s_i + \tfrac{1}{4}\, v_I \cdot v_I \quad \text{(mod 2)}$$

Equivalently,

$N_{IJ}\, v_I \cdot v_J = 0$ (mod 4) for $I \neq J$,

$N_{II}\, v_I \cdot v_I = 0$ (mod 4) if I odd

0 (mod 8) if I even

# of real fermions simultaneously periodic for any 3 $v_{I,}\ v_J,\ v_K$ is even

A physical state with charge vector Q_α from sector $\alpha = a_J\, v_J$ must satisfy

$v_I \cdot Q_\alpha = a_J\, k_{IJ} + v_{I,1}$ (mod 2), for each I

Some gauge group/matter questions regarding FFHSL:

- What gauge group products are possible?
- What is the probability for a given gauge group product?
- What is the probability for a given observable (hidden) sector gauge factor or factors?
- What is the probability for given class of hidden sector matter?
- Can the more probable hidden sector matter classes satisfy requirements for dark matter source?

- If a U(1) is present, what it the probability it is anomalous?
- Anomalous U(1) critically effects phenomenology of a model!!!
- Forces flat direction VEVS

Prior analysis of FFHSL gauge group statistics from order 2 models by *randomly* generating order 2 FFHS models [6]. As discussed therein statistics from randomly generating models very susceptible to floating correlations: Some models have many more equivalent representations than others. More likely to hit the former models in "random" searches.

We are therefore performing a systematic study of the gauge group parameter space of the FFHSL using an efficient linearized re-expression of constraints which allows complete search up to chosen, well defined limits (layer/order). Model production and analysis currently at a rate of 10^{12} models per year, with further rate improvements underway.

I. Free Fermionic Heterotic Gauge Models

For D= 4

Layer 1:* We generated all order 2 to order 32 gauge models (2,237387,779 total) with a single twisted gauge basis sector. A gauge sector is defined as one with all anti-periodic left-moving components. (Any dimension D, Layer L model automatically includes the all-periodic sector and the standard SUSY sector (unless the SUSY sector is generated by one of the gauge sectors).

For models with N = 4 (= N_{max}) SUSY (cp. to $k_{S,v}$ = 0), 68 distinct models, with no new models past order 22. (Each model can be classified by its embedded N_{max} gauge model.)

For models with SUSY broken from N=4 to N = 0 (cp. to $k_{S,v}$ = 1), 527 distinct models, with no new models past order 24. 50 models came with both N=4 and N= 0 versions.

Layer 2: No new gauge group models so far! All are redundant with Layer 1 models.

For D = 4, we will generate layer by layer, order by order, all possible basis sets of unique gauge sectors up to about layer 22, order 24.

The D =10 study was performed as an examination of degeneracy identification (after automatic exclusion of those from charge conjugation and basis vector permutations), especially from SO(44) rotations.

Model	ST SUSY	Model	ST SUSY
SO(32)	1	SO(32)	0
$E_8 \otimes E_8$	1	$SO(16) \otimes SO(16)$ 1 128 128 1 16 16	0
$SO(8) \otimes SO(24)$ 8 24 8 24	0	$SO(16) \otimes E_8$ 128 1 128 1	0
$SU(2) \otimes SU(2) \otimes E_7 \otimes E_7$ 1 2 1 56 1 2 56 1 2 1 1 56 2 1 56 1	0	$SU(16) \otimes U(1)$ 120 120 120 120	0

Also: the layer 1 model degeneracies suggest a relationship between gauge group, GSO projection matrix, and the basis vector. We found the ratio of $(v_1)^2/k_{11}$ is roughly constant for each gauge group, although there are few deviations, namely when $k_{11} = 0$.

Layer 1 GUT Groups Found:

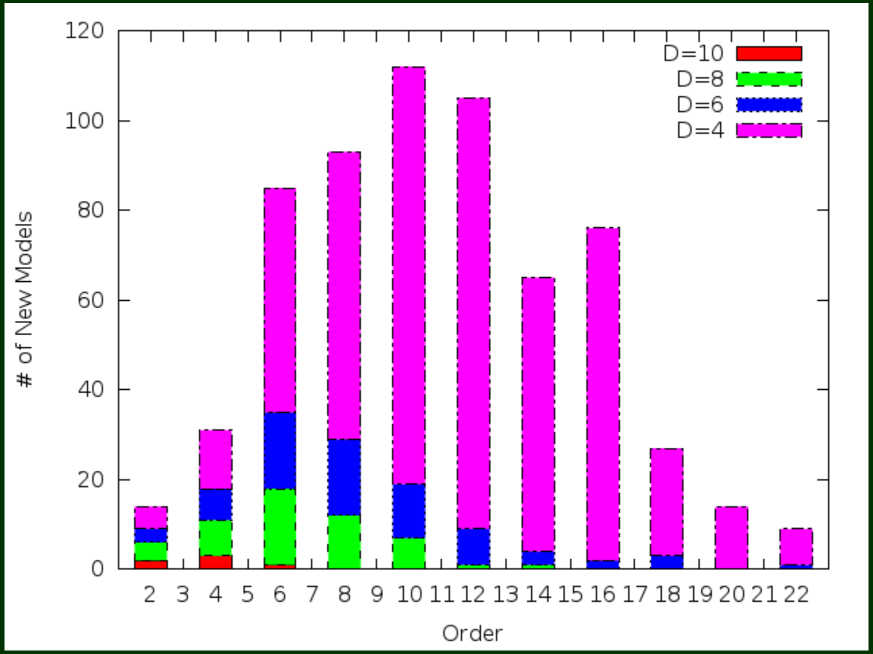
	D=4		D=5	D=6		D=7	D=8		D=9	D=10	
Group	N=0	N=4	w/gtino	N=0	N=2	w/gtino	N=0	N=1	w/grav	N=0	N=1
D5	70	9	7	10	2	2	4	0	1	0	0
A4	105	4	2	4	0	0	0	0	0	0	0
A4 U1	105	4	2	4	0	0	0	0	0	0	0
A4 U1 U1	105	4	2	4	0	0	0	0	0	0	0
A3 A1 A1	123	5	2	7	0	0	3	0	0	0	0
A2 A1 A1	74	0	0	0	0	0	0	0	0	0	0
A2 A1 U1	74	0	0	0	0	0	0	0	0	0	0
E6	48	6	4	8	2	2	4	1	1	0	0
UNIQUE	509	68	40	73	18	16	50	13	9	6	2
N=0,4											
Matches	50			18			6			1	
TOTAL	63726244		12449890	29079534		5565174	12493632		2320915	4953930	

D = 4, N = 0 model with A4 A8 E6 was found

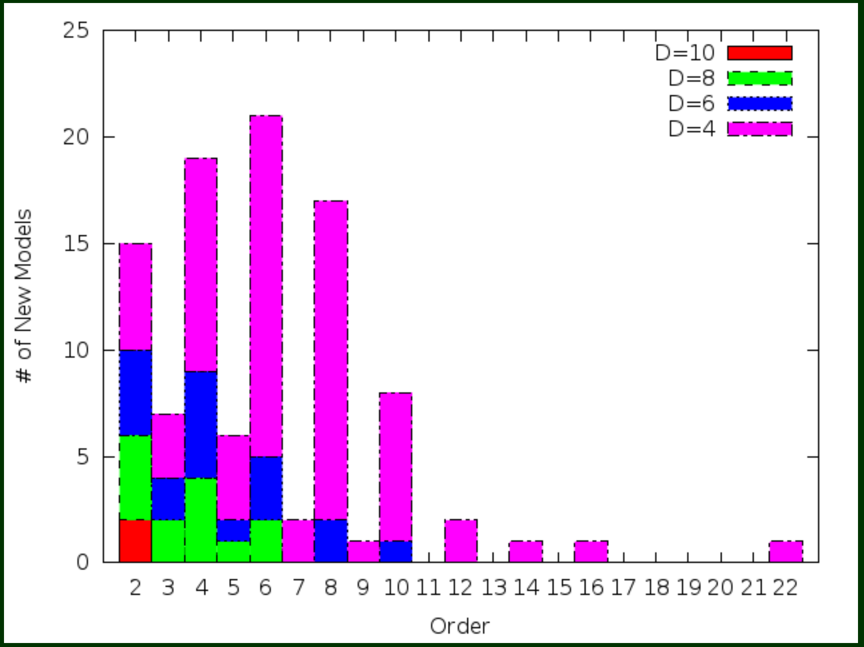
D = 4, N = 1 models with E6 and A4 was found in our NAHE extension study

Both findings are counterexamples to conjectures in Dienes et al. [6] based on order-2 random studies that E6 may not be allowed by modular invariance to occur with A4.

New Gauge Models Per Order for D = 4 to 10:

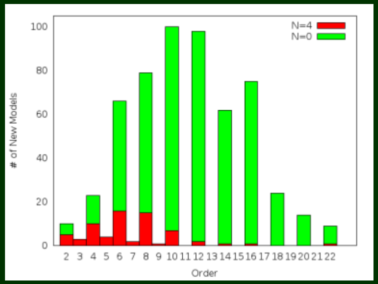


N = 0

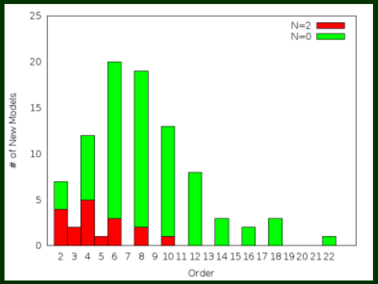


$N = N_{max}$

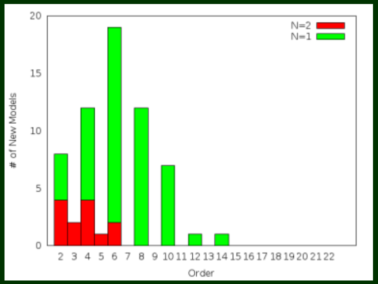
New Models Per Order for D = 4 to 10 with N = 0, N_{max}



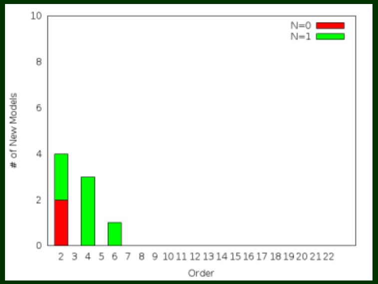
D= 4



D= 6



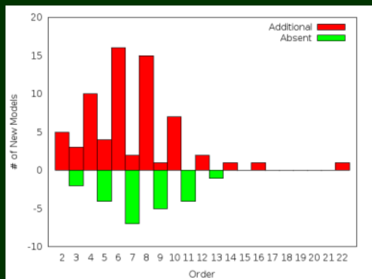
D= 8*



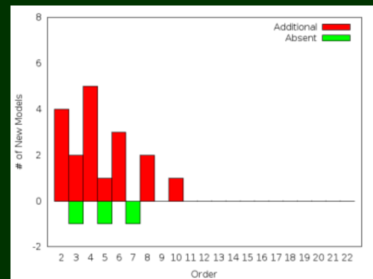
D= 10

*That FFHS yields N_{max} = 1 SUSY for D = 8 was shown in Chaudhuri et al., PRL 75 (1995) 2264.

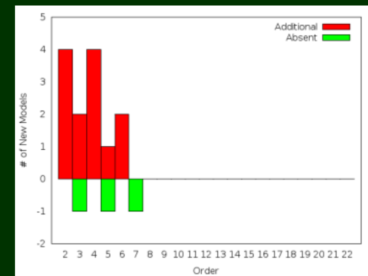
New Models and Absent Models Per Order for D = 4 to 10 with N = 0, N_{max}



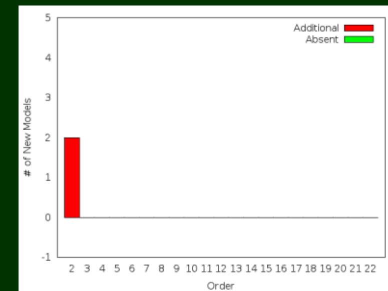
D= 4, N = 4



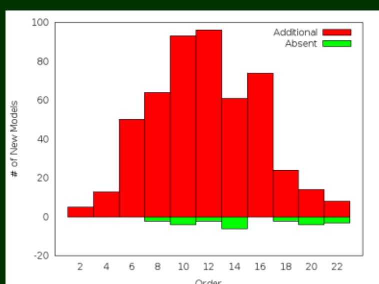
D= 6, N = 2



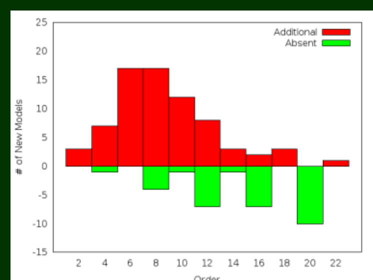
D= 8, N = 1*



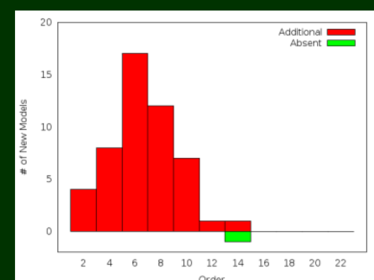
D= 10, N = 1



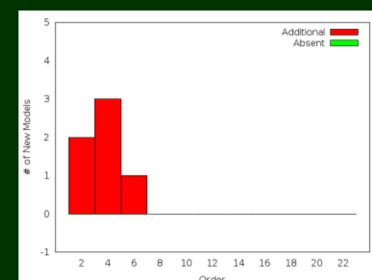
D= 4, N = 0



D= 6, N = 0



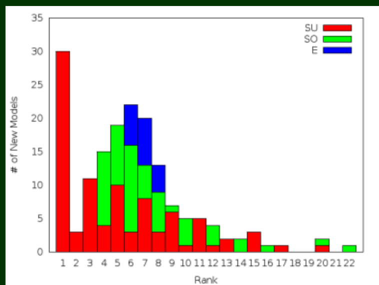
D= 8, N = 0



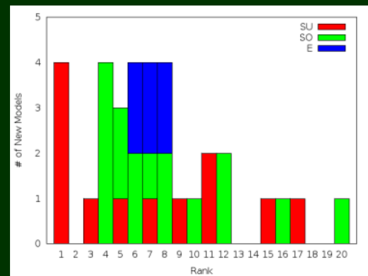
D= 10, N = 0

*That FFHS yields N_{max} = 1 SUSY for D = 8 was shown in Chaudhuri et al., PRL 75 (1995) 2264.

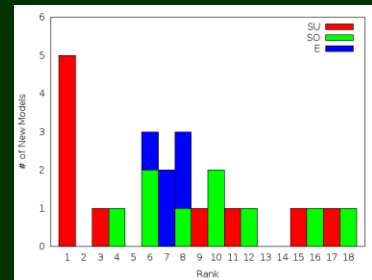
Gauge Group Factors for D = 4 to 10 with N = 0, N_{max}



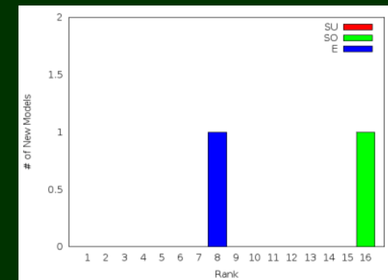
D= 4, N = 4



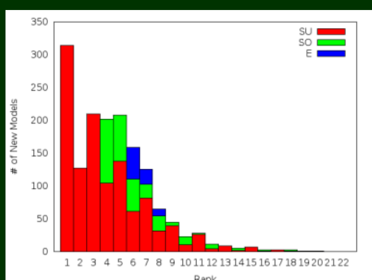
D= 6, N = 2



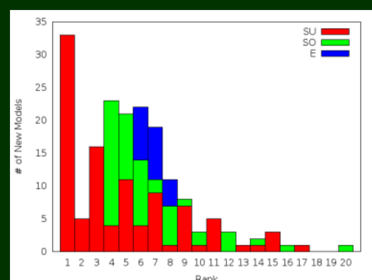
D= 8, N = 1*



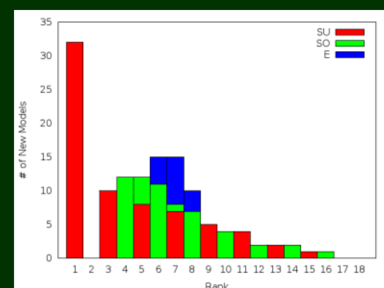
D= 10, N = 1



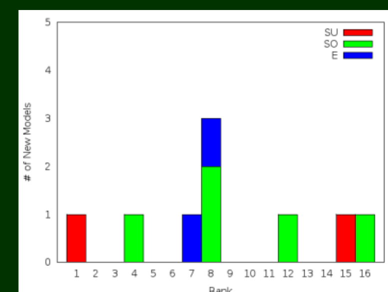
D= 4, N = 0



D= 6, N = 0



D= 8, N = 0



D= 10, N = 0

*That FFHS yields N_{max} = 1 SUSY for D = 8 was shown in Chaudhuri et al., PRL 75 (1995) 2264.

II. Systematic study of expected (most probable) phenomenology of NAHE-based Left-Right Symmetric SU(4) x $SU(2)_L$ x $SU(2)_R$, flipped SU(5), NMSSM's

Modular invariance constraints limit number of additional basis vectors that can be added to the NAHE set, typically to around only 4 or 5 more.

(i) NAHE subset

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar{\psi}^{1,\ldots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\ldots,8}$
1	1	1	1	1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0	0	0	0,...,0
$\mathbf{b}_1$	1	1	0	0	1,...,1	1	0	0	0,...,0
$\mathbf{b}_2$	1	0	1	0	1,...,1	0	1	0	0,...,0
$\mathbf{b}_3$	1	0	0	1	1,...,1	0	0	1	0,...,0

	$y^{3,\ldots,6}$	$\bar{y}^{3,\ldots,6}$	$y^{1,2},\omega^{5,6}$	$\bar{y}^{1,2},\bar{\omega}^{5,6}$	$\omega^{1,\ldots,4}$	$\bar{\omega}^{1,\ldots,4}$
1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
S	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}_1$	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
$\mathbf{b}_2$	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
$\mathbf{b}_3$	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

With the NAHE choice of generalized GSO coefficients:

$$C\binom{\mathbf{b}_i}{\mathbf{b}_j} = C\binom{\mathbf{b}_i}{\mathbf{S}} = C\binom{\mathbf{1}}{\mathbf{1}} = -1\,.$$

N = 1 SUSY

Gauge Group: SO(10) x $SO(6)^3$ x E_8^{hid}

Massless Matter: 3 x 16 = 48 MSSM generations (16 copies of each of the 3 16-plet generations)

Need additional basis vectors to break SO(10) to

(i) SO(6)xSO(4)

needs new basis vector w/ $\bar{\Psi}_{1,2,3} = 1$ & $\bar{\Psi}_{4,5} = 0$ (or vice-versa)

(ii) SU(5) needs new basis vector w/ $\bar{\Psi}_{1 \text{ to } 5} =$ rational $\neq 1$

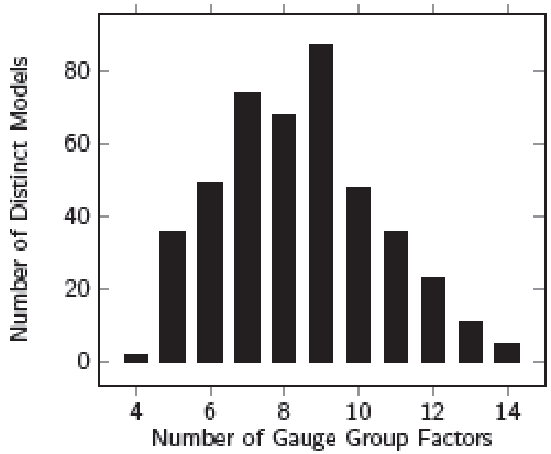
(iii) SU(3) x SU(2) needs basis vector type (i) and basis vector type (ii)

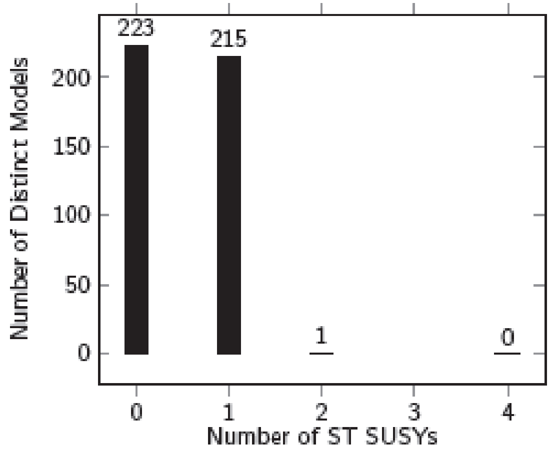
Systematically adding to NAHE set by Layer/Order process:

Extended NAHE Results, Layer 1-Order 2

- There were 439 distinct models out of 1,945,088 total models.
- In addition, 9.5% of the models without rank-cuts and 13% of the models with rank-cuts were removed as duplicates.

GUT Group	% of Models
E_6	0.2278%
$SO(10)$	36.45%
$SU(5) \otimes U(1)$	0%
PS	55.35%
LR	0%
MSSM	0%

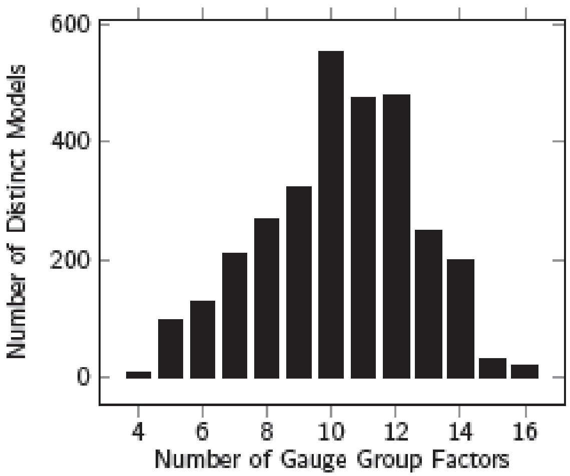


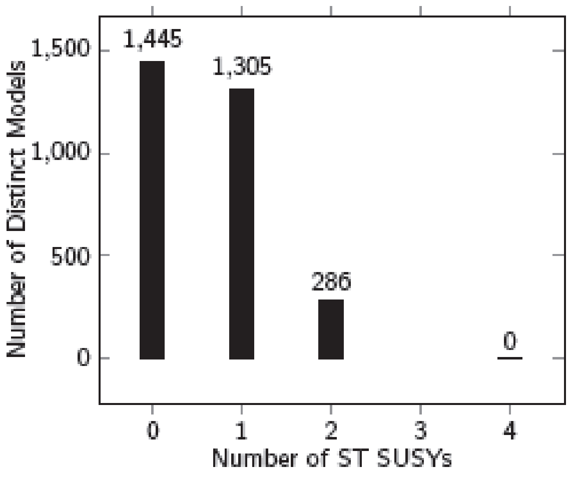


Extended NAHE Results, Layer 1-Order 3

- There were 3,036 distinct models out of 373,152.
- Based on the estimates from the O2L1 models without rank-cuts, the systematic uncertainty for these models is 10%.

GUT Group	% of Models
E_6	6.36%
$SO(10)$	21.71%
$SU(5) \otimes U(1)$	17.89%
PS	54.28%
LR	20.69%
MSSM	25.53%





Summary of These NAHE Extensions

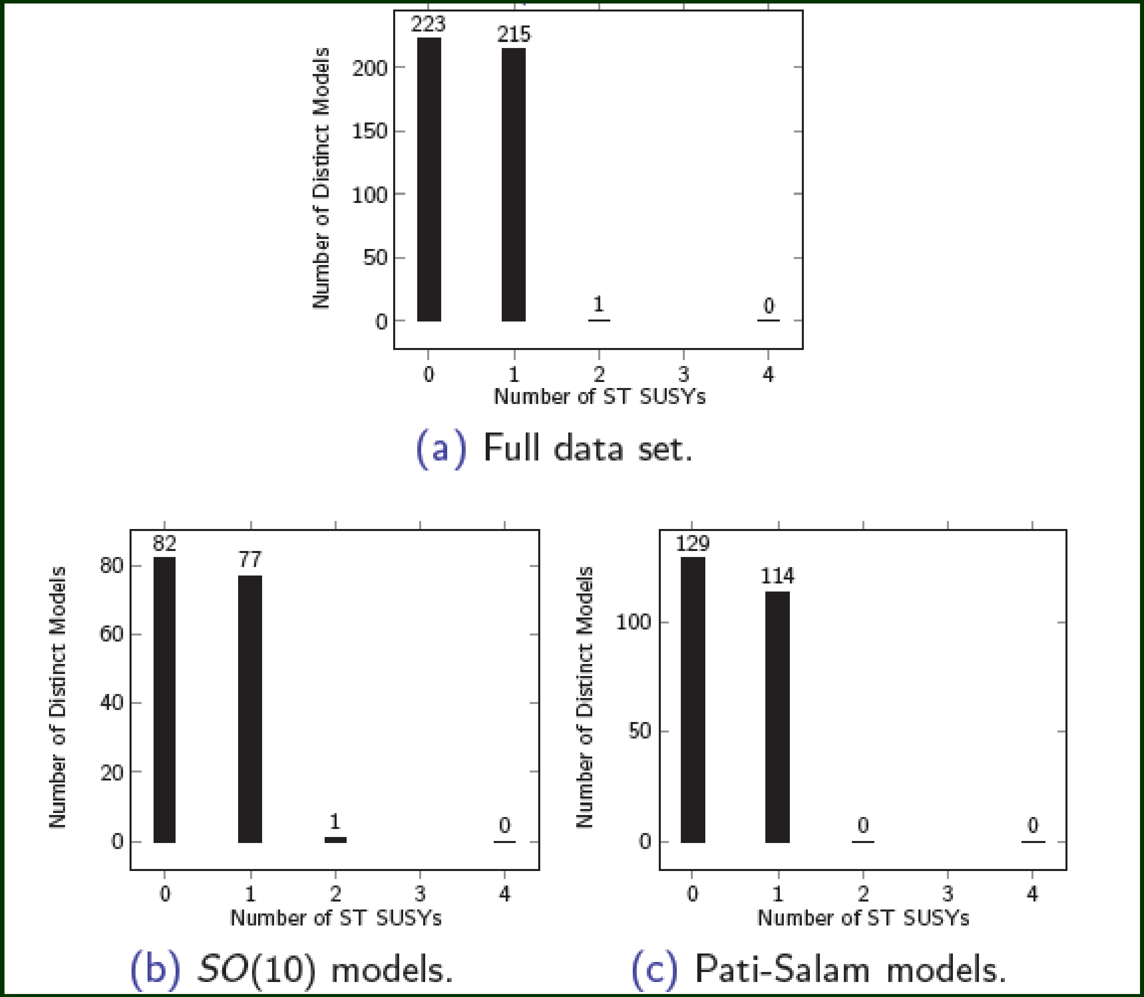
GUT	Chiral Generations?	3 Generations?
O2L1 $SO(10)$	Yes	No
O2L1 Pati-Salam	No	No
O3L1 E_6	Yes	No
O3L1 $SO(10)$	Yes	No
O3L1 $SU(5) \otimes U(1)$	Yes	Yes
O3L1 Pati-Salam	Yes	No
O3L1 L-R Symmetric	Yes	Yes
O3L1 MSSM	Yes	Yes

One Example: A Three-Generation Flipped-SU(5) Model

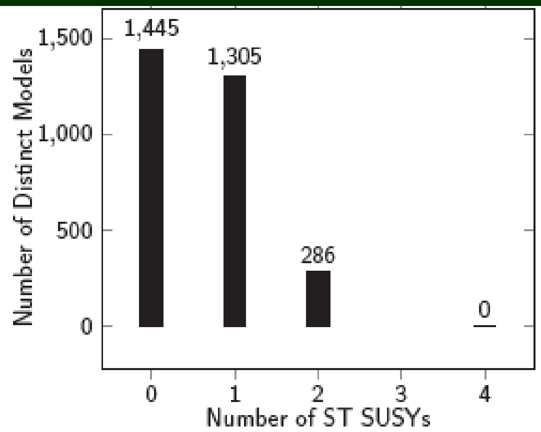
QTY	$SU(3)$	$SU(3)$	$SU(4)$	$SU(5)$	$SU(7)$
2	1	$\overline{3}$	1	5	1
1	1	1	4	5	1
2	1	1	1	$\overline{5}$	1
3	1	1	1	5	1
1	1	1	$\overline{4}$	5	1
2	1	3	1	$\overline{5}$	1
1	$\overline{3}$	1	1	$\overline{10}$	1
2	$\overline{3}$	1	1	$\overline{5}$	1

- This model also has five $U(1)$ gauge groups, and N=1 ST SUSY.
- There are 14 extra 5's and 8 extra $\overline{5}$'s.

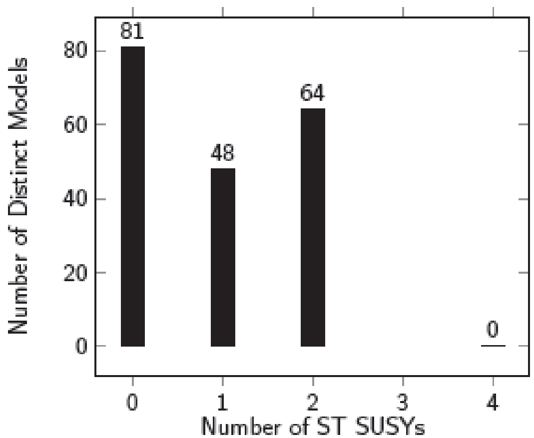
Spacetime SUSY Distributions
Layer 1, Order 2 Extensions



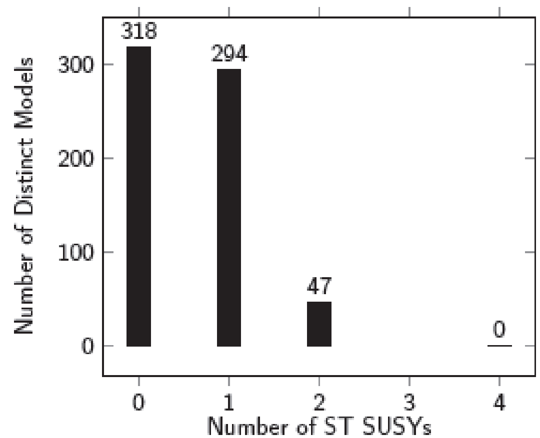
Spacetime SUSY Distributions
Layer 1, Order 3 Extensions



(d) Full data set.



(e) E_6 models.



(f) $SO(10)$ models.

III. Variation of NAHE-based models: work inspired by

R Donagi & K Wengland [7]
-- advanced correspondence between orbifolds and free fermion models.
-- showed different possible T^6/Z_2xZ_2 symmetries.
One provides geometric interp. for FFHS models equiv to orbifolds
Another does not.

B. Dundee, J. Perkins, G.C. [8]
-- showed that NAHE-based mirror models have an inherent difficulty:
Extra $U(1)_{i=1 \text{ to } 3}$ generational charges spread out too much to not interfere (overlap) with cp. Mirror Generation Charges.
GSO projections forced to act differently on observable and hidden gauge groups, inherently breaking mirror gauge group symmetry.

$$[SO(6)xSO(4)]^o \text{ x } SO(6)xSO(4)]^h \rightarrow [SU(4)_C xSU(2)_L]^o \text{ x } [SO(10) \text{ x } SU(2)^R]^h$$

NAHE Variation

NAHE-Based model use all 12 fermions of compact dirs. for U(1)'s
Doesn't allow for mirror models

	X_1		X_2		X_3		X_4		X_5		X_6		
	y_1	w_1	y_2	w_2	y_3	w_3	y_4	w_4	y_5	w_5	y_6	w_6	
Gen 1:					1		1		1		1		Z_2^A
Gen 2:	1		1							1		1	Z_2^B
Gen 3:		1		1		1		1					$1 + Z_2^A x Z_2^B$

NAHE Variation allowing for mirror models (2^{nd} & 3^{rd} Gen. Sectors Modified):

	y_1	w_1	y_2	w_2	y_3	w_3	y_4	w_4	y_5	w_5	y_6	w_6	
Gen 1:					1		1		1		1		$Z_2^{A'}$
Gen 2:	1		1						1		1		$Z_2^{B'}$
Gen 3:	1		1		1		1						$Z_2^{A'} x Z_2^{B'}$

Mirror generations have corresponding w_i charges

Rotation of charges into NAHE variation enhances

$SO(10) \times U(1) \rightarrow E_6$

Giving full gauge group

$SO(10) \times SO(6)^3 \times E_8 \rightarrow E_6 \times U(1)^5 \times SO(22)$

16 (1 generation) + 10 (higgs/higgs-bar) + singlet → 27

3*8 copies of (16 + 16-bar) in NAHE → 3*4(27 + 27-bar)

Addition of mirror matter sectors transforms this into

$E_6 \times U(1)^5 \times SO(22) \rightarrow E_6 \times U(1)^{10} \times E_6$

Hidden sector communicates to Observable sector via shadow $U(1)^{10}$

Offers new class of FFHS models, including true mirror-models.

NAHE Variation

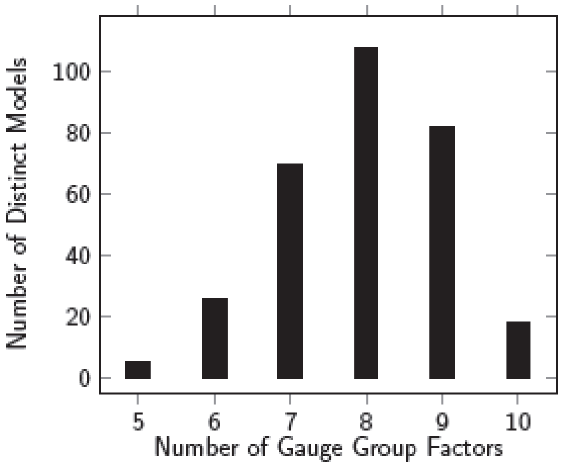
QTY	$SO(22)$	E_6
30	22	1
15	1	27
90	1	1
15	1	$\overline{27}$

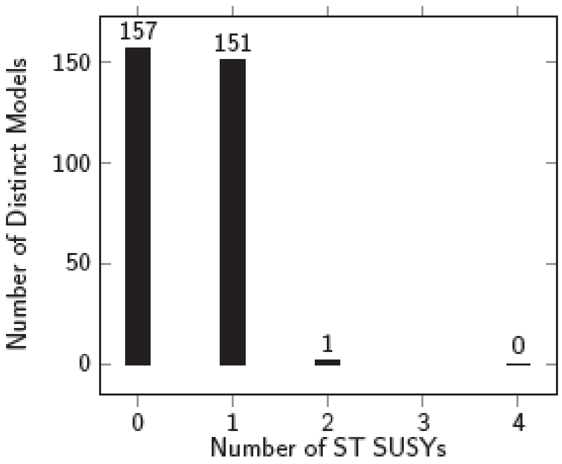
- The NAHE variation also has five $U(1)$'s and $N = 1$ ST SUSY.
- It also allows for models with "mirroring."

Layer 1, Order 2 Extensions

- There were 309 unique models out of 1,315,328 total consistent models.
- 2% of the models without rank-cuts were duplicates, while none of the models with rank-cuts were duplicates.

GUT Group	% of Models
E_6	32.69%
$SO(10)$	40.45%
$SU(5) \otimes U(1)$	0%
PS	0%
LR	0%
MSSM	0%

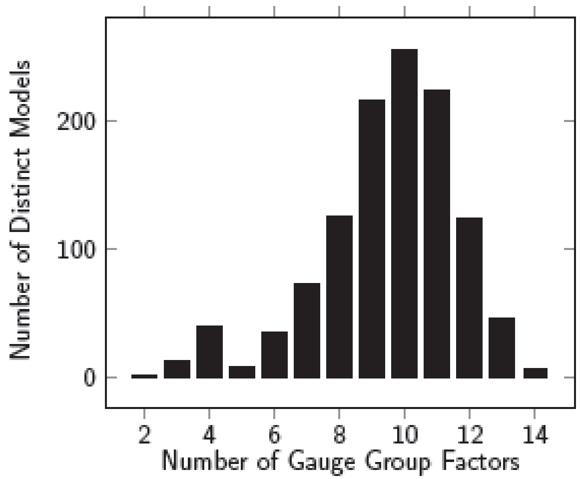


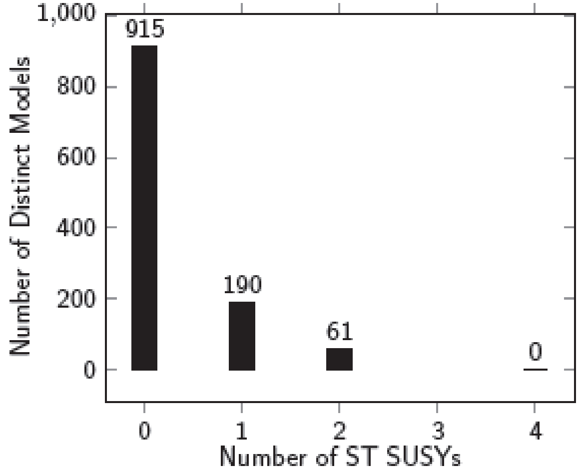


Layer 1, Order 3 Extensions

- There were 1,166 distinct models out of 442,272.
- The estimated systematic uncertainty for models in this data set is 2%.

GUT Group	% of Models
E_6	5.832%
$SO(10)$	23.24%
$SU(5) \otimes U(1)$	14.15%
PS	10.72%
LR	5.232%
MSSM	5.403%





Summary of These NAHE-Variation Extensions

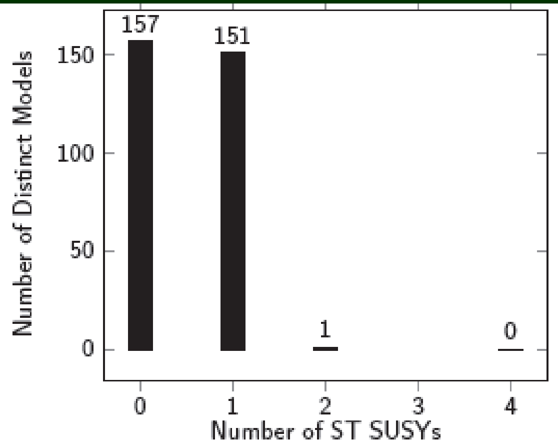
GUT	Chiral Generations?	3 Generations?
O2L1 E_6	Yes	No
O2L1 $SO(10)$	Yes	No
O3L1 E_6	No	No
O3L1 $SO(10)$	No	No
O3L1 $SU(5) \otimes U(1)$	No	No
O3L1 Pati-Salam	No	No
O3L1 L-R Symmetric	No	No
O3L1 MSSM	No	No

Example Mirror Model

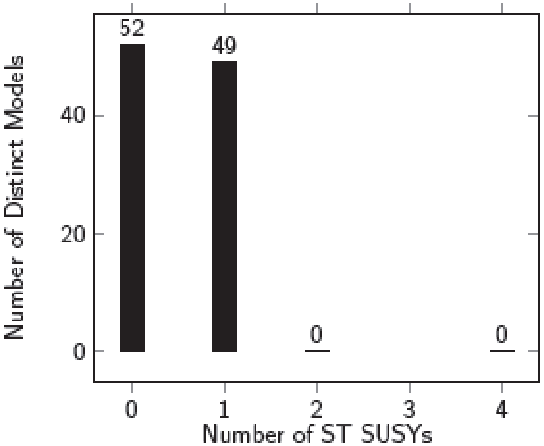
QTY	$SO(14)$	E_6	E_6
12	14	1	1
6	1	27	1
6	1	1	27
6	1	1	$\overline{27}$
6	1	$\overline{27}$	1

- This model has three $U(1)$ gauge groups and N=2 ST SUSY.
- There were no models which had completely mirrored matter representations.

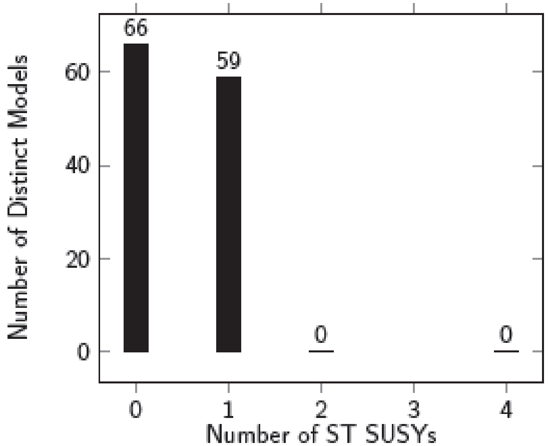
Spacetime SUSY Distributions
Layer 1, Order 2 Extensions



(g) Full data set.

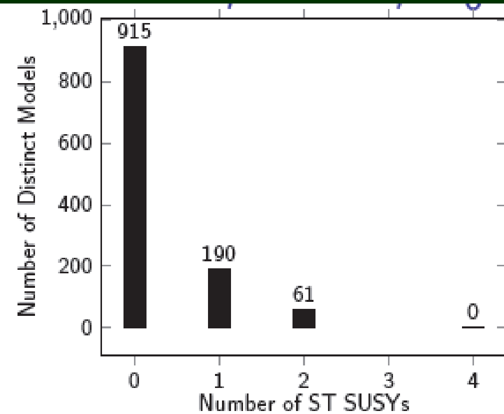


(h) E_6 models.

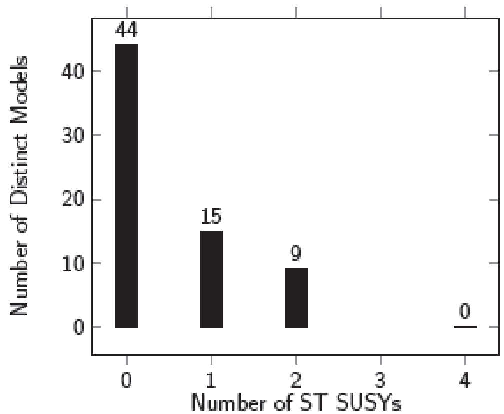


(i) $SO(10)$ models.

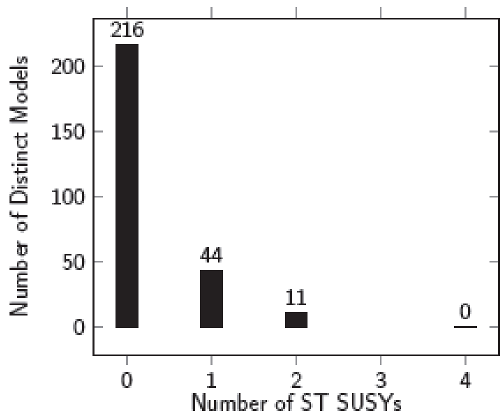
Spacetime SUSY Distributions
Layer 1, Order 3 Extensions



(j) Full data set.

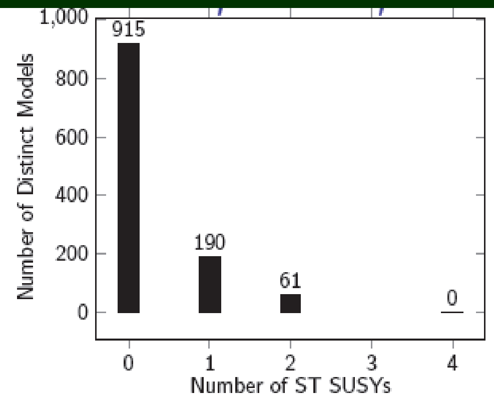


(k) E_6 models.

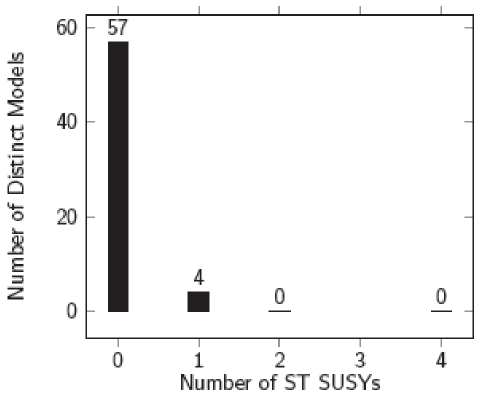


(l) $SO(10)$ models.

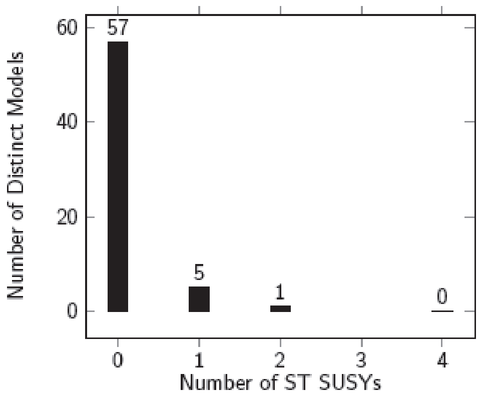
Spacetime SUSY Distributions
Layer 1, Order 3 Extensions



(m) Full data set.



(n) Left-Right Symmetric models.



(o) MSSM models.

Summary

➢The goals of the U.S. String Vacuum Project (SVP) are: (i) the enumeration and classification of string vacua; (ii) the development of a detailed understanding of those string vacua with realistic low-energy phenomenologies; (iii) the development of more explicit connections between string vacua and LHC data and phenomenology generally; and (iv) statistical studies across the entire Landscape of string vacua. Accordingly, the SVP values both direct enumeration and statistical studies, through which experience is gained in the construction and analysis of phenomenologically viable string models and their connections to the Standard Model and ultimately to experiment. We believe that the research presented herein contributes to SVP goals (i), (ii), and (iv).

➢Much phenomenology remains to be discovered in FFHSL, especially with higher order $Z_{N>2}$ twisted basis sectors.

Our current research focuses on probabilities of particular phenomenological features being found within the FFHSL

Research group is performing systematic, efficient studies of

-- gauge group stats of high layer/order models for ~ 10^{12} models/yr

-- NAHE class extensions

-- NAHE variation class extensions that especially include mirror models

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